Christoph Marquardt, Gerd Leuchs & Klaus Mantel
**Experimental physics 3 for physicists**
FAU Erlangen–Nuremberg, winter term 2018/19
https://mpl.mpg.de/en/leuchs/lehre/skripte.html

# Problem sheet 4
22./23.11.2018

## Problem 1: Fourier spectrum of a square pulse

a) Determine the Fourier transform $\tilde{f}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}t\, f(t)\, e^{-\mathrm{i}\omega t}$ of the rectangular pulse

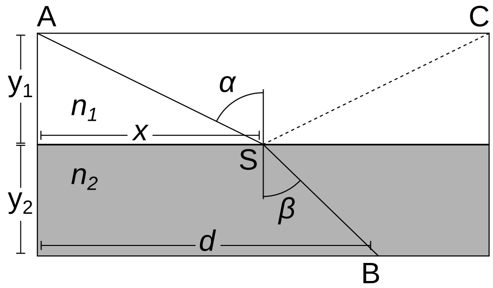
$$f(t) = \begin{cases} f_0, & \text{when } 0 \leq |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

b) Determine the Fourier amplitude at the origin, i.e. for angular frequency $\omega = 0$.

## Problem 2: Fermat's principle

Show that the directions of reflected and refracted light beams at an interface of two different media, can be obtained from *Fermat's principle*, i.e. in both cases the light "chooses" the path that minimizes the optical path length $\int dsn(s)$, where $n(s)$ is the refractive index at the point $s$.

*Hint:* Consider only straight segments $\overline{\text{AS}}$, $\overline{\text{SB}}$ and $\overline{\text{SC}}$.




## Problem 3: Wave properties of an electron

An electron is accelerated by the voltage **a)** $U_a = 13.6\,\text{V}$, **b)** $U_b = 1\,\text{MV}$.

- What is velocity of the electron in each case?
- What is the de Broglie wavelength $\lambda = h/p$ of the electron?

## Problem 4: Probability density of matter waves

As discussed in lectures, wave function of a free particle satisfies the differential equation

$$\mathrm{i}\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_x^2\psi.$$

a. show that the time derivative of the squared modulus $\psi$ is given by the expression:

$$\partial_t |\psi|^2 = \partial_t(\psi \cdot \psi^*) = \frac{\hbar}{\mathrm{i}2m}\partial_x(\psi\partial_x\psi^* - \psi^*\partial_x\psi)$$

(Hint: the differential equation for $\psi^*$ is obtained from the wave differential equation by the complex conjugation.)

b. show for a plane wave $\psi = \exp(\mathrm{i}kx - \mathrm{i}\omega t)$ that the following equation holds:

$$\frac{\hbar}{\mathrm{i}2m}(\psi\partial_x\psi^* - \psi^*\partial_x\psi) = -\frac{\hbar k}{m}\psi\psi^*.$$

Then using the fact, that $v = \frac{\hbar k}{m}$, one can get the continuity equation for the matter wave of a free particle:

$$\partial_t\rho = -\partial_x(v \cdot \rho) \text{ where } \rho = \psi\psi^*.$$