Quantum error correction for the toric code using deep reinforcement learning

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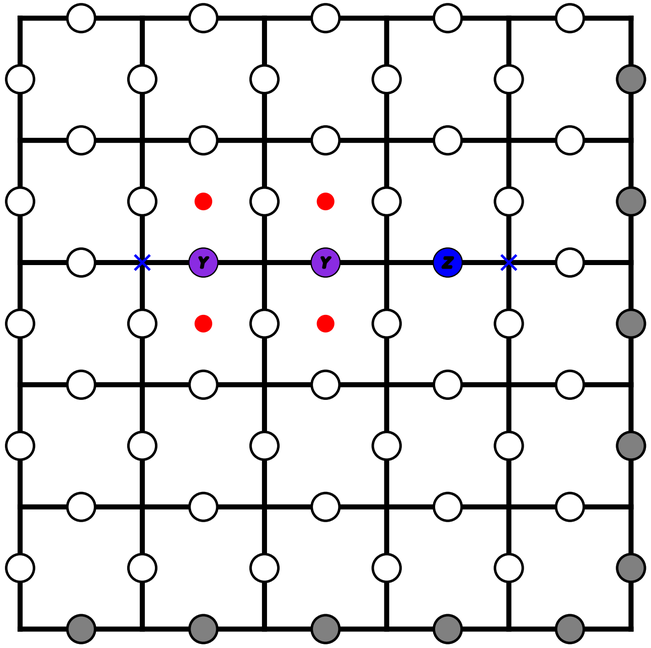
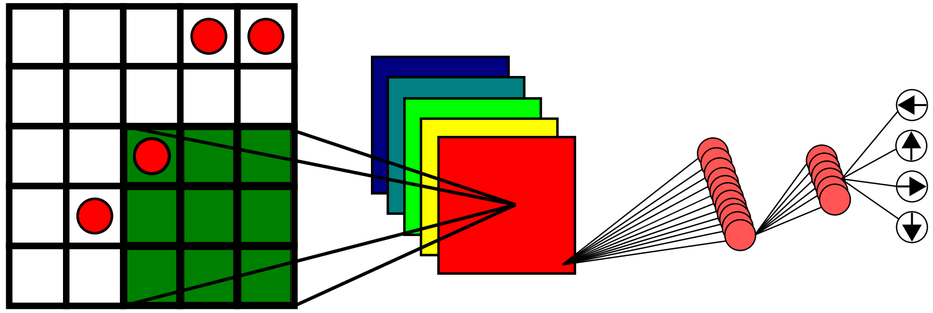
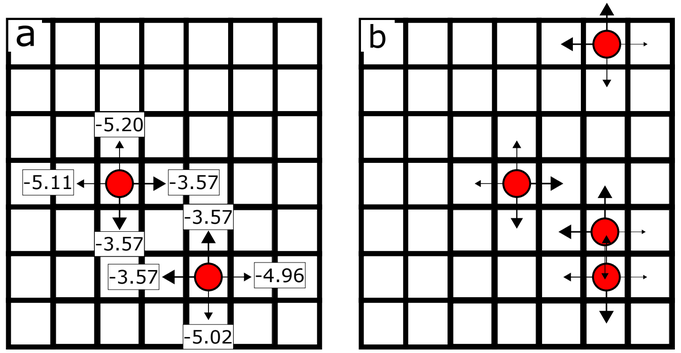






Philip Andreasson, Joel Johansson, Simon Liljestrand, MG, arXiv:1811.12338

Mattias Eliasson, David Fitzek, MG, in progress

Bottom line

We do the “simplest” error correction problem for a topological code

- Periodic boundary conditions
- No measurement noise/perfect syndrome
- only bit flip noise (initially, can also do depolarizing)

Still challenging for reinforcement learning: deep Q-networks needed

Allows for easy benchmark

The toric code

Fault-tolerant quantum computation by anyons

A.Yu. Kitaev*

L.D. Landau Institute for Theoretical Physics, 117940, Kosygina St. 2, Germany

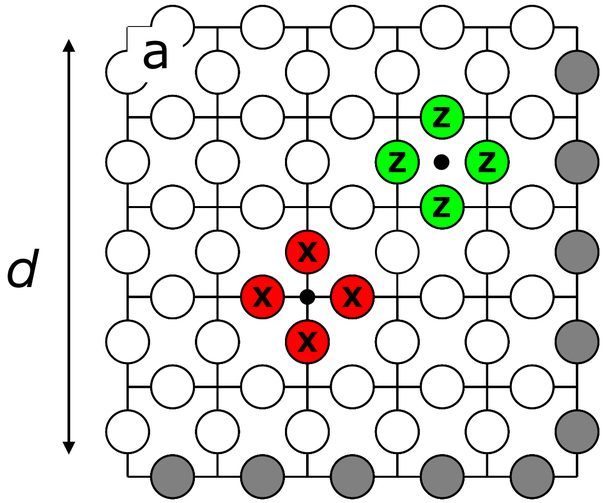
Received 20 May 2002

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 43, NUMBER 9 SEPTEMBER 2002

Topological quantum memory[a)]

Eric Dennis[b)]
Princeton University, Princeton, New Jersey 08544

Alexei Kitaev,[c)] Andrew Landahl,[d)] and John Preskill[e)]
Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125



$$H = -\sum_{\alpha} \hat{P}_\alpha - \sum_{\nu} \hat{V}_\nu$$

$$\hat{P}_\alpha = \prod_{i \in \alpha} \sigma_i^z$$

$$\hat{V}_\nu = \prod_{i \in \nu} \sigma_i^x$$

Plaquette and Vertex stabilizers (parity checks)

$2d^2$ physical qubits, $2d^2$-2 independent stabilizers

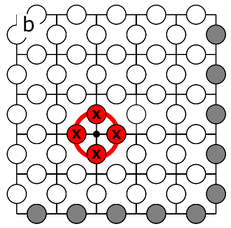
Ground state

consider:

$$| \uparrow\uparrow\uparrow \cdots \rangle$$

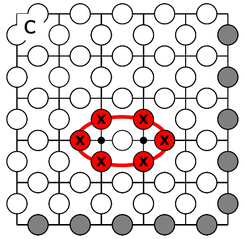
plaquette operator ground state:

act with vertex op:



still a plaquette ground state

act with two vertex op:



still a plaquette ground state

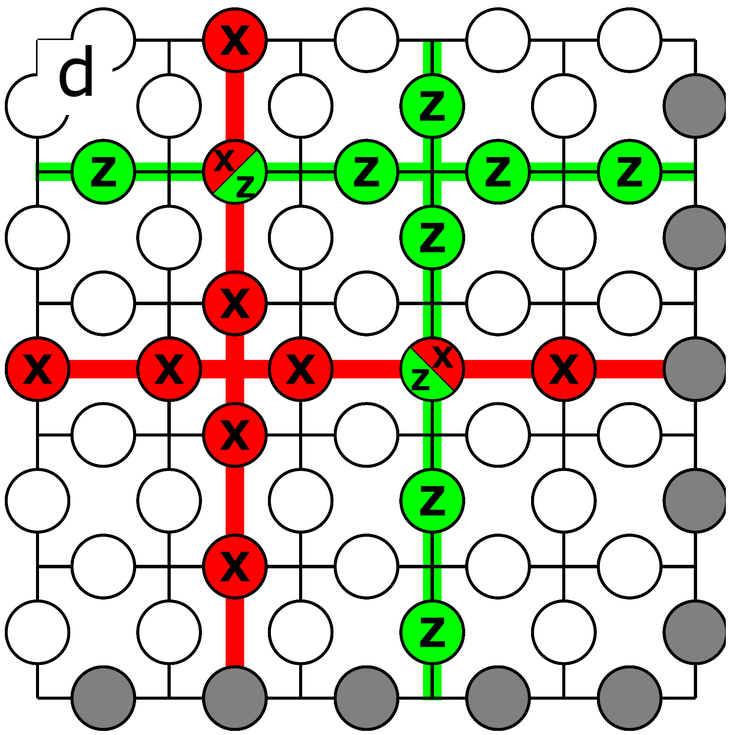
GS is symmetric superposition of all *trivial* loops:

$$|\mathrm{GS}_0\rangle = \sum_{i \in \text{all trivial loops}} \mathrm{loop}_i | \uparrow\uparrow\uparrow \cdots \rangle$$

highly entangled

Ground state degeneracy

Non-trivial loops (encircling torus) X_1, X_2
are not products of vertex operators.



Four ground states/The logical qubit

$$\{|\mathrm{GS}_0\rangle, X_1|\mathrm{GS}_0\rangle, X_2|\mathrm{GS}_0\rangle, X_2X_1|\mathrm{GS}_0\rangle\}$$

Distinguished by ±1 eigenvalues of Z_1 and Z_2.

Corresponding to $2(d^2-1)$ independent stabilizers on $2d^2$ physical qubits.

Topologically protected qubit

Non-trivial loops=Logical bit-flip operators
Requires at least *d* physical bit-flip errors
code distance *d*

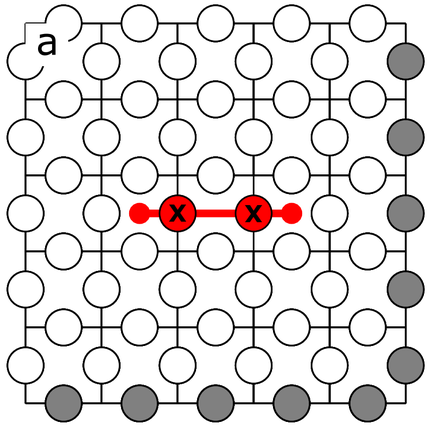
Error correction

consider bit-flip errors

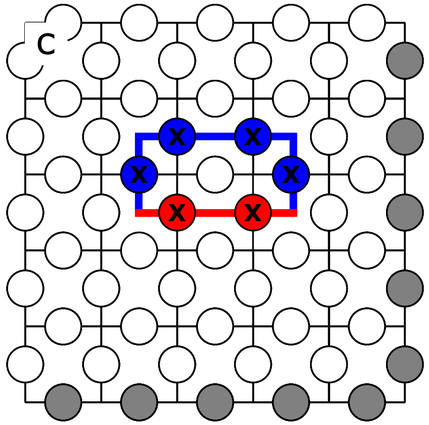
The *syndrome* (defects/bad plaquettes), is quantum non-demolition measurement

Ex.

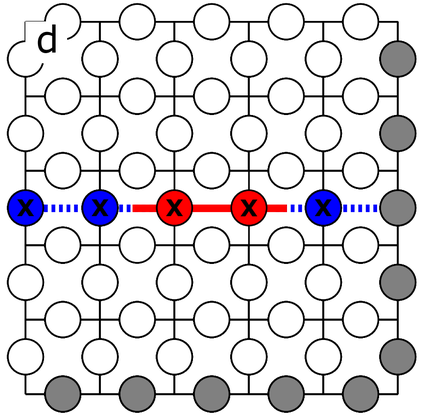
two neighbouring bit flip errors,
two defects



proper error correction
trivial loop



failed error correction
non-trivial loop



Standard algorithm to suggest error correcting strings:
Minimum Weight Perfect Matching (MWPM)/Blossom

J. Edmunds, 1965

Find shortest total correction string. (Which is the most likely)

Error models

Depolarizing

- **(1-p) no error**
- **p/3 X**
- **p/3 Y=XZ**
- **p/3 Z**

Uncorrelated

- **$(1-p)^2$ no error**
- **p(1-p) X**
- **p^2 Y=XZ**
- **p(1-p) Z**

Bit- and phase-flip errors
(i.e. plaquette and vertex errors).
are independent. Corrected separately.

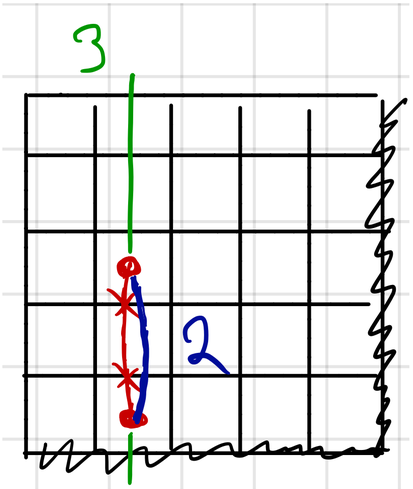
MWPM is (near) optimal

Minimum Weight Perfect Matching Low-p fail rate for bit-flip errors

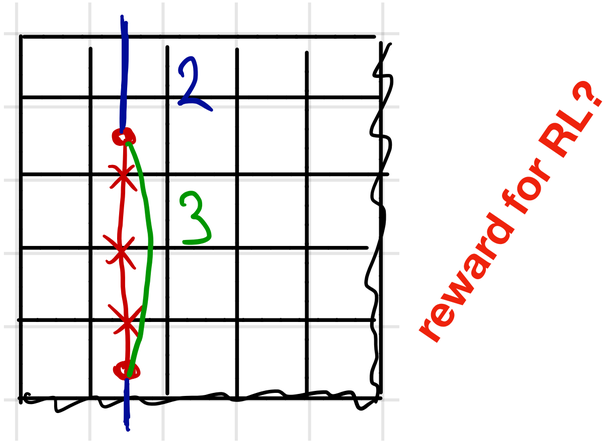
For p -> 0 we only need to consider error chains with minimal number of errors that can give failed error correction

Consider *d*=5:

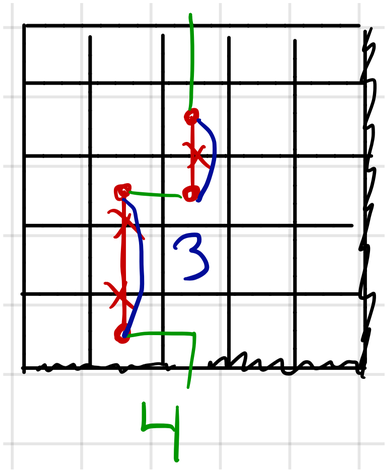
Two errors is always corrected successfully



Three errors in a row always gives failed error correction



Three errors not in a row always gives successful correction



MWPM asymptotic (lowest order in *p*) fail rate is:

$$p_L = 2d\binom{d}{\lceil d/2 \rceil} p^{\lceil d/2 \rceil}$$

Mats Granath, MLQT, Erlangen 2019

Deep reinforcement learning/Deep Q-learning

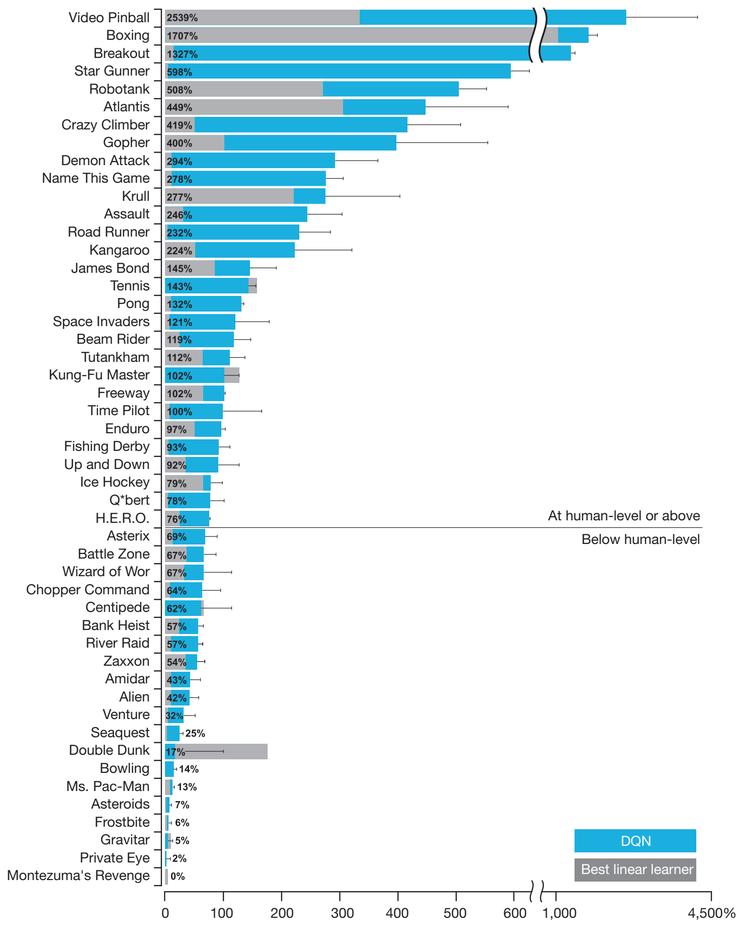
LETTER

2015

doi:10.1038/nature14236

Human-level control through deep reinforcement learning

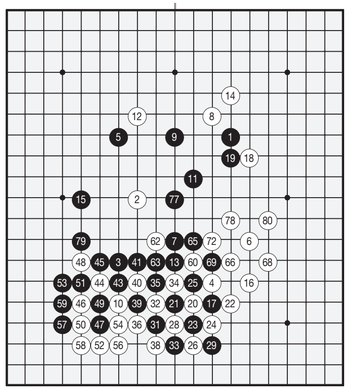
Volodymyr Mnih[1]*, Koray Kavukcuoglu[1]*, David Silver[1]*, Andrei A. Rusu[1], Joel Veness[1], Marc G. Bellemare[1], Alex Graves[1], Martin Riedmiller[1], Andreas K. Fidjeland[1], Georg Ostrovski[1], Stig Petersen[1], Charles Beattie[1], Amir Sadik[1], Ioannis Antonoglou[1], Helen King[1], Dharshan Kumaran[1], Daan Wierstra[1], Shane Legg[1] & Demis Hassabis[1]



2017

Mastering the game of Go without human knowledge

David Silver[1]*, Julian Schrittwieser[1]*, Karen Simonyan[1]*, Ioannis Antonoglou[1], Aja Huang[1], Arthur Guez[1], Thomas Hubert[1], Lucas Baker[1], Matthew Lai[1], Adrian Bolton[1], Yutian Chen[1], Timothy Lillicrap[1], Fan Hui[1], Laurent Sifre[1], George van den Driessche[1], Thore Graepel[1] & Demis Hassabis[1]



AlphaStar 2019

Mats Granath, MLQT, Erlangen 2019

Q-learning

- Agent in an environment described by a **state** *s*.
- Agent takes **actions** *a* to move between states, *s* -> *s*'.
- **Reward** (positive or negative) *r* is given depending on state/action.
- Agent learns **policy**, *π(s,a)*, to navigate environment for optimal accumulated reward (return) by exploring.

Q-function (action-value fcn) *Q(s,a)* **quantifies expected return from taking action** *a* **in state** *s* **and subsequently following the optimal policy.**

$$Q(s,a) = r + \gamma \max_{a'} Q(s',a')$$

γ<1 is discounting factor, better to get reward now than later

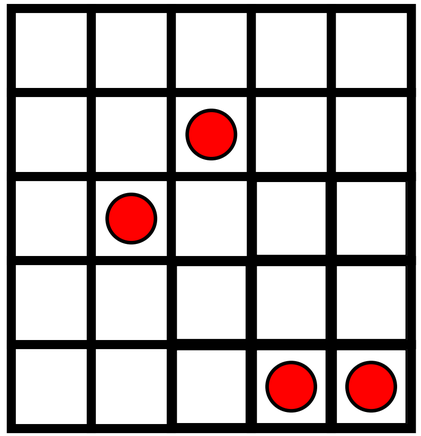
Explore to get reward and learn Q => optimal policy

Difficult if big world with many states and actions

Use Artificial Neural Network to represent Q-function
Deep Q-learning

Q-learning for the toric code

state is a syndrome
action is a bitflip=cardinal move of defect
reward, r=-1 per move (i.e. we aim to learn MWPM)



State space is very big
number of ways of placing N_S defects on d^2 sites:

$$\binom{d^2}{N_s} \approx \binom{49}{20} \sim 10^{13}$$

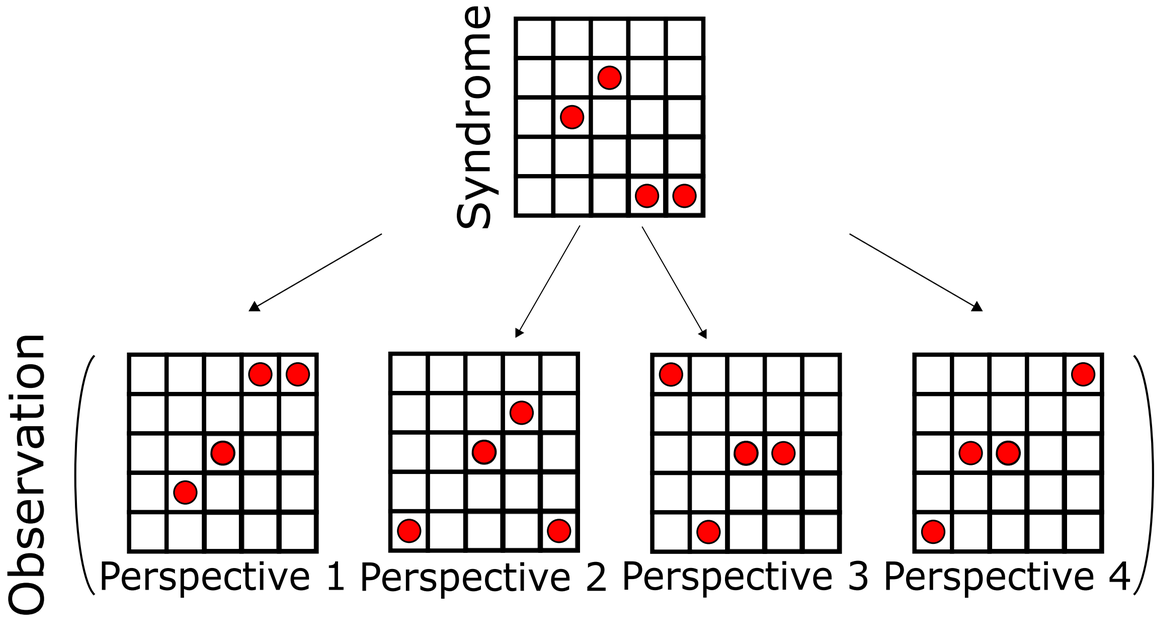
for d=7 and p=10%

Use deep Q-learning

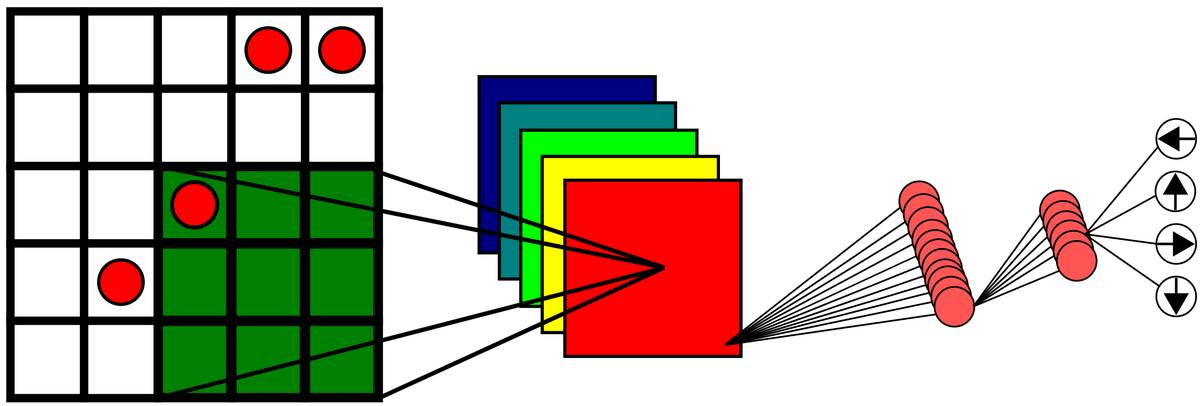
Mats Granath, MLQT, Erlangen 2019

Efficient implementation of Q-network

Use translational and rotational symmetry
to center each defect.



Convolutional NN



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Deep Q-network

Network gives Q-values for the 4 movements of the **central** defect.
Crucial simplification, fixed number (4) actions, and doesn't have to learn about boundaries.

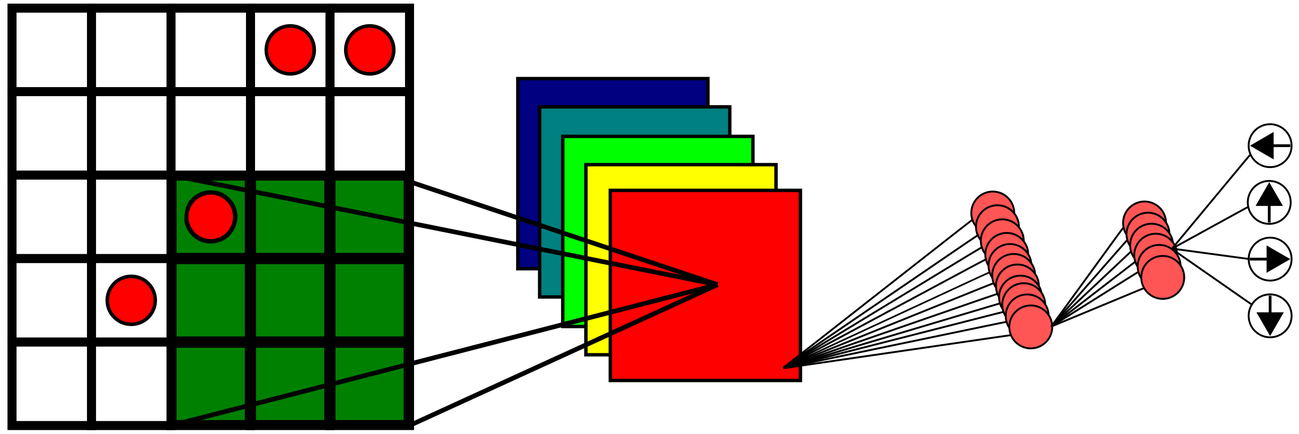


Table 2: Network architecture d=7.

#	Type	Size	# parameters
0	Input	7x7	
1	Conv.	512 filters; 3x3 size; 2-2 stride	5 120
2	FC	256 neurons	1 179 904
3	FC	128 neurons	32 896
4	FC	64 neurons	8 256
5	FC	32 neurons	2 080
6	FC (out)	4 neurons	132
			1 228 388

Experience replay is crucial for training

Significant reduction in number of parameters.
Size of state space for d=7, and N_S=20 defects (10% error)

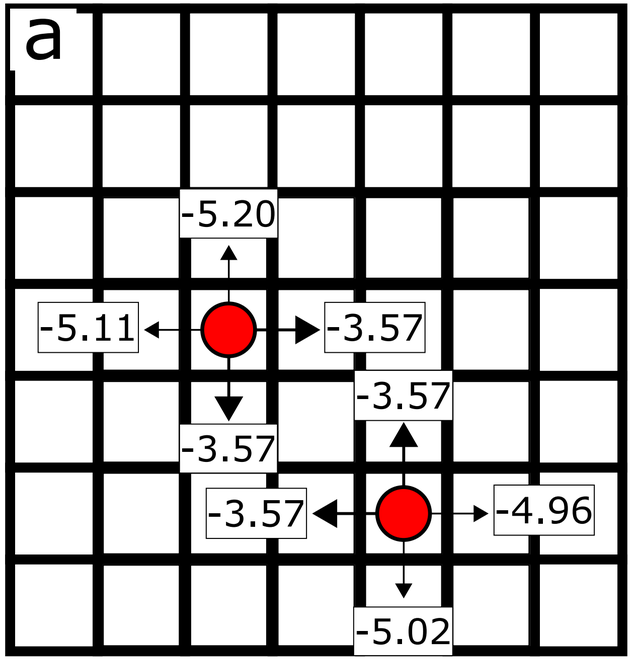
$$\binom{d^2}{N_s} \approx \binom{49}{20} \sim 10^{13}$$

Results. Converged Q-network.

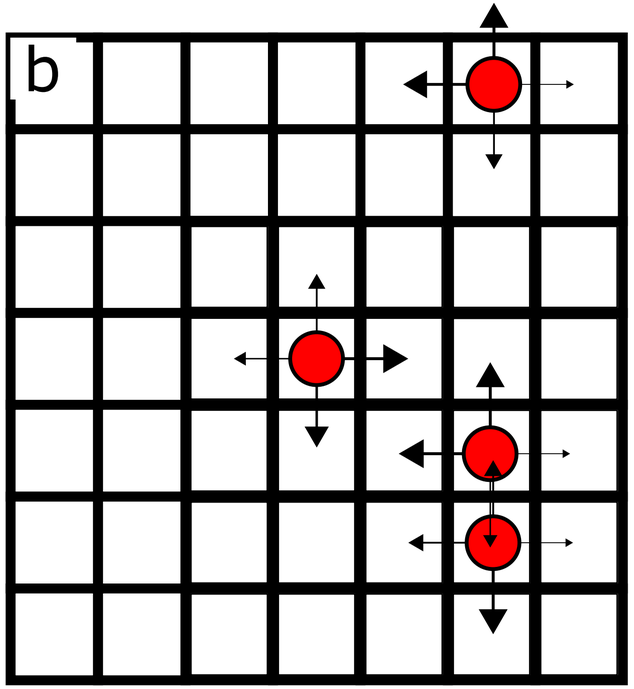
Examples:

Large arrow=Large Q-value for that action

4-steps to elimination



Shortest total path (MWPM)



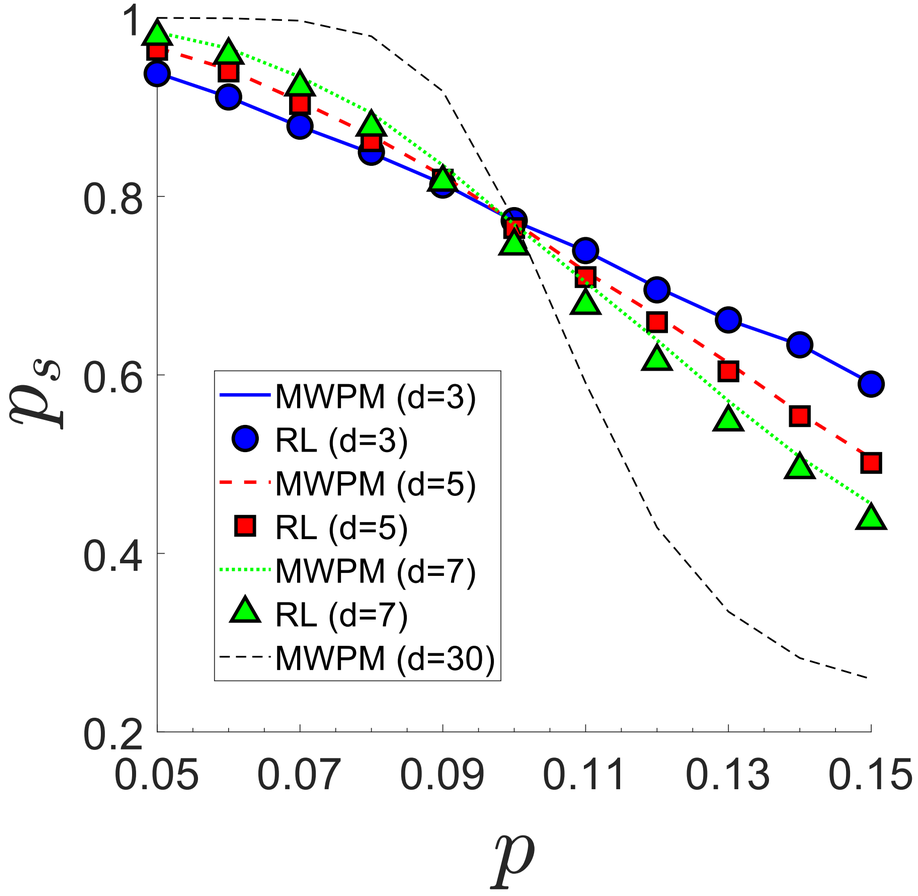
$$R = -1 - \gamma - \gamma^2 - \gamma^3 = -3.62$$

$$\gamma = 0.95$$

(semi-) quantitatively correct Q-values

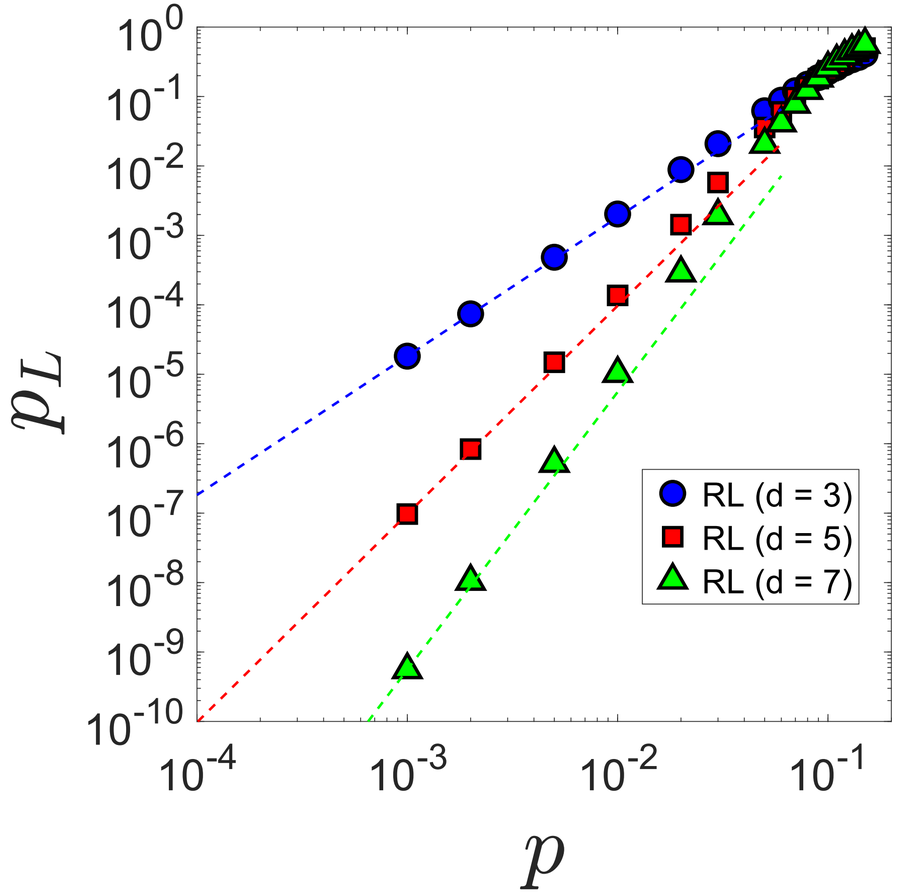
Results

Logical success-rate, large error rates close to MWPM



bit flip error rate

Logical fail-rate, small error rates identical to MWPM



Philip Andreasson, Joel Johansson, Simon Liljestrand, Mats Granath, arXiv:1811.12338

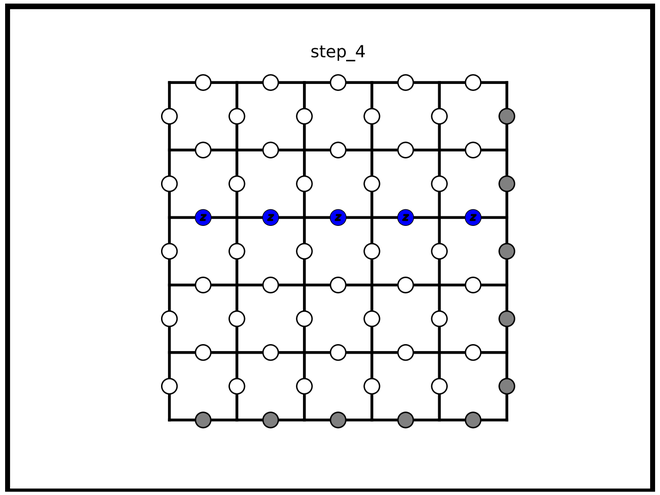
Fits asymptotic form for small p:

$$p_L = 2d\binom{d}{\lceil d/2 \rceil} p^{\lceil d/2 \rceil}$$

Depolarizing noise, work in progress

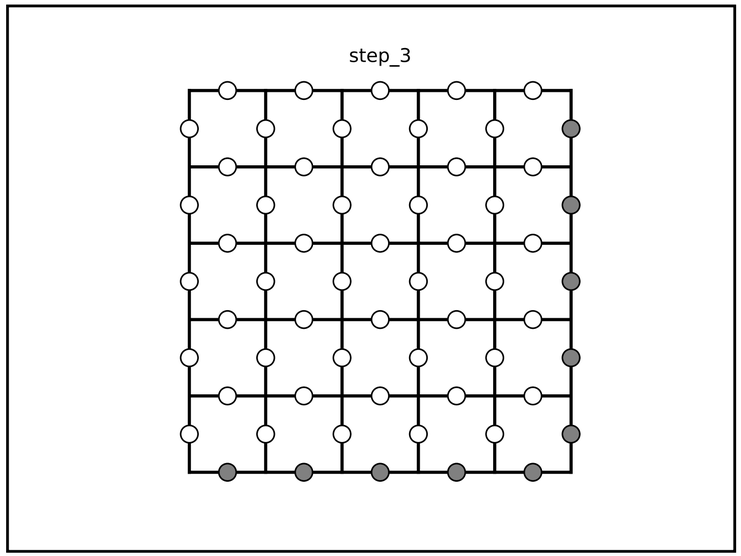
Example syndrome

MWPM



logical phase-flip

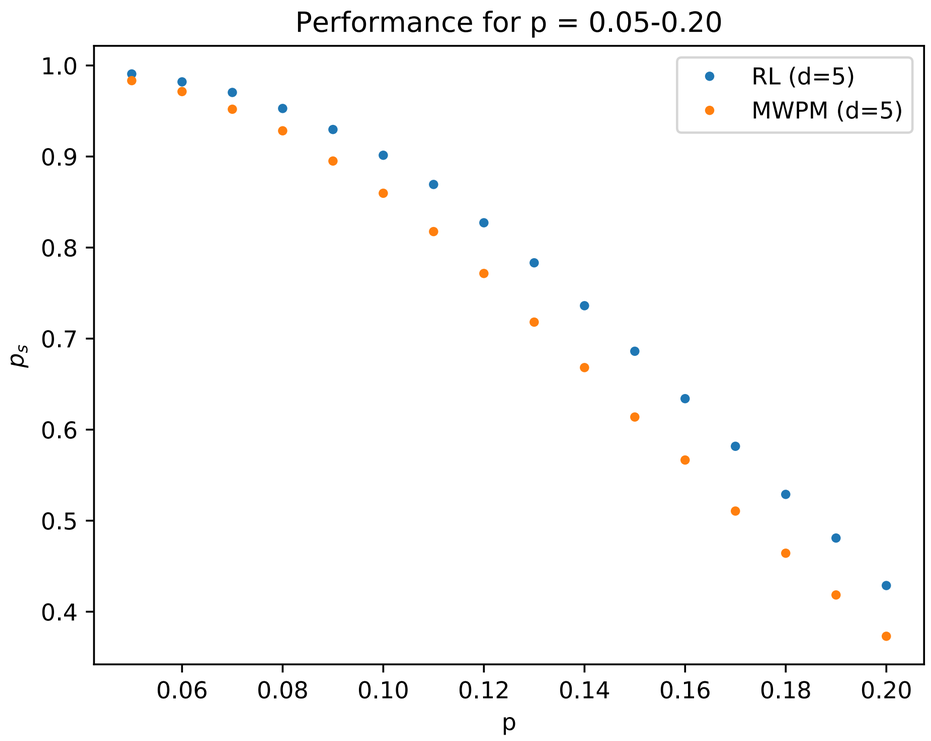
Reinforcement trained solver
reward=annihilation of complete syndrome + small intermediate reward



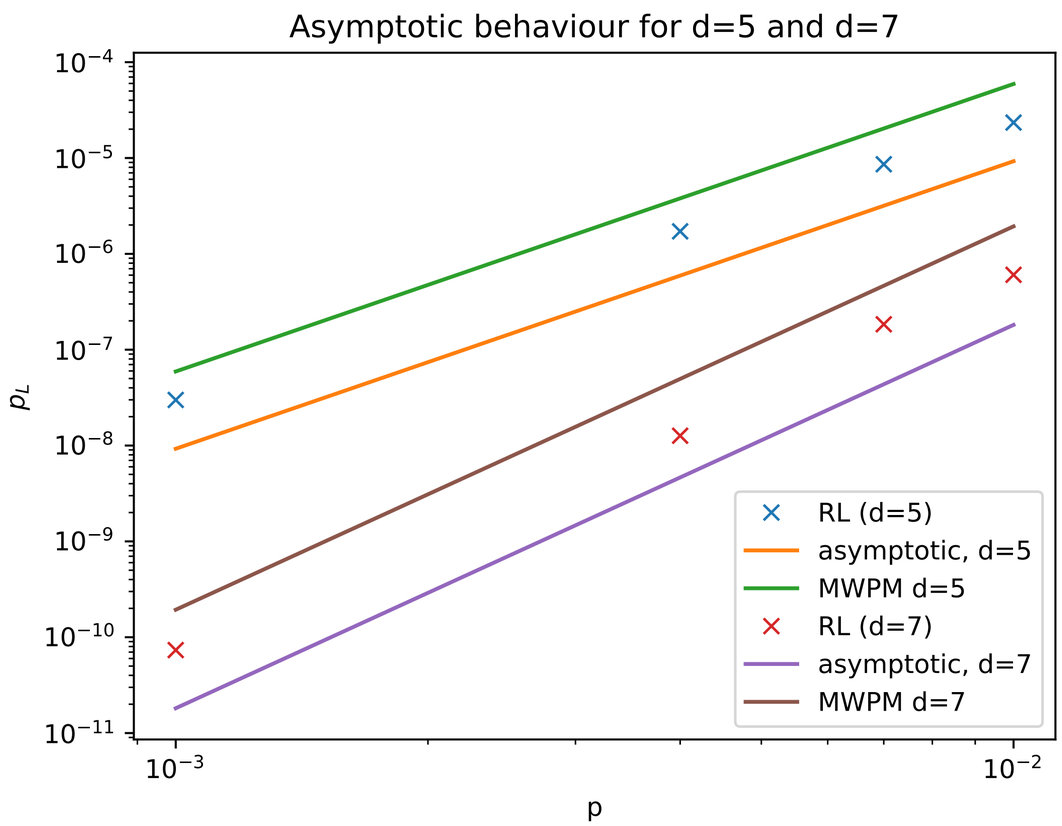
No logical operation

The agent can use Y to take advantage of correlations between bit-flip and phase-flip errors

Preliminary performance of RL solver for depolarizing noise



Outperforms MWPM



Deep Q-networks

distance 5 code

Layer (type)	Output Shape	Param #
Conv2d-1	[-1, 128, 5, 5]	2,432
Conv2d-2	[-1, 128, 5, 5]	147,584
Conv2d-3	[-1, 120, 5, 5]	138,360
Conv2d-4	[-1, 111, 5, 5]	119,991
Conv2d-5	[-1, 104, 5, 5]	104,000
Conv2d-6	[-1, 103, 5, 5]	96,511
Conv2d-7	[-1, 90, 5, 5]	83,520
Conv2d-8	[-1, 80, 5, 5]	64,880
Conv2d-9	[-1, 73, 5, 5]	52,633
Conv2d-10	[-1, 71, 5, 5]	46,718
Conv2d-11	[-1, 64, 3, 3]	40,960
Linear-12	[-1, 3]	1,731

Total params: 899,320

trained on desktop GPU for 5 hours (using PyTorch)

distance 7 code

Layer (type)	Output Shape	Param #
Conv2d-1	[-1, 200, 7, 7]	3,800
Conv2d-2	[-1, 190, 7, 7]	342,190
Conv2d-3	[-1, 189, 7, 7]	323,379
Conv2d-4	[-1, 160, 7, 7]	272,320
Conv2d-5	[-1, 150, 7, 7]	216,150
Conv2d-6	[-1, 132, 7, 7]	178,332
Conv2d-7	[-1, 128, 7, 7]	152,192
Conv2d-8	[-1, 120, 7, 7]	138,360
Conv2d-9	[-1, 111, 7, 7]	119,991
Conv2d-10	[-1, 104, 7, 7]	104,000
Conv2d-11	[-1, 103, 7, 7]	96,511
Conv2d-12	[-1, 90, 7, 7]	83,520
Conv2d-13	[-1, 80, 7, 7]	64,880
Conv2d-14	[-1, 73, 7, 7]	52,633
Conv2d-15	[-1, 71, 7, 7]	46,718
Conv2d-16	[-1, 64, 5, 5]	40,960
Linear-17	[-1, 3]	4,803

Total params: 2,240,739

trained on desktop GPU for 12 hours

Unnecessarily deep?

Conclusions

Deep Q-learning works well for error correction on *toric* code.
Can match or even outperform MWPM (for moderate code distance)

But, does require quite deep Q-networks

Periodic boundaries important for our approach.

Future challenges:

- **Larger code distances**
- **Improve reward scheme, use actual success or failure of error correction**
- **Include syndrome measurement error.** (R. Sweke et al, arXiv:1810.07207)
- **Surface code with boundaries. (Tougher due to lack of translational invariance)**

Philip Andreasson, Joel Johansson, Simon Liljestrand, Mats Granath, arXiv:1811.12338

Mattias Eliasson, David Fitzek, MG, in progress