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Experimental physics 3 for physicists
FAU Erlangen–Nuremberg, winter term 2018/19
https://mpl.mpg.de/en/leuchs/lehre/skripte.html

Problem sheet 1
25./26.10.2018

Dates

Thursdays, 12:00-14:00

Tobias Frank	SRLP 0.179

Fridays, 10:00-12:00

Alexander Otterpohl	HF
Alexandra Popp	SRLP 0.179
Stefan Richter	SR 02.779
Thomas Dirmeier	SR 01.779
Kevin Günthner	SR 00.103

Problem 1: How small is an optical wavelength?

How many wavelengths of green light ($\lambda = 555\,\mathrm{nm}$) fit into a path corresponding to the thickness of a sheet of paper ($d = 0.1\,\mathrm{mm}$)?

Problem 2: Properties of plane waves

Using Maxwell's equations, prove the following properties of plane waves:

a. $\vec{E}$ and $\vec{B}$ are perpendicular to the wave vector $\vec{k}$,

b. $\vec{E}$ is perpendicular to $\vec{B}$.

Problem 3: Group velocity of a wave-packet

A *wave-packet* propagating in x-direction can be represented as a superposition of monochromatic waves

$$A(x,t) = \int_0^\infty a(k) e^{i(kx-\omega(k)t)} dk$$

For each wave there is a relation between its wave vectors k and angular frequency ω, prescribed by a dispersion relation $\omega = \omega(k)$ (which in turn is defined by a propagation medium). Also wave vectors (and angular frequencies, correspondingly) span some interval around a central value k_0 (respectively, ω_0).

Show that the characteristic propagation speed of the wave-packet (i.e. *group velocity* of the overall wave-packet shape) can be calculated via the expression

$$v_g = \partial\omega/\partial k \quad \text{at} \quad \omega = \omega_0$$

What is the group velocity in the vacuum?

Hint: Use the first order expansion of the dispersion relation around the central wave-vector: $\omega(k) = \omega(k_0) + \frac{\partial\omega}{\partial k}|_{k=k_0}(k - k_0)$

Problem 4: Does a charged particle at rest lose energy via radiation?

The Poynting vector gives both absolute the value of the energy flux of an electromagnetic field in [W m^{-2}] at the point of interest and the direction of the flux. In vacuum, the Poynting vector of a wave can be calculated with the expression $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$.

Check that a charged particle at rest in vacuum does not emit energy using the Maxwell equations in vacuum and calculating the divergence of the Pointing vector.

Hint: $\nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b})$.