



Problem 1: Brewster's angle

Show that, when an incident electromagnetic field falls onto an interface at Brewster's angle α_B (lecture §3.1), then the direction of the reflected (vanishing) light is perpendicular to the direction of the transmitted light.

Hints: In the lecture, the condition $n_2^2 \cos^2 \alpha_B = n_1^2 \sin^2 \alpha_B$ or $\tan \alpha_B = \frac{n_2}{n_1}$ was derived. We also have $\cos(90^\circ - x) = \sin x$.

Problem 2: Polarization filter built with a stack of plane parallel glass plates

The degree of polarization of a light can be quantitatively characterized via the expression

$$p = \frac{|I_{\perp} - I_{\parallel}|}{I_{\perp} + I_{\parallel}}$$

where I_{\perp}, I_{\parallel} are intensities of two orthogonally polarized components of the light. For example, for non-polarized light $I_{\perp} = I_{\parallel}$ and the polarization degree equals to zero.

An unpolarized light beam is incident at the Brewster angle on a stack of plane-parallel glass plates ($n_g = 1.5$), each separated by a layer of air ($n_a = 1.0$). The thickness of the glass plates and layers is greater than the coherence length of the light, so that the light reflected from different surfaces add up incoherently (and interference effect does not play a role).

Show that the transmitted light will be polarized. Estimate the effect (i.e. how good is the polarization filter) calculating the polarization degree of the light after a single plate/ten plates.

Hint: Use the following formula for the transmittance coefficient: $T^{\perp/\parallel} = \left| \frac{n_2 \cos \alpha_2}{n_1 \cos \alpha_1} \right| \cdot |t^{\perp/\parallel}|^2$, where $t^{\perp/\parallel} = \frac{E_g^{\perp/\parallel}}{E_e^{\perp/\parallel}}$. For simplicity, do not consider multiply reflected beams.

Problem 3: Frustrated total internal reflection

As it was shown in lectures there is a total internal reflection phenomenon that happens when a light strikes an interface with a less optically dense medium at an angle larger than a critical one. Then the light does not pass through and is reflected. Fresnel equations say the reflection is total (no transmission).

In fact there is still the evanescent transmitted light that penetrates into the lower refractive index medium.

- a. How deep the evanescent field penetrates into the medium? What is the intensity distribution of the evanescent field depending on the distance from the boundary?
- b. Consider the optical setup at the Figure. Assume that under the given distance d between the prisms 50% of the light power is reflected. What happens to this ratio when the distance is increased by $1\mu\text{m}$? (Assume a wavelength of the light 500 nm , the index of refraction of prisms is $n = 1.5$, angle of incidence 45° and negligible back reflection at the second interface.)

