Lectures 1-10

1. During what time (on the order of magnitude) sunlight is polarized?
2. The same for a LED emitting at 500 nm with the bandwidth 30 nm.
3. What should be the thickness of a quartz plate whose optic axis is oriented at $\chi = \pi/4$, so that it converts H-polarized radiation of Ti-sapphire laser with the central wavelength 800 nm and the pulse duration 100 fs
   a) into V-polarized radiation;
   b) into unpolarized radiation. The birefringence of quartz can be assumed to be 0.01.
4. How many real parameters define a generic two-mode beamsplitter? A phase plate? A rotator?
5. What happens with the Jones vector when the Stokes vector is rotated by π around the σ_3 axis? Write the Jones matrix.
6. Are polarization transformations commutative? Justify your statement.
7. Can there be a set of phase plates or rotators that would transform any polarization state into an orthogonal one?
8. (a) What polarization state will not be changed by any rotator? (b) What rotator would not change any arbitrary polarization state?
9. How can one make a $\frac{\lambda}{6}$ plate out of a $\frac{\lambda}{2}$ plate and a $\frac{\lambda}{3}$ plate?
10. There are 2 QWPs for the radiation of a YAG:Nd laser with the wavelength 1064 nm. Will they work as a single HWP? Will one such plate work as a HWP for the second harmonic of this radiation?
11. Alice and Bob are both building experimental setups, where 5 half-wave plates should be oriented with optic axes horizontally. Both made it slightly inaccurate, but Alice put all plates shifted clockwise by 1°, and Bob placed odd plates 1° clockwise shifted and even plates 1° anticlockwise shifted. Who will get polarization closer to H? The initial polarization is H.
12. Prove that $S_3 = 2\,\mathrm{Im}\{E_H^* E_V\}$ can be equivalently written as $S_3 = I_L - I_R$.
13. Draw a setup for the measurement of $(S_1 + S_2)/\sqrt{2}$.
14. Draw a setup for the measurement of $(S_1 + S_3)/\sqrt{2}$.
15. By what angle will the plane of oscillations for a Foucault pendulum in Erlangen be shifted after 24 hours?
16. Find the interference visibility for states polarized horizontally and at an angle $\pi/8$. How can the second state be prepared?
17. H- polarized light is transformed into a D-polarized state by (a) a rotator and (b) a $\lambda/2$ plate. Find the geometric phase shift in both cases.
18. Right-hand polarized light is transmitted through a $\lambda/2$ plate with the optic axis at angle χ and then, through another $\lambda/2$ plate with the optic axis at $-\chi$. Find the geometric phase shift.
19. A state has the Jones vector $\vec{e} = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ e^{i\pi/4}\sqrt{\frac{2}{3}} \end{pmatrix}$. Calculate the Stokes vector of this state and show it on the Poincare sphere.
20. What will be the geometric phase shift for this state after a transformation with a rotator with $\delta = \pi$?
21. Write the Fresnel equations and find the principal values of n for the following form of dielectric tensor: $\varepsilon_x = \varepsilon_y = 3, \varepsilon_z = 4$. Find the phase velocities and the

group velocities for light propagating orthogonally to the optic axis. Assume that there is no group-velocity dispersion.

22. The components of the dielectric tensor are $\varepsilon_x = 2.3, \varepsilon_y = 2.7, \varepsilon_z = 3$. In what plane are the optic axes?
23. For the beta-barium borate (BBO) crystal, the ordinary and extraordinary refractive indices at 532 nm are, respectively, 1.675 and 1.556. Is it a negative or positive crystal? Find the refractive indices for a wave propagating at 20 degrees to the optic axis.
24. For the bismuth triborate (BiBO) crystal, the refractive indices at 532 nm are 1.79, 1.82, 1.96. Find the angle between the optic axes.
25. For the same crystal, the refractive indices at 1.5 μ are 1.75, 1.77, and 1.90. Does the angle between the optic axes change with the wavelength?
26. In calcite, the ordinary and extraordinary refractive indices at 532 nm are 1.66 and 1.49, respectively. Find the walk-off angle for the wavevector at $\pi/4$ to the optic axis.
27. For a calcite crystal of length 2 cm, cut at $\pi/4$ to the optic axis, what will be the distance between the ordinary and extraordinary beams at the output?
28. Find the phase and group velocities for this case, assuming no group-velocity dispersion.
29. Derive the expression for the walk-off angle (Lecture 5) $\tan(\alpha \pm \vartheta) = \pm \frac{n_o^2}{n_e^2} \tan\vartheta$
30. For light at 532 nm, 3.5 mm of quartz cut orthogonal to the optic axis will rotate the polarization by 90°. Find the circular birefringence of quartz at this wavelength.
31. Find the bandwidth of a zero-order quartz HWP. Consider the bandwidth as the distance between the wavelengths where $\delta = 0.4\pi$ and $\delta = 0.6\pi$. The central bendwidth is 500 nm.
32. Calculate the minimal thickness of a quartz plate such that it is a HWP for 532 nm (YAG:Nd laser) and a QWP for 633 nm (He-Ne laser).
33. Calculate the tilt angle of the interface of the Wollaston prism, for the angle between the two output beams (outside of the crystal) to be 20°.
34. How can one find out whether a beam is polarized vertically or horizontally using only a thin fused silica plate?
35. Using such a plate, find out how your smartphone screen is polarized (during the problem class).
36. Find the angle of oo->e phase matching for SHG from 1064 nm to 532 nm in lithium niobate crystal. The refractive indices are given in the lecture.
37. What is the walk-off angle in this case?
38. Find the wavevector mismatch for ee->e SHG ($\theta = \pi/2$) from 1064 nm to 532 nm in lithium niobate crystal. Is there walk-off in this case?
39. Find the effective nonlinearity for the ee->e phasematching ($\theta = \pi/2$) from 1064 nm to 532 nm in lithium niobate crystal and compare it with the ooe effective nonlinearity. The nonlinear susceptibility tensor for 3m point group is

$$d_{3m} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix},$$

and the components are

$$d_{22} = 2.5\frac{\text{pm}}{\text{V}}, d_{31} = -4.6\frac{\text{pm}}{\text{V}}, d_{33} = -41.7\frac{\text{pm}}{\text{V}}.$$

40. Find the expression for the effective nonlinearity for oe->e (type-II) SHG [and of course, the reverse process, PDC] for 3m point group.
41. Draw a setup for measuring the combination of the Stokes operators S_1+S_2 for a single photon.
42. Calculate $[S_1,S_3]$.
43. Find the eigenstates of $\hat{S}_2$ and $\hat{S}_3$.
44. Calculate the Stokes parameters and the degree of polarization for the state $|\Psi\rangle = |2\rangle_H$. (This is the state generated through collinear degenerate type-I SPDC.)
45. Find the commutator $[\mathbf{S}^2,S_1]$.
46. Find the orientation of the beta barium borate (BBO) crystal providing type-II collinear degenerate phase matching for SPDC from 400 nm pump. The refractive indices of BBO at 400 nm are n_0(400 nm)=1.69, n_e(400 nm)=1.57; at 800 nm, n_0(800 nm)=1.66, n_e(800 nm)=1.55.
47. Find the photon number for mode A in the Bell state $|\Psi^{(+)}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)$.
48. Find the degree of polarization for mode A in the same state.