ITMO UNIVERSITY

ITMO UNIVERSITY

Speedup problem for quantum walks and quantum annealing algorithms implementation

Alexander Alodjants

Alexander_ap@list.ru

Saint Petersburg

Plan of Talk

- Introduction to Classical Random (CRW) and Quantum Walks (QW)
- Machine Learning approach to speedup problem for Random Walks algorithms
- Results and conclusions

ITMO UNIVERSITY

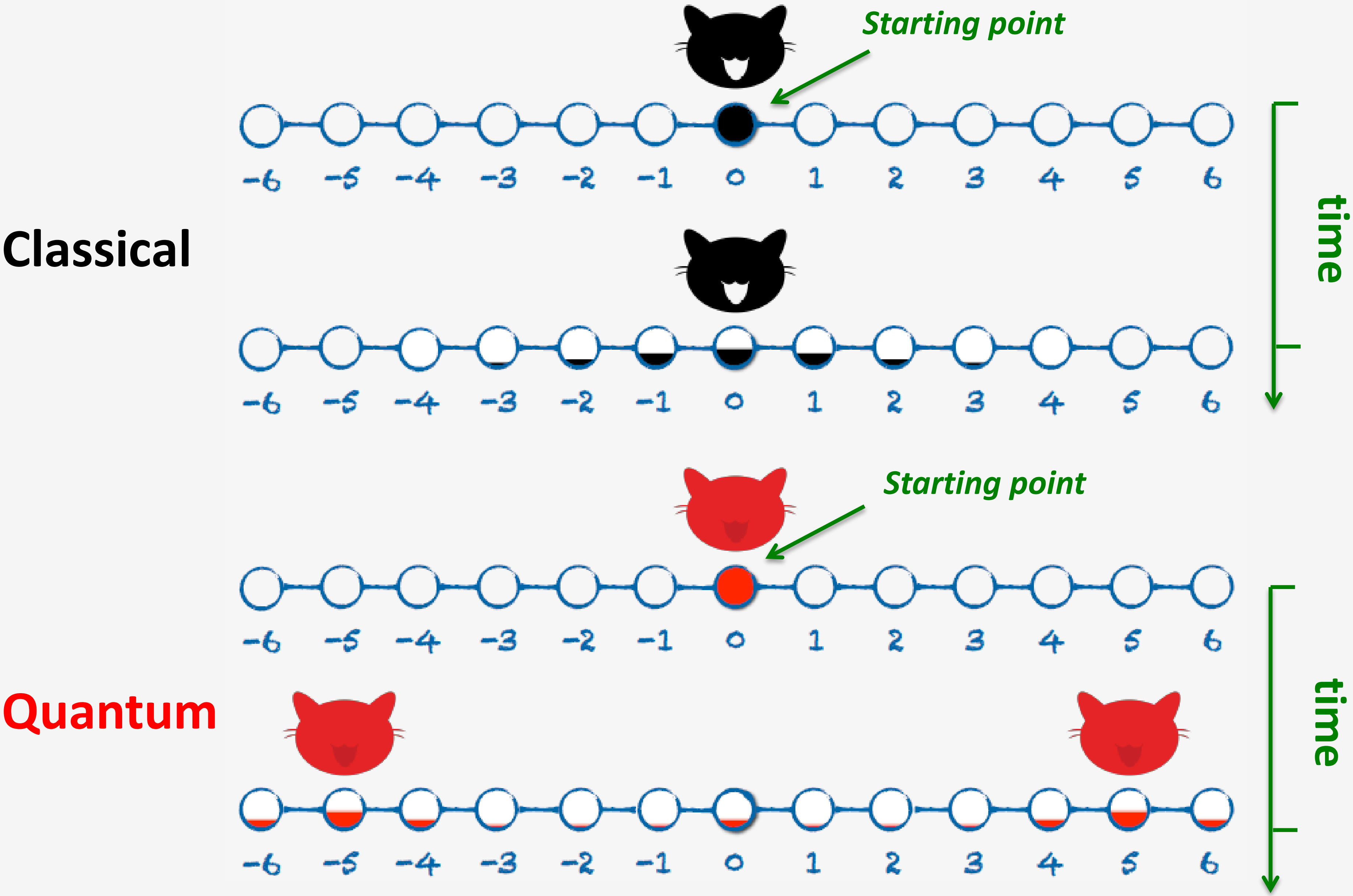
5100 PROGRAMM FOR ENHANCING THE COMPETITIVENESS

Welcome to Highly Non-classical University!

- 2 nd University in Russia and in top 150 of QS WUR in Computer Science & info Systems,
- Seven-time Champions in ICPC,
- 2 nd in 5-100 Federal target program ranking for Russian Universities.

RoboCup Open Russia-2019

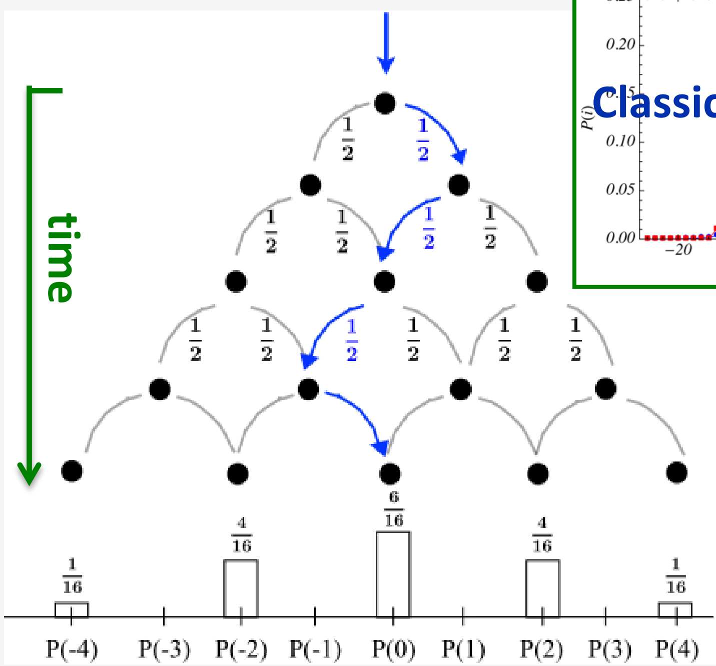
Classical and quantum walks paradigm

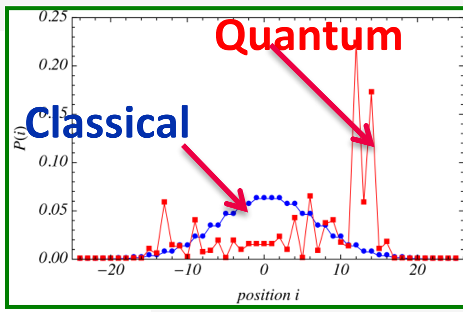


Classical and Quantum Discrete-time Walks

Classical random - Galton Board

N. Chernov , D. Dolgopyat, PR . 99, 030601 (2007).



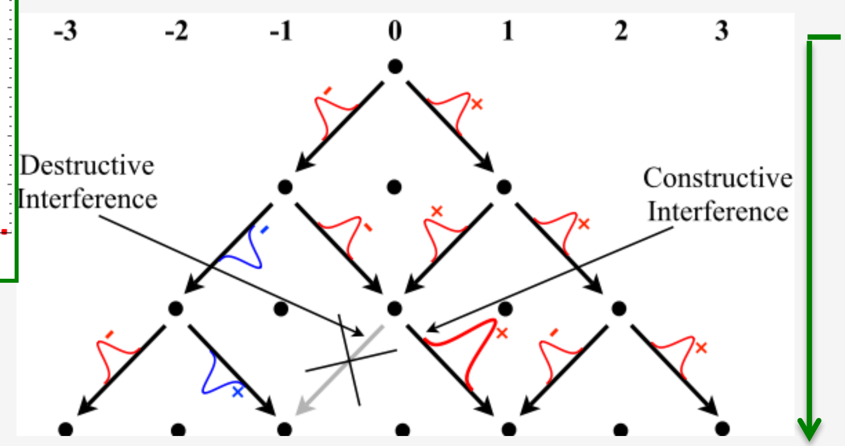


Quantum walks – Coined QW

Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A 48, 1687–1690 (1993).

Translation operator

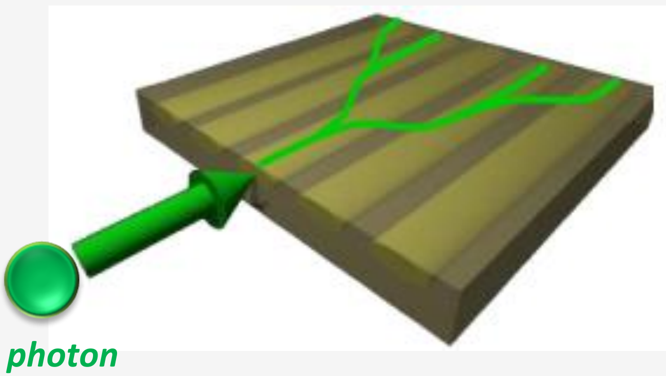
$$S = \sum_{x\in\mathbb{Z}} \left(|x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| + |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow|\right).$$

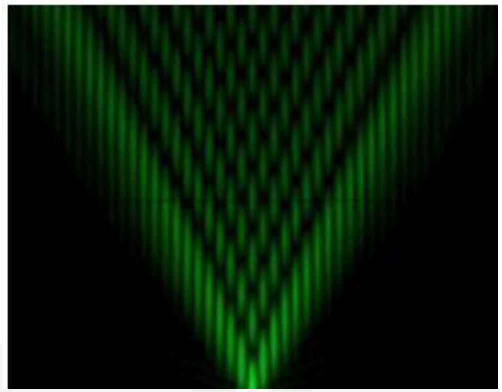


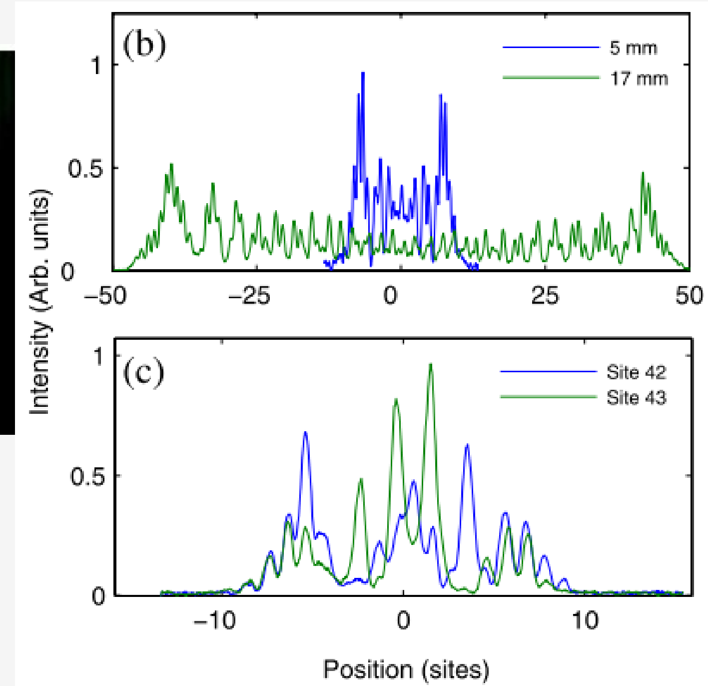
The wave packets symbolize the probability Amplitudes for the states to be occupied. The ± -signs correspond to the polarization state. Positive amplitudes are in red color, negative amplitudes are blue.

Experiment on QW with photons

Optical coupled waveguides







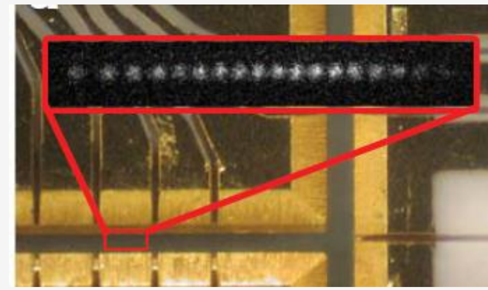
Y. Silberberg, et al, PRL 100, 170506 (2008)

The observed output pattern of light intensity after short (blue) and long (green) propagation in a periodic lattice. This well-known pattern is one of the hallmarks of the ballistic propagation of QWs. (c) Output patterns of light intensity resulting from injection of light into two adjacent single waveguides (sites 42 and 43) of a disordered lattice. The different patterns observed demonstrate the high sensitivity of the QW to the initial conditions in this case.

WQ as a tool box

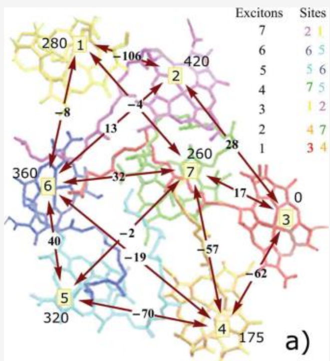
✤ **Quantum (search) algorithms. Speedup is approaches to** $O(\sqrt{N})$

N. Shenvi, J. Kempe, K. B. Whaley, Phys. Rev.A 67, 052307 (2003)



✤ **Quantum computing**

A. M. Childs, PRL 102, 180501 (2009); Science 339, 791 (2013)

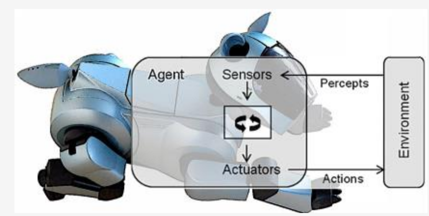


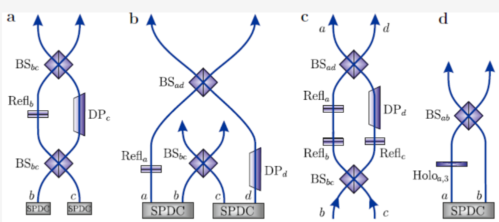
✤ **Quantum transport in biophotonics (FMO complexes)**

M. Mohseni, et al., The J. of chemical physics 129, 174106 (2008)

✤ **Quantum AI**

G.D. Paparo, et al., Phys. Rev. X 4 (3), 031002 (2014)

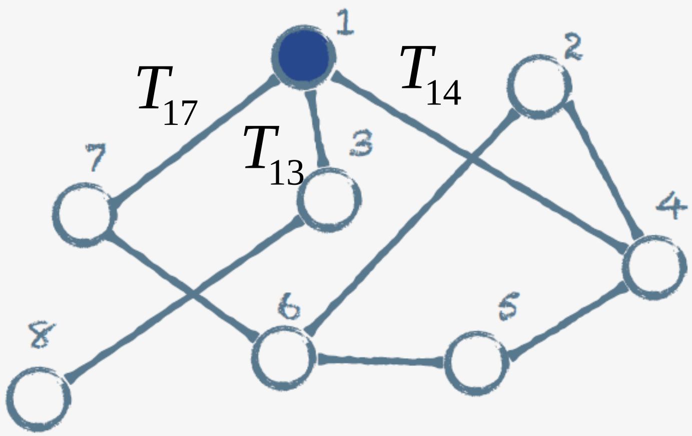




✤ **Design of new quantum experiments**

Alexey A. Melnikov, et al PNAS *115(6):1221* (2018)

Classical Random Walks (CRW)



T_{ij} Transition matrix elements that defines dynamics

$T_{ij} = T_{ji}$ $T_{ii} = 0$ (Without loops)

Lets ϵ probability of elementary transition

Probability distribution changes as

$$p(t) - p(t-1) = \epsilon(T - I)p(t-1)$$

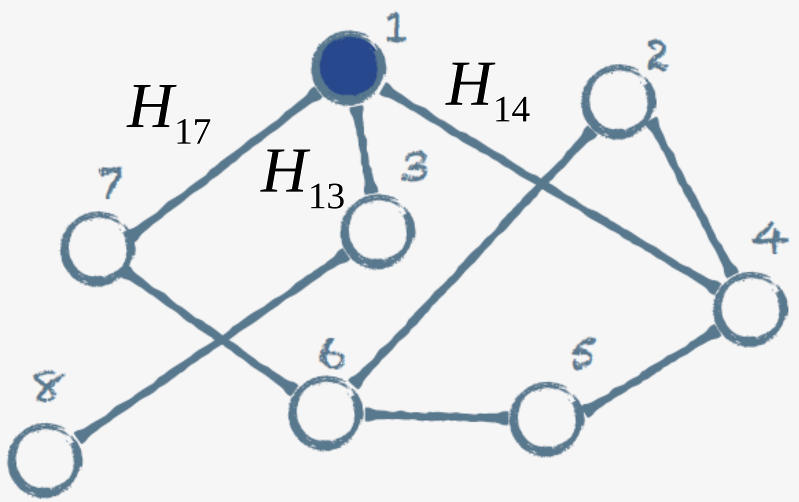
Discrete time random walks

$$p(t) = Tp(t-1)$$

Continuous time random walks, $\epsilon \to 0$

$$\frac{d}{dt}p(t) = (T - I)p(t)$$

Quantum Random Walks (QW)



H_{ij} Is Hamiltonian matrix elements

$$H_{ij} = H_{ji}, \quad H_{ii} = 0$$

Hopping Hamiltonian is

$$H = \hbar J \sum_{(i,j)} |i\rangle\langle j|$$

Discrete time random walks

$$H = H_{\text{particle}} \otimes H_{\text{coin}}$$

$$\text{Shift}\,|x\rangle|\uparrow\rangle = |x+1\rangle|\uparrow\rangle,$$

$$\text{Shift}\,|x\rangle|\downarrow\rangle = |x-1\rangle|\downarrow\rangle$$

$$|\psi(t)\rangle = \text{Shift}\,(I \otimes \text{Had})\,|\psi(t-1)\rangle$$

Continuous time random walks

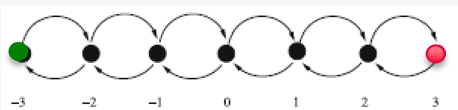
$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

A. M. Childs, Comms in Math. Phys. 294, 2 (2010)

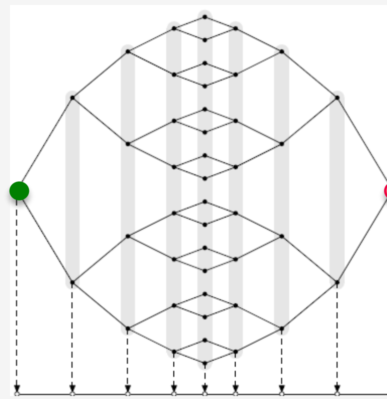
Speedup problem

QW are quadratically faster than CW on:

- line

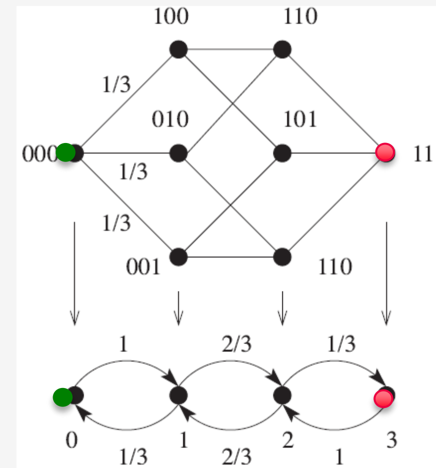


- glued tries



A. Ambainis, Int. J. of Quant. Info. Vol. 1, 507 (2003)

- hypercube

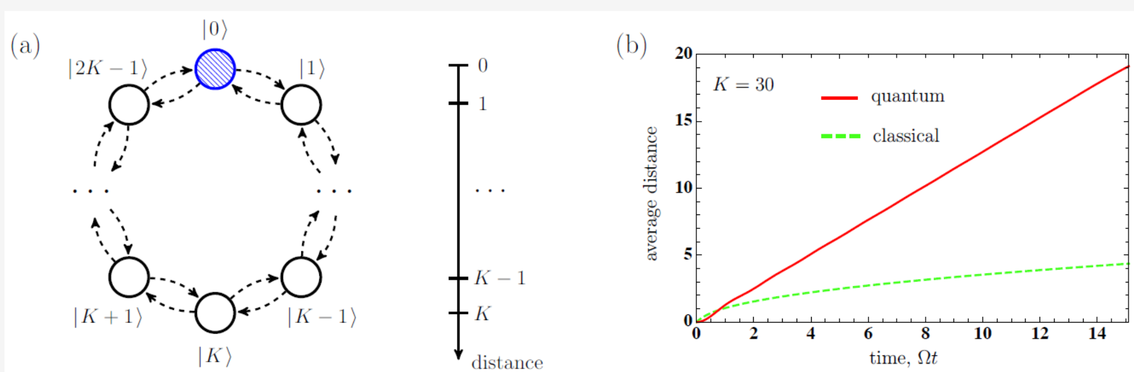


J Kempe, Contemp. Phys., 44:4, 307 (2003)

● is starting point

● Is final point

- circles



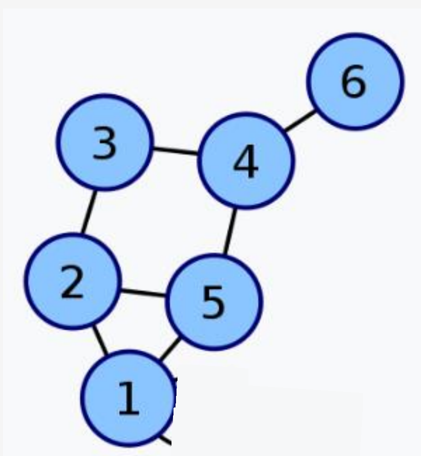
A. A. Melnikov, A. Alodjants, L. Fedichkin, Hitting time for quantum walks of identical particles, SPIE, 2018

However!

- Positions of input and output points are important,
- There exist “dark” areas (due to quantum destructive interference) where particle disappear,
- Specifics of quantum measurement.

How we can detect speedup of random walk for arbitrary graph?

Final vertices



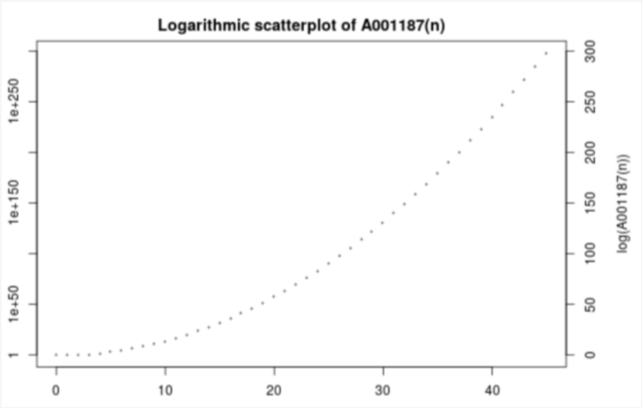
Initial vertices

Adjacency (A)- matrix A_{ij}

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

We consider:
- Undirected graphs,
- Connective graphs

Number of graphs



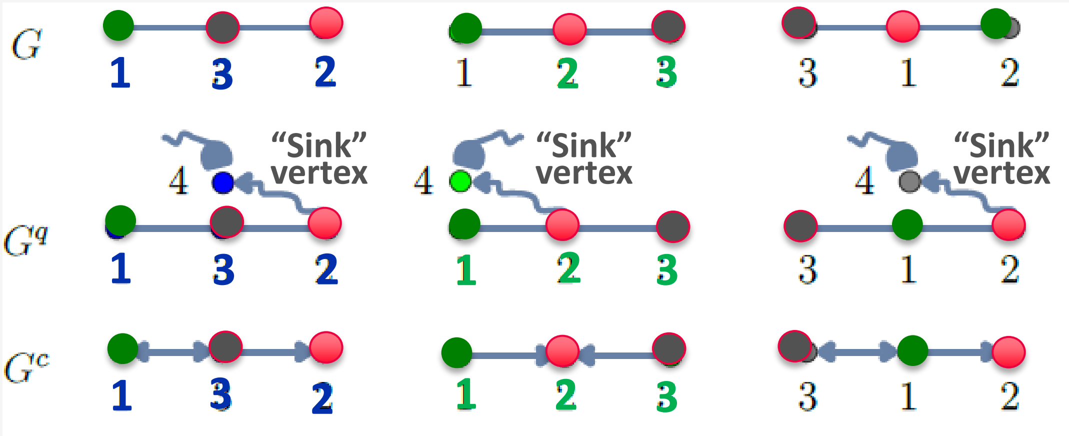
Number of vertices

The On-Line Encyclopedia of Integer Sequences® (OEIS®)

Alexey A. Melnikov, Leonid E. Fedichkin, and Alexander Alodjants, arXiv:1901.10632v1 [quant-ph] 30 Jan 2019

Toy Model of QWs

Quantum is faster **Classical is faster**



Simple graph sample on the line

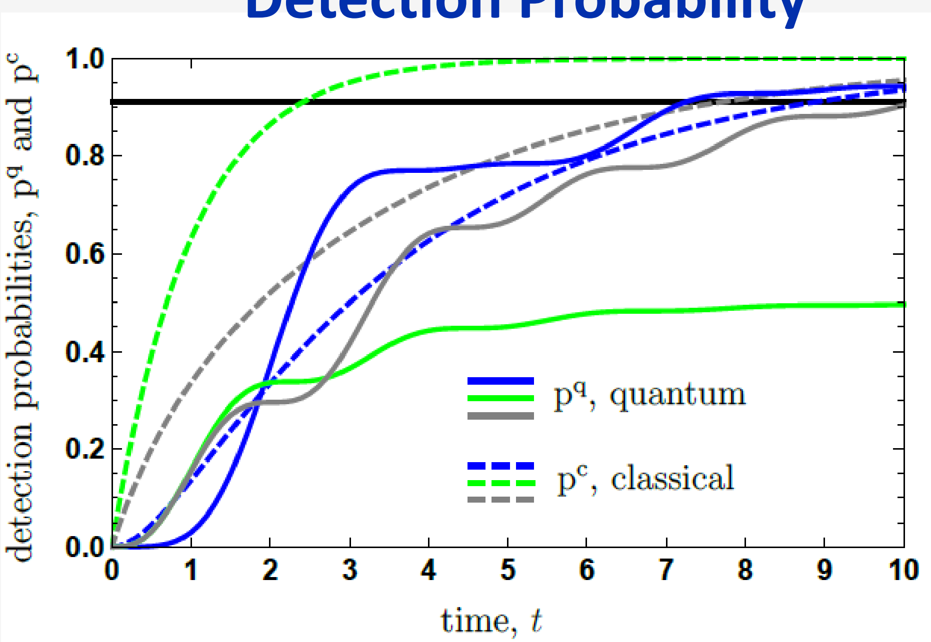
QW

CRW $\mathrm{p}(t) = \mathrm{e}^{(T-I)t}\mathrm{p}(0) = \mathrm{e}^{-t}\mathrm{e}^{Tt}\mathrm{p}(0),$

$\mathrm{p}(0) = (1, 0, \ldots, 0)^{\mathrm{T}}$ is a probability vector

Corresponding to a classical particle initially located in v =1.

Detection Probability



The Master Equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\frac{i}{\hbar}[H, \rho(t)] + \gamma\left(L\rho(t)L^{\dagger} - \frac{1}{2}\{LL^{\dagger}, \rho(t)\}\right)$$

$\mathcal{H} = \hbar A^q.$

A^q is $(n+1) \times (n+1)$ matrix

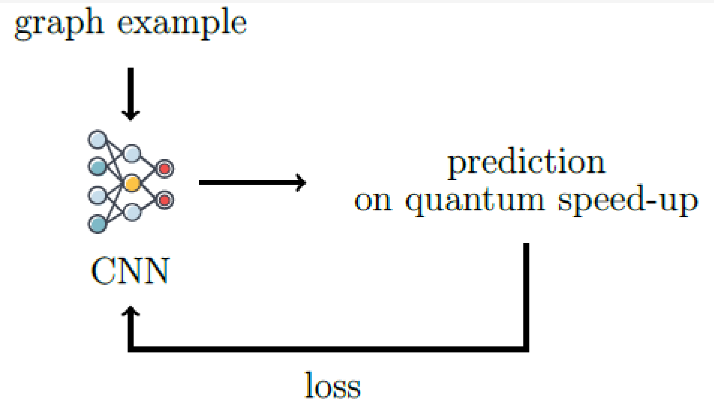
- γ characterizes decay from the final state **2** to the output ("sink") vertices **4**

$L = |n+1\rangle\langle n|$

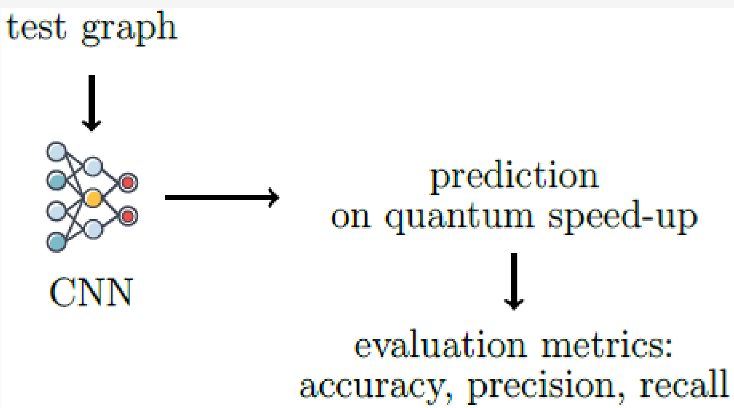
ITMO UNIVERSITY

The ML approach for detecting quantum speedup

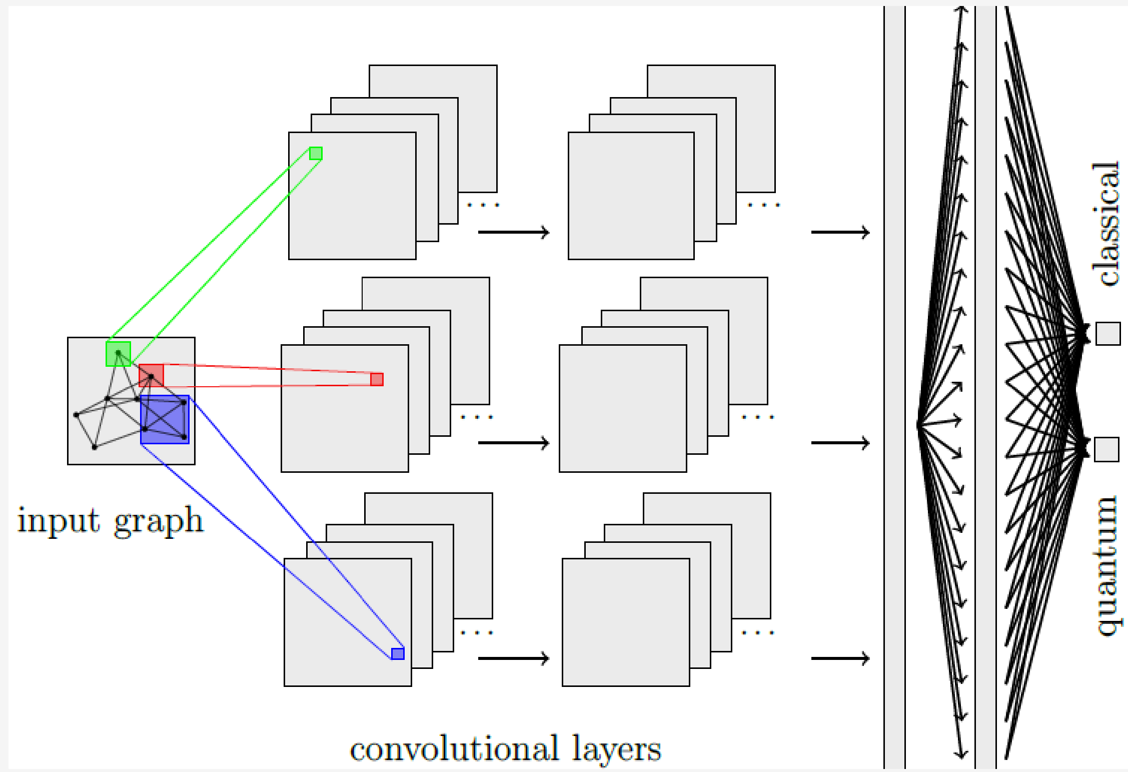
Training of convolutional neural network



Testing of convolutional neural network



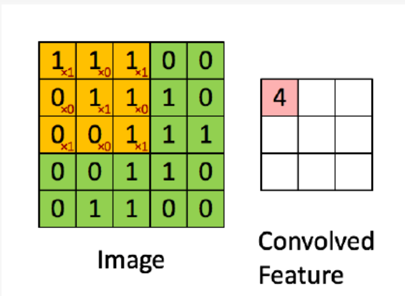
Example of CNN architecture

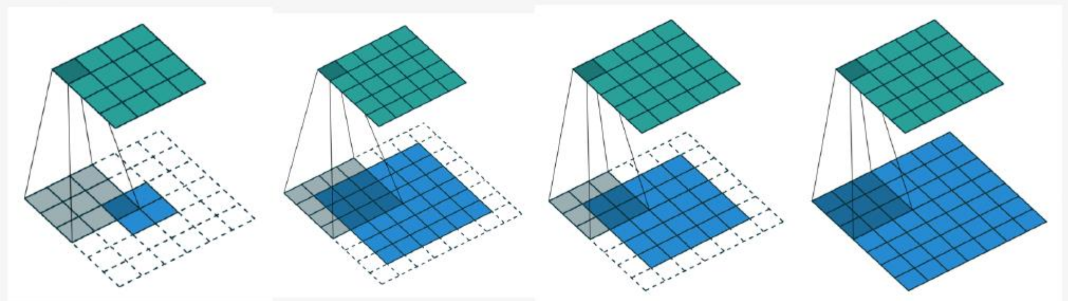


The number of filters are taken from set of experiences

Filtering

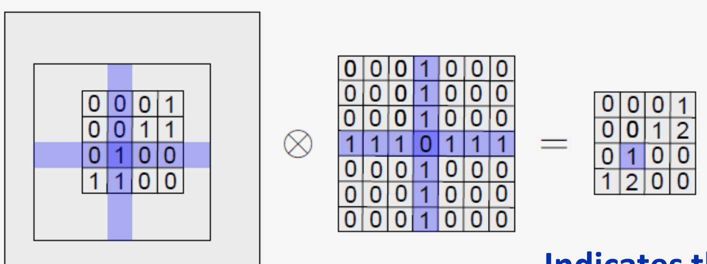
Convolution procedure as usual





Convolutional filters and procedure that we use

Edge to edge filtering

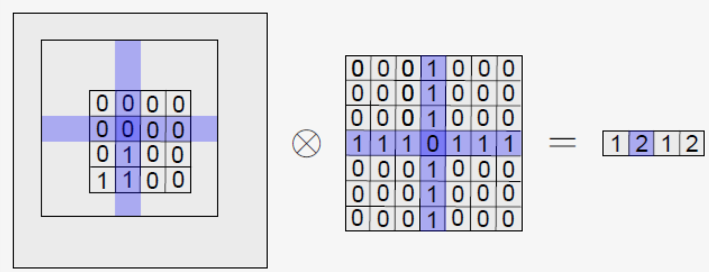


Initial A-matrix

filter

Indicates the number of edge-to edges

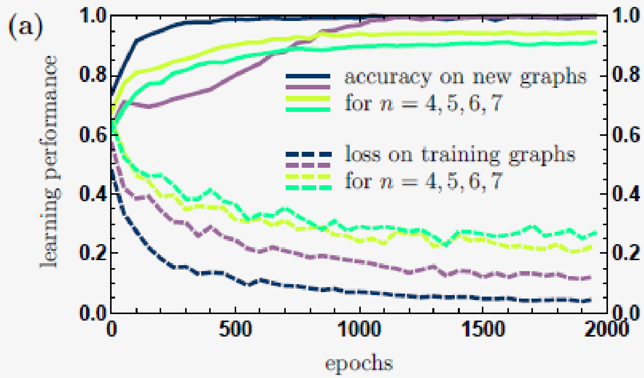
Edge to vertices filtering

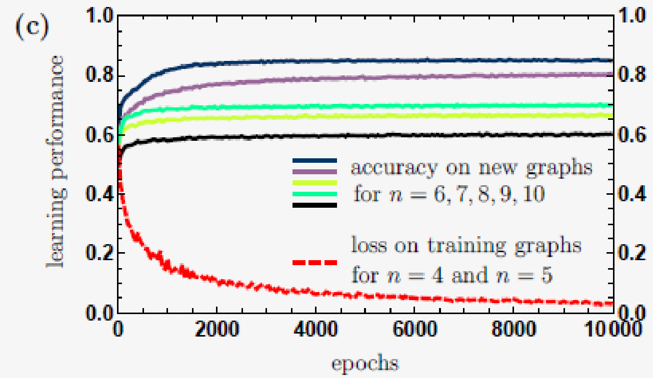


"Halve" of A-matrix

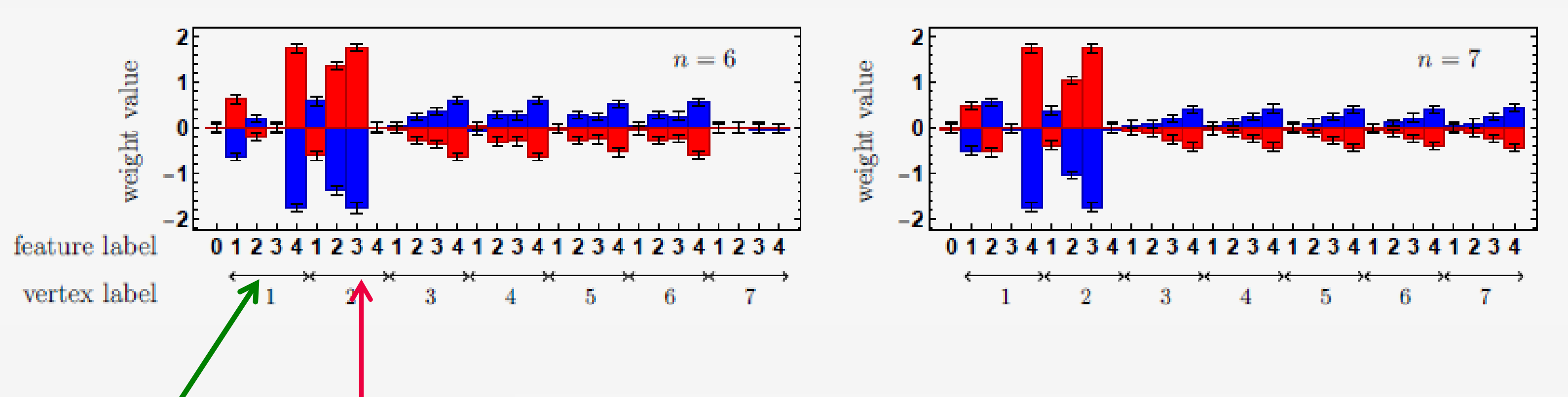
Learning performance

Accuracy of prediction with test samples





These results are the average over 100 independent CNNs Losses are defined through cross-entropy

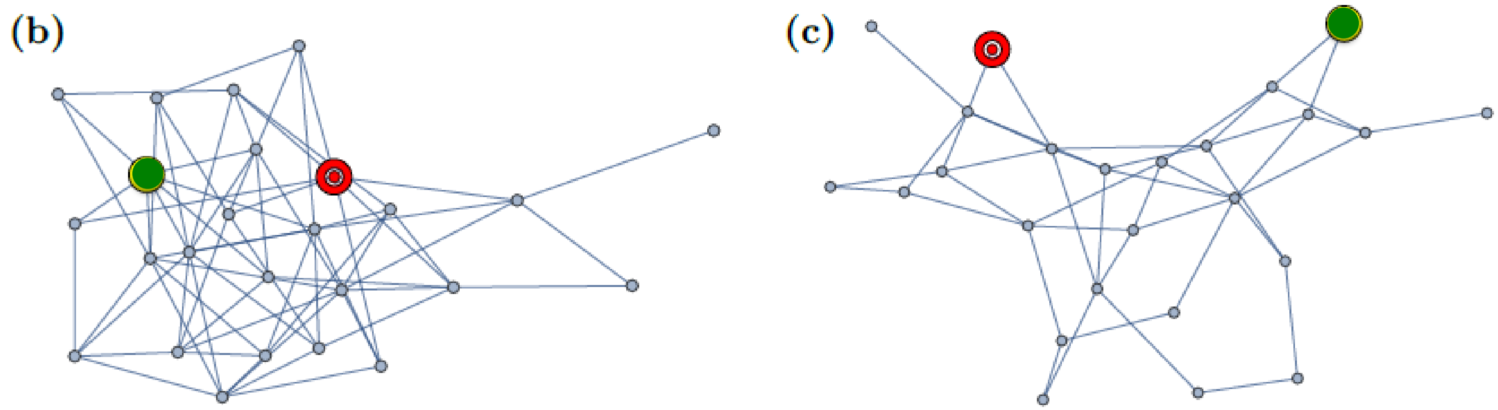


Initial vertices **Final vertices**

contribution to "quantum" class   contribution to "classical" class

Mean squared deviation is shown as a vertical line for each bar. The zeroth component of the feature vector is the bias. The first feature for each vertex corresponds to the number of edges this vertex has. The second feature to the total number of neighboring edges of all edges leading to the vertex. The third feature gives one if the vertex is connected to the initial vertex by an edge, and zero otherwise. The fourth feature does the same relative to the target vertex.

CNN predictions for large graphs



Classical walker is faster

Quantum walker is faster

Conclusions

- ✤ We propose convolutional neural network paradigm for speedup detection of random walks on the graphs,
- ✤ Detection is 90% and more for test graphs taken on the line,
- ✤ Training in small graphs allowed the neural network to build a model which works on graphs of higher dimension,
- ✤ Neural network recognized about 25% of "quantum" graphs using random graphs samples. Moreover, when the network said "quantum", it was right in 90% cases

Publications

- ✓ Alexey A. Melnikov, Leonid E. Fedichkin, and Alexander Alodjants, **Detecting quantum speedup by quantum walk with convolutional neural networks** arXiv:1901.10632v1 [quant-ph] 30 Jan 2019

For quantum annealing pls, look

- ✓ M. Lebedev, D. Dolinina, K.B. Hong, T. Lu, A.V. Kavokin, A.P. **Alodjants**. **Exciton-polariton Josephson junctions at finite temperatures** // Scientific Reports - 2017, Vol. 7, pp. 9515

Thank you for attention!