Lecture 10. Seeding.

Seeding of nonlinear processes and related effects like photon addition and nonlinear subtraction, stimulated emission tomography and brightness measurement.

1. *Seeding.*

In this lecture we will consider stimulation of nonlinear optical processes, commonly called seeding, by sending a coherent beam (seed) to the input. The processes we will describe will be parametric down-conversion (four-wave mixing) and frequency up-conversion.

Seeded non-degenerate (phase insensitive) optical parametric amplifier, NOPA.
Consider the three-wave interaction described in Lecture 2 (Fig.1), and assume that there is an input beam at the idler frequency ω_1 (Fig.1, the same as Fig.3 of Lecture 2). However, unlike

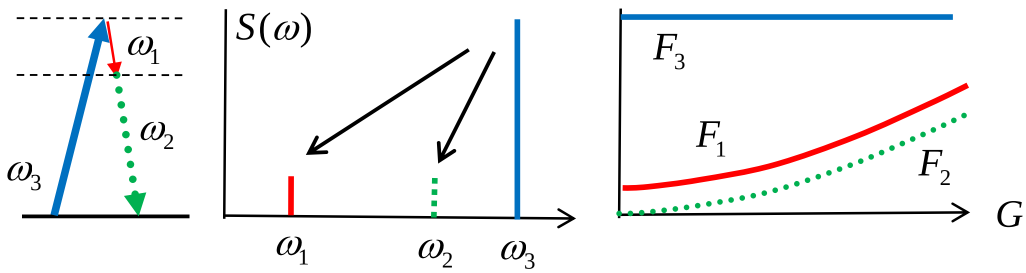

Fig.1

in Lecture 2, we now use the quantum description. The pump power being constant (undepleted pump), we can write the Bogolyubov transformations again:

$$
\begin{aligned}
a_1 &= a_{10}\cosh G + a_{20}^{+}\sinh G, \\
a_2 &= a_{20}\cosh G + a_{10}^{+}\sinh G.
\end{aligned} \tag{1}
$$

Let us find the output numbers of photons in both modes provided that there is a certain quantum state at input 1, and a vacuum at input 2:

$$\langle N_1\rangle = \langle a_1^{+}a_1\rangle = \langle (a_{10}^{+}\cosh G + a_{20}\sinh G)(a_{10}\cosh G + a_{20}^{+}\sinh G)\rangle,$$
$$\langle N_2\rangle = \langle a_2^{+}a_2\rangle = \langle (a_{20}^{+}\cosh G + a_{10}\sinh G)(a_{20}\cosh G + a_{10}^{+}\sinh G)\rangle.$$

The averaging is over a vacuum state in mode 2 and over some input quantum state in mode 1:

$$
\begin{aligned}
\langle N_1\rangle &= \langle\Psi|\langle 0|(a_{10}^{+}\cosh G + a_{20}\sinh G)(a_{10}\cosh G + a_{20}^{+}\sinh G)|0\rangle|\Psi\rangle = \\
&= \langle\Psi|\langle 0|a_{10}^{+}a_{10}\cosh^2 G + (a_{20}a_{10} + a_{10}^{+}a_{20}^{+})\sinh G\cosh G + a_{20}a_{20}^{+}\sinh^2 G)|0\rangle|\Psi\rangle = \\
&= \langle a_{10}^{+}a_{10}\rangle\cosh^2 G + \sinh^2 G = \langle N_{10}\rangle\cosh^2 G + \sinh^2 G, \\
\langle N_2\rangle &= \langle\Psi|\langle 0|(a_{20}^{+}\cosh G + a_{20}\sinh G)(a_{20}\cosh G + a_{10}^{+}\sinh G)|0\rangle|\Psi\rangle = \\
&= \langle\Psi|\langle 0|a_{20}^{+}a_{20}\cosh^2 G + (a_{10}a_{20} + a_{20}^{+}a_{10}^{+})\sinh G\cosh G + a_{10}a_{10}^{+}\sinh^2 G)|0\rangle|\Psi\rangle = \\
&= \langle N_{10}\rangle\sinh^2 G + \sinh^2 G.
\end{aligned} \tag{2}
$$

This will be valid for any input state. The mean number of photons in mode 1 is amplified by a factor of $\cosh^2 G = 1+\sinh^2 G$, and also the 'vacuum noise', squeezed vacuum appears, with the mean number of photons $\sinh^2 G$. Note that this is exactly the consequence of the quantum description: in classical description, this term is absent. In mode 2, we will also have squeezed vacuum but also the number of photons converted from mode 1. This situation is fully described in Fig.1, right. The input state will be amplified regardless of its phase; in particular, a coherent state will be always amplified. This is why the amplifier is called phase insensitive.

Seeded degenerate (phase sensitive) parametric amplifier.
Consider now the same situation, but with a single-mode amplifier (degenerate optical parametric amplifier, DOPA). The Bogolyubov transformation takes the form

$$a = a_0 \cosh G + a_0^+ \sinh G, \qquad (3)$$

and we assume a state $|\Psi\rangle$ at the input. The output number of photons is

$$\begin{aligned}\langle N \rangle &= \langle \Psi | (a_0^+ \cosh G + a_0 \sinh G)(a_0 \cosh G + a_0^+ \sinh G) | \Psi \rangle = \\ &= \langle \Psi | a_0^+ a_0 \cosh^2 G + (a_0 a_0 + a_0^+ a_0^+) \sinh G \cosh G + a_0 a_0^+ \sinh^2 G) | \Psi \rangle = \\ &= \langle N_0 \rangle \cosh^2 G + (\langle N_0 \rangle + 1) \sinh^2 G + \langle a_0^2 + (a_0^+)^2 \rangle \sinh G \cosh G.\end{aligned} \qquad (4)$$

Coherent state at the input. It is interesting to consider a state $|\Psi\rangle = |\alpha\rangle = |\alpha_0 e^{i\varphi}\rangle$. Then,

$\langle a_0^2 + (a_0^+)^2 \rangle = 2|\alpha_0|^2 \cos(2\varphi)$, and

$$\langle N \rangle = \sinh^2 G + |\alpha_0|^2 [\cosh^2 G + \sinh^2 G + 2 \sinh G \cosh G \cos(2\varphi)],$$

which means that there will be amplification or deamplification depending on the phase. This is where the term 'phase sensitive amplifier comes from. In the case of deamplification, the part in square brackets goes to nearly zero (actually, to 1), but there will still remain the squeezed vacuum part.

2. ***Single photon addition, squeezing, subtraction.***

It is interesting to consider a single photon at the input. Let it be first a single photon at the input of a phase-insensitive amplifier.

First of all, at large parametric gain, $G >> 1$, $\cosh^2 G \approx \sinh^2 G$, and (2) becomes

$$\langle N_1 \rangle = \langle N_2 \rangle = (\langle N_{10} \rangle + 1) \sinh^2 G = 2 \sinh^2 G.$$

In other words, seeding a strongly pumped parametric amplifier with a single photon increases the mean number of photons at the output by a factor of 2.

If the amplifier is phase sensitive, we use (3) and obtain

$$\langle N \rangle = (2\langle N_0 \rangle + 1) \sinh^2 G, \qquad (5)$$

which means that the number of photons is increased by a factor of 3.

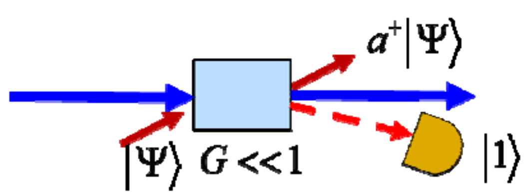

Fig.2

Single photon addition. Consider now a low-gain non-degenerate amplifier seeded by an arbitrary state; the state at the input is $|\Psi\rangle|0\rangle$ (a state $|\Psi\rangle$ at input 1 and nothing at input 2, see Fig.2). Then, the Hamiltonian

$$H = i\hbar\Gamma a_1^+ a_2^+ + h.c.$$

creates at the output a state

$$|\Psi'\rangle = |\Psi\rangle|0\rangle + \Gamma t(a_1^+ a_2^+ + h.c.)|\Psi\rangle|0\rangle = |\Psi\rangle|0\rangle + \Gamma t a_1^+ |\Psi\rangle|1\rangle. \quad (6)$$

This means that if a detector in the output mode 2 detects a photon, then it means that a photon was added to the state $|\Psi\rangle$ in the output mode 2. This operation indeed implements the photon creation operator a^+.

This technique was developed in the group of Marco Bellini [V. Parigi, A. Zavatta, M. Kim, and M. Bellini, Science 317, 1890–1893 (2007).]

Photon subtraction. Even simpler is to realize the subtraction of a single photon from a state. (It is not based on seeding a parametric amplifier but we consider it for completeness.) The scheme is sketched in Fig.3. A weakly reflecting beamsplitter is put on the way of a beam with a state $|\Psi\rangle$. If the detector in the reflected arm detects a photon, the transmitted arm contains the state $a|\Psi\rangle$. In other words, the input state gets a photon subtracted.

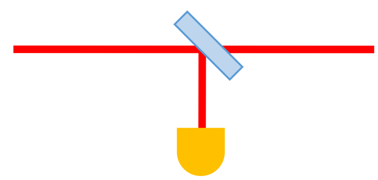

Fig.3

This can be strictly proved using the beamsplitter transformation with r<<1.

Nonlinear single-photon subtraction. In some cases, one would like to subtract a photon not just with a beamsplitter, but using a nonlinear process. Then, the same geometry as in Fig. 2 should be used, except that the Hamiltonian should be of up-conversion:

$$H = i\hbar\Gamma a_1 a_2^+ + h.c. \quad (7)$$

Then, the state at the output is

$$|\Psi'\rangle = |\Psi\rangle|0\rangle + \Gamma t(a_1 a_2^+ + h.c.)|\Psi\rangle|0\rangle = |\Psi\rangle|0\rangle + \Gamma t a_1 |\Psi\rangle|1\rangle. \quad (8)$$

This can be useful, for instance, to subtract a photon from a state within a certain given mode.

3. Brightness measurement.

The number of photons per mode is an important characteristic of any light source. This is, for instance, what distinguishes a laser from a lamp: both can have the same power, for instance a table lamp emits about 200 lumen, which is equivalent to about 150 mW at 532 nm. But the effect you can produce with a 150 mW coherent 532 nm laser is much stronger: you can, for instance, observe nonlinear effects.

The difference is in the number of photons per mode, which is proportional to the brightness, which is power per unit area per unit solid angle per unit frequency interval.

From the second one of equations (2), one can see that this number of photons per mode in a source one can find by seeding a non-degenerate parametric amplifier with this source and comparing the output signal in mode 2:

$$\langle N_2\rangle = [\langle N_{10}\rangle + 1]\sinh^2 G. \quad (9)$$

The ratio of the signals in the presence and in the absence of external seeding provides the brightness of the source. This method, called absolute measurement of brightness, has been developed in the 1970-s [D.N. Klyshko, Sov. J. Quant. Elect. 7 (1977) 591; G.Kh. Kitaeva, A.N. Penin, V.V. Fadeev, Yu.A. Yanait, Sov. Phys. Dokl. 24 (1979) 564] and is now used in metrology.

4. Stimulated-emission tomography.

Recently, a method has been proposed to measure the squared modulus of the two-photon amplitude $F(\omega_s, \omega_i)$ or $F(q_s, q_i)$ without registering coincidences. The idea [M. Liscidini and J. E. Sipe, 'Stimulated Emission Tomography,' PRL 111, 193602 (2013)] is to seed an SPDC source by a coherent beam. This situation is also described by (9) but with the gain value being low. In the presence of the seed, the output intensity will increase a lot, and by scanning the seed beam in the angle and frequency one can reconstruct the two-photon amplitude from the output intensity distribution.