Lecture 8. From nonlinear optical effects to squeezing.

Squeezing obtained from nonlinear optical effects: PDC and FWM, Kerr effect. Passing from quadrature to twin-beam squeezing and vice versa. Obtaining sub-Poissonian statistics through feedforward technique and through second harmonic generation. The role of loss.

1. Quadrature squeezing.

In this lecture we will continue discussing how nonclassical light can be produced through PDC and FWM. Unlike in the previous lecture, now we focus on high-gain effects and bright light. Therefore the criteria for nonclassicality will be different now: squeezing, sub-Poissonian photon statistics, and sub-shot-noise photon-number correlations (noise reduction).
Parametric down-conversion. Consider the degenerate Hamiltonian,

$$\hat{H} = i\hbar\Gamma(a^+)^2 + h.c. \qquad (1)$$

In Lecture 4 we have derived that this Hamiltonian makes the quadratures evolve as

$$\hat{q} = e^G \hat{q}_0,\ \hat{p} = e^{-G}\hat{p}_0, \qquad (2)$$
$$G \equiv 2\Gamma t.$$

Through this evolution, an input vacuum state becomes squeezed vacuum, see Fig.2 in Lecture 4. One can describe it by acting on the input vacuum state by the evolution operator,

$$\exp\{\frac{1}{i\hbar}\int dt\hat{H}\} = \exp\{\frac{G}{2}(a^+)^2 + h.c.\} \equiv \hat{S}(G),$$

where we introduced the squeezing operator $\hat{S}(G)$. Then the squeezed state can be written as

$$|\Psi\rangle = \hat{S}(G)|vac\rangle. \qquad (3)$$

To observe this squeezing, one should perform homodyne tomography and measure the Wigner function (Lecture 5). The nonclassical feature is then the uncertainty of a quadrature being below the shot noise. A measure of this nonclassicality is quadrature squeezing in dB, defined as

$$10\log\frac{\Delta p^2}{\Delta p_0^2} = 10\log[e^{-2G}].$$

(We assume here that it is the p quadrature that is squeezed; in the general case it can be any quadrature.)
The record quadrature squeezing achieved now is -15 dB [H. Wahlbruch et al., PRL 17, 110801 (2016)]. It means that the variance of the squeezed quadrature was $10^{1.5} \approx 32$ times smaller than the shot-noise limit, and the uncertainty was $\sqrt{32} \approx 5.6$ times smaller than the shot-noise limit (Fig.1 schematically shows the corresponding Wigner function). The mean number of photons in this squeezed vacuum was $\sinh^2 G$, with $G = \ln(5.6) \approx 1.7$, which makes about 7 photons. This is a relatively weak state of light, but its detection was made possible by using a strong local oscillator.

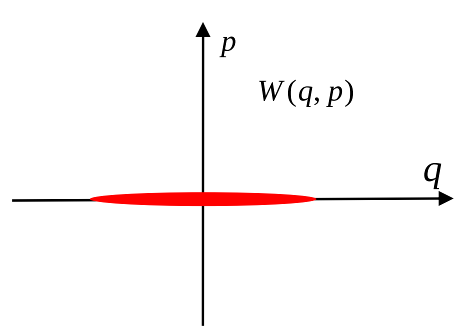

Fig. 1

It is worth mentioning that in such experiments, PDC occurs in a cavity, which sets the spectrum of the squeezed vacuum (should be resonant with the cavity) and provides a single frequency and angular mode.

At the same time, one can obtain, instead of a squeezed-vacuum state, a squeezed coherent state, which has the same squeezing but a larger number of photons. There are 2 ways to do it.

1. To send to the nonlinear crystal not the vacuum (no input beam, except the pump) but a coherent state $|\alpha\rangle$. What we obtain then can be understood from the evolution (2); see also Fig.2 of Lecture 4. The state will then be written as

$$|\Psi\rangle = \hat{S}(G)|\alpha\rangle.$$

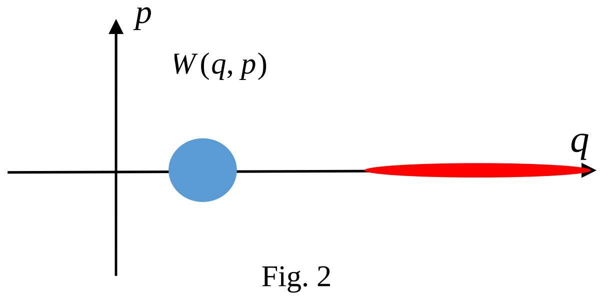

Fig. 2

The corresponding Wigner functions are shown in Fig.2: in blue for the initial coherent state and in red for the resulting displaced squeezed state.

2. The other way is to overlap the squeezed vacuum with a coherent state on a beamsplitter. In order not to destroy the squeezing, the transmission of the beamsplitter for the squeezed vacuum should be very high; the coherent light will then be attenuated but remain coherent. This procedure will then displace the squeezed vacuum state along the amplitude of the coherent state. Figure 3 shows the case where the displacement is along the squeezing direction. Blue circle and ellipse show the initial states (the coherent state is shown already attenuated after the beamsplitter) and the red ellipse shows the displaced squeezed state. One can say that this state has a ‘coherent carrier’ (the red ‘stick’).

This situation is interesting because it allows one to prepare bright sub-Poissonian light. It is useful for any measurements involving direct detection because the accuracy of photon-number measurement can be very high. And the state shown in Fig.3 (red ellipse) indeed corresponds to sub-Poissonian light, as it is squeezed in the amplitude -> in the number of photons.

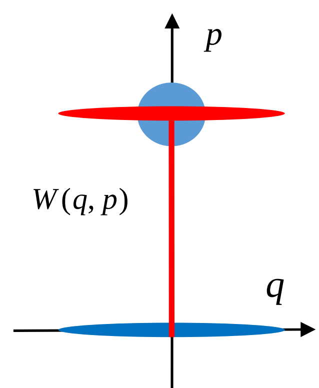

Fig. 3

Kerr squeezing. Consider now a material with cubic nonlinearity. Is it possible to observe quadrature squeezing in this case? Clearly, Hamiltonian (1) can be only realized if the pump is at the same wavelength as the squeezed light. At first sight, it is a problem because one cannot get rid of it. But this is only a problem if one needs photon pairs, not bright light. In the case of bright light, one can consider the pump as the ‘coherent carrier’ for squeezed vacuum, as in the previous example.

In a material with a large cubic nonlinearity, the Kerr effect will indeed lead to squeezing (Fig.4). To see this, consider the evolution of a coherent state in the course of its propagation through such a material (for instance, a fibre). Points corresponding to higher amplitudes will have a larger refractive index and larger phase delays. As a result, the uncertainty region will evolve and become compressed (grey ellipse) in some direction, corresponding to a certain quadrature (red arrows). Note that it is not the amplitude, because the Kerr effect leaves the total amplitude uncertainty constant. Rather, there appears some correlation (coupling) between the amplitude and the

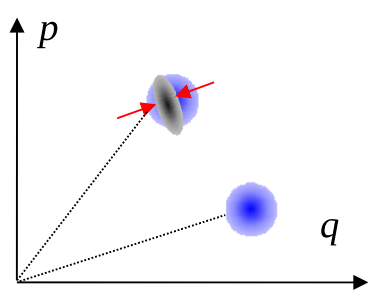

Fig.4

phase, - that is why the shape is tilted.

To obtain amplitude squeezing, one can overlap this beam with some coherent one and shift the ellipse in such a way that the squeezing becomes amplitude squeezing, as in Fig.3.

2. Two-mode squeezing and entanglement.

Consider now the two-mode Hamiltonian

$$\hat{H} = i\hbar\Gamma a_1^+ a_2^+ + h.c. \tag{4}$$

We know that it leads to sub-shot-noise photon-number correlations, with the nonclassical feature

$$NRF \equiv \frac{Var(N_1 - N_2)}{\langle N_1 + N_2 \rangle} < 1. \tag{5}$$

But there is another nonclassical feature one can observe for the state at the output of such a nonlinear material (two-mode squeezer). Recall the two-mode Bogoliubov transformations (Lecture 4),

$$a_1 = ua_{10} + va_{20}^+,$$
$$a_2 = ua_{20} + va_{10}^+,$$
$$u \equiv \sinh G, v \equiv \cosh G.$$

By introducing quadratures, $a_{1,2} = \hat{q}_{1,2} + \hat{p}_{1,2}$, we get

$$q_1 + ip_1 = u(q_{10} + ip_{10}) + v(q_{20} - ip_{20}),$$
$$q_2 + ip_2 = u(q_{10} + ip_{10}) + v(q_{20} - ip_{20}).$$

This leads to 4 equations for the quadratures (we take the real and imaginary parts separately):

$$q_1 = uq_{10} + vq_{20},$$
$$q_2 = uq_{20} + vq_{10}$$
$$p_1 = up_{10} - vp_{20},$$
$$p_2 = up_{20} - vp_{10}.$$

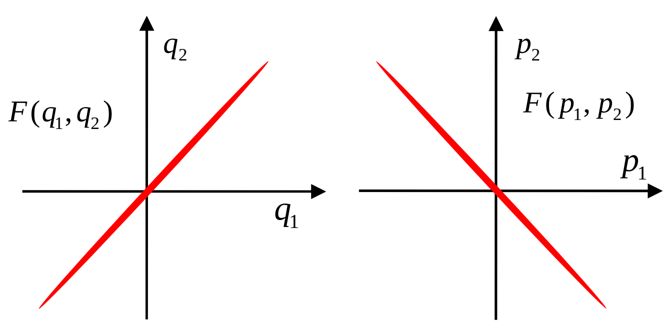

Fig. 5

Let us see what happens with the sums and differences of the quadratures. Then we get

$$q_1 + q_2 = e^G(q_{10} + q_{20}),$$
$$q_1 - q_2 = e^{-G}(q_{10} - q_{20})$$
$$p_1 + p_2 = e^{-G}(p_{10} + p_{20}),$$
$$p_1 - p_2 = e^G(p_{10} - q_{20}).$$

It means that the sum of position quadratures, $q_1 + q_2$, and the difference of the momentum quadratures, $p_1 - p_2$, will get anti-squeezed, while the difference $q_1 - q_2$ and the sum $p_1 + p_2$, will get squeezed:

$$\Delta(q_1 + q_2) = \Delta(p_1 - p_2) = e^G \Delta(q_{10} + q_{20}) = e^G \sqrt{\Delta(q_{10} + q_{20})^2} = e^G \sqrt{2}\Delta q_{10} = e^G \sqrt{2}/2 >> 1,$$
$$\Delta(q_1 - q_2) = \Delta(p_1 + p_2) = ... << 1.$$

At the same time, taken separately, each quadrature will not be squeezed. But there will be correlation between the quadratures. This exactly means *entanglement for quadratures,* and we can draw the probability amplitude distributions $F(q_1,q_2)$ and $F(p_1,p_2)$ (Fig.5) similar to how we drew $F(k_1,k_2)$ at Lecture 7.

3. Transformations between two-mode and quadrature squeezing.

One can pass from quadrature to two-mode squeezing and vice versa. Indeed, if we introduce new modes, characterized by new annihilation operators,

$$a_s = \frac{1}{\sqrt{2}}[a_1 + a_2], \quad a_d = \frac{1}{\sqrt{2}}[a_1 - a_2],$$

and the corresponding creation operators, then Hamiltonian (4) can be rewritten:

$$\hat{H} = i\hbar\frac{\Gamma}{2}[(a_s^+)^2 - (a_d^+)^2] + h.c.$$

It means that for the new modes s,d there will be quadrature squeezing; moreover, if $\Gamma > 0$, the squeezed quadratures will be $\hat{p}_s, \hat{q}_d$, while $\hat{q}_s, \hat{p}_d$ will be anti-squeezed.

4. Obtaining sub-Poissonian pulsed light.

Feedforward technique. Very similar to the heralded preparation of single photons is the so-called feedforward technique where one of the bright pulsed twin beams (2 in Fig.6) is registered by a detector and then the other one (beam 1) is transmitted only if the detector measures a certain number of photons, within some bounds. This detector of course does not need to count photons, but just integrates their number (one can say, measures the total energy). For beam 1, sub-Poissonian statistics will then be observed: $\Delta N^2 < \langle N \rangle$, or the Fano factor will be $F < 1$.

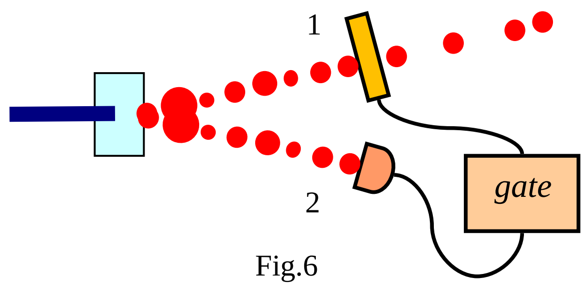

Fig.6

Sub-Poissonian light from SHG. The same effect can be achieved through the second harmonic generation (SHG). Indeed, due to SHG, pairs or groups of photons will be removed from the pump beam, and the pump will become anti-bunched. The same effect can be illustrated by a simple diagram on the phase plane (Fig.6). The initial pump state (blue) becomes squeezed in the amplitude, because intensity peaks are suppressed.

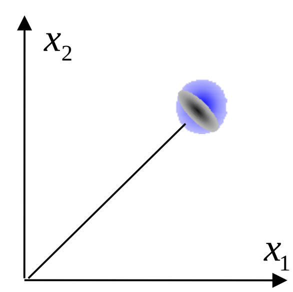

Fig.6

5. The role of losses.

All these bright-light nonclassical effects (quadrature squeezing, sub-shot-noise photon-number correlations, sub-Poissonian statistics) are very sensitive to losses. As an example, consider quadrature squeezing. Losses, or non-unity quantum efficiency η of a detector can be represented as a beamsplitter having transmissivity $t = \sqrt{\eta}$ and reflectivity $r = \sqrt{\eta - 1}$. Then, the annihilation operator a at the output will be expressed through the annihilation operator a_0 at the input as

$$a = \sqrt{\eta}a_0 + \sqrt{1-\eta}a_v,$$

where a_v is the vacuum annihilation operator. By taking the Hermitian and anti-Hermitian parts, we see that the quadratures will change as

$$q = \sqrt{\eta} q_0 + \sqrt{1-\eta} q_v,$$

$$p = \sqrt{\eta} p_0 + \sqrt{1-\eta} p_v.$$

For the variances, one can then write

$$\Delta q^2 = \eta \Delta q_0^2 + (1-\eta) \Delta q_v^2,$$

$$\Delta p^2 = \eta \Delta p_0^2 + (1-\eta) \Delta p_v^2.$$

Suppose that initially, the p-quadrature was squeezed. After the loss, its squeezing will be given by

$$\Delta p^2 / \Delta p_v^2 = \eta \Delta p_0^2 / \Delta p_v^2 + 1 - \eta.$$

Even if the squeezing was infinite initially, $\Delta p_0^2 = 0$, after loss it will be $1-\eta$, limited by the quantum efficiency.

This behavior of nonclassical features is different from the case of ‘faint’ light and nonclassicality signs based on the normalized correlation functions, which we considered in Lecture 7.

Home task: using the same model of loss, calculate the effect of 50% quantum efficiency on a beam of light with the Fano factor 0.1.

Books:

1. Bachor, Ralph, A Guide to Experiments in Quantum Optics.