Lecture 1.

Introduction: the role of polarization in quantum and nonlinear optics. Brief history of the polarization optics.
Classical polarization state. The role of higher moments. ‘Hidden polarization’ effect.

1. The role of polarization in modern optics. It is difficult to overestimate it: just look at the website of any company selling optical components and see the section ‘polarization optics’. In nonlinear optics, polarization of light is crucial for understanding phase matching and for the analysis of the tensor properties of different nonlinear susceptibilities. Structured light is obtained with spatial light modulators, which usually employ liquid crystals, the polarization of light being the underlying effect. Moreover, spatial light modulators are the main element of beamers, which we now see in every lecture room. The fact that liquid crystal elements use polarization can be seen by simply looking at your mobile phone through a polarizer.

Finally, and most importantly, polarization plays the central role in quantum optics. The main reason is that qubits are so easily realized in the form of polarized photons $\alpha|H\rangle + \beta|V\rangle$. This state is similar to the one of a spin ½ particle, $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$. Photons are the best carriers of information: they do not easily interact with anything; this means that they can propagate relatively far without being lost or scattered. This is why one can use polarized photons for quantum key distribution – this will be the subject of our last lecture.

2. Polarization of light was probably known already to ancient Vikings. Most probably, they used calcite (also called island spar) for navigation. With this ‘sunstone’, as they called it, they managed to find the position of the sun in the sky on a cloudy day. Indeed, light is not polarized when it comes directly from the Sun, but it is partly polarized when scattered. When light is partially polarized, the two images of an object seen through a calcite crystal are of different strength, at least for some orientation of the crystal. In unpolarized light, the two images will have the same strength, no matter how one rotates the crystal. As such objects, the Vikings could have used some details of clouds.

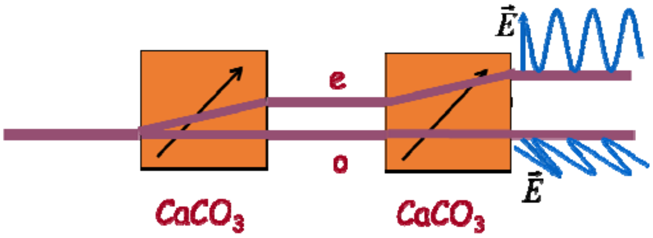


Fig.1

Systematic study of polarization probably started in the 17th century: in 1669, Rasmus Bartholin discovered double refraction in island spar (Fig. 1). This behavior (birefringence, walk-off) will be the subject of Lecture 7. Much later, in 1690, Huygens proved that polarization is a property of light: in another similar block of calcite, there is no further splitting. In 1808, Malus discovered that reflection leads to the polarization of light: a blazed reflection from a dielectric surface, it is polarized. (It is believed that he was looking at the reflections from the windows of Palais du Luxembourg in Paris through a calcite crystal.) The Brewster law was formulated in 1812.

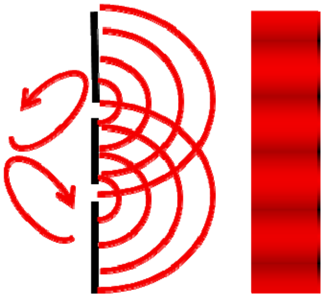


Fig. 2

Interference of light was studied extensively by Fresnel and Young, and in 1816 - 1817 they obtained a very important result: beams that are polarized orthogonally do not interfere. Indeed, imagine the double-slit experiment by Young and let the polarization in front of a slit or after it be rotated. Interference will disappear. Young and Fresnel only considered linear polarizations, H and V, but the same is true also for right and left circularly polarized modes (Fig. 2).

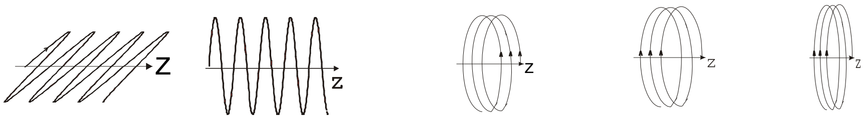
This is part of the general property: polarization, like frequency and direction, defines a mode.

In 1852, Stokes introduced 4 parameters for the description of polarization of light. We will consider them in Lecture 3. The Poincare sphere (Lecture 4), which we will use intensely in this course, was proposed in 1892. The same mathematical formalism was used by Bloch later, in the 20th century, for the description of a two-level system like spin ½ or an atom. The Jones matrices were used starting from 1940 and the Müller matrices, starting from 1943.

This classical polarization optics, as well as crystal optics, closely related to it, was very important for the development of nonlinear optics in the 1960s. In the first paper on second harmonic generation, by Franken et al. (PRL 7, 118, 1961), it was stressed that the efficiency depends on the polarization of the pump and the orientation of the crystal. Later, the phase matching conditions were formulated, and at that time they were satisfied only through the choice of different polarization. This will be the subject of Lecture 8.

In the 1950s – 1960s, polarization became one of the most important tools in the emerging quantum optics. In 1957, Bohm proposed an interpretation of the Einstein-Podolsky-Rosen (EPR) paradox in a Stern-Gerlach experiment with spin ½ particles. Due to the direct analogy with photons, such experiments were indeed carried out later, but with photons rather than with spin ½ particles. In 1964, Bell formulated inequalities that could be tested in such experiments, and these inequalities were being tested starting from 1982 and until 2015, with polarized photons. Finally, quantum key distribution (QKD) was first proposed (Bennett and Brassard, 1984) based on polarized photons. A whole lecture (11) will be devoted to QKD. Although other types of encoding were used since then, up to now satellite quantum communication is still carried out using polarization.

3. Classical description of polarization. Polarization is introduced as the way the electric vector oscillates. Indeed, imagine that we can take a snapshot showing us the ‘trajectory’ of the electric field vector:



H V R L elliptic

Fig.3

Then we can distinguish between different types of polarization (Fig.3): horizontal, vertical, right-hand and left-hand circular, elliptical. The mathematical description of all these polarization states will be considered in Lecture 2.

Note that monochromatic light is always polarized. Indeed, for a monochromatic wave,
$E_H(t)=E_{0H}\exp(-i\omega t+ikz)$, $E_V(t)=E_{0V}\exp(-i\omega t+ikz+i\varphi)$.

But if the amplitudes E_{0H} and E_{0V} have independent fluctuations, then one can speak of partially polarized light. In this case, the picture looks as shown in Fig.4 (imagine that we are looking right into the beam).

Polarized beam partially polarized beam non-polarized beam

Fig.4

The state of polarization is given by the coherence matrix, containing second moments of the fields:

$$K = \begin{pmatrix} \langle E_H^* E_H \rangle & \langle E_H^* E_V \rangle \\ \langle E_V^* E_H \rangle & \langle E_V^* E_V \rangle \end{pmatrix}$$

The averaging here is made according to statistical optics, over time. In quantum optics, there is a similar matrix, containing mean values of various operators:

$$\begin{pmatrix} \langle a^+ a \rangle & \langle a^+ b \rangle \\ \langle b^+ a \rangle & \langle b^+ b \rangle \end{pmatrix}$$

Here, **a**, **b** are the photon creation operators for modes H and V.

This coherence matrix contains only second-order moments of the field. This, however, turns out to be not the complete information about polarization. For instance, here is an example of how second moments fail to reveal some effect, called ‘hidden polarization’. Imagine unpolarized light prepared like shown in Fig.5: a polarized beam is split, then one half is delayed by a time larger than the coherence time. Then both halves are combined.

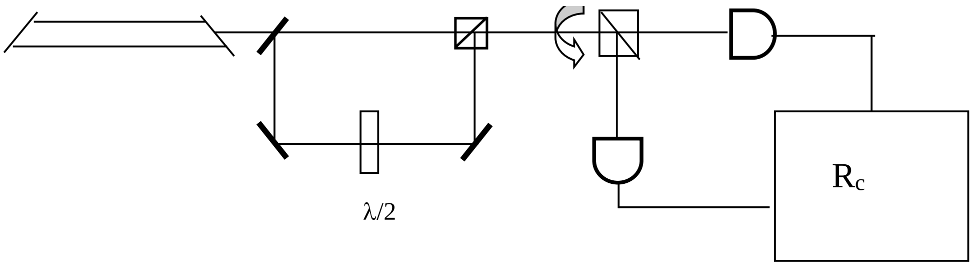


Fig. 5

If one simply looks at the beam through a polarizing prism, one will see constant intensities at its output regardless of its orientation (Fig. 6, black line). The conclusion should be: ‘light is not polarized’.

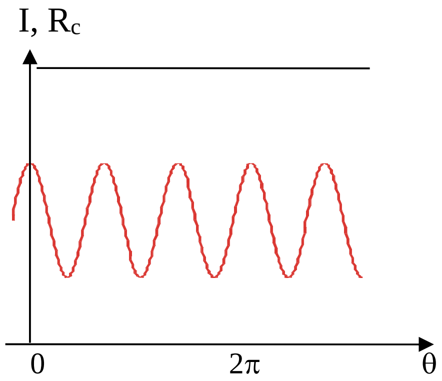


Fig. 6

However, if two detectors are placed in both outputs of the prism, with a coincidence circuit afterwards, then coincidence counting rate will depend on the orientation of the prism. One can learn the initial direction of the polarization.

Interestingly, if instead of this classical source one uses two-photon light, with the photons in pairs polarized orthogonally, the oscillations in the coincidence rate will have 100% visibility (Fig.7). This is because for every such a pair, there is no constant phase difference between vertically polarized field (photon A) and horizontally polarized field (photon B). The phase difference is completely uncertain. But there is correlation of photon numbers: every horizontally polarized photon has a match that is vertically polarized. This effect will be considered in Lecture 10.

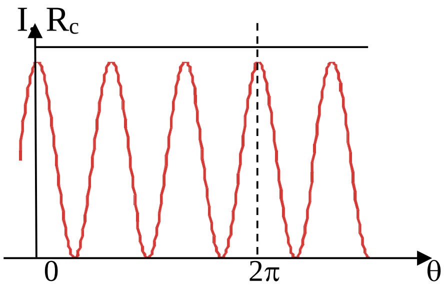


Fig. 7

Literature:

Born and Wolf, Principles of Optics

https://www.nature.com/news/2011/110131/full/news.2011.58.html