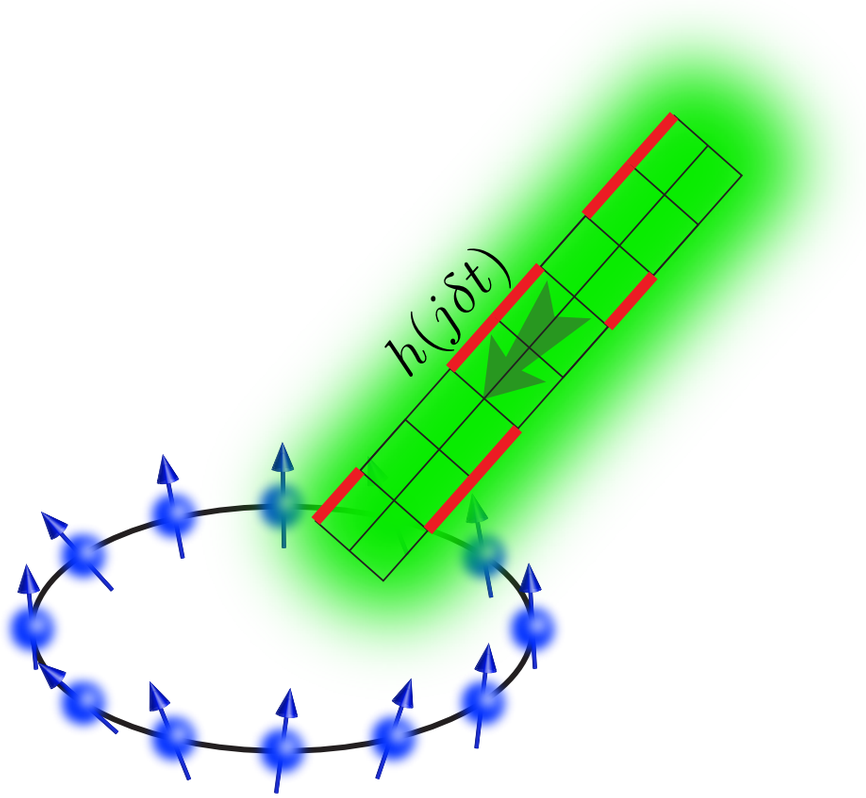
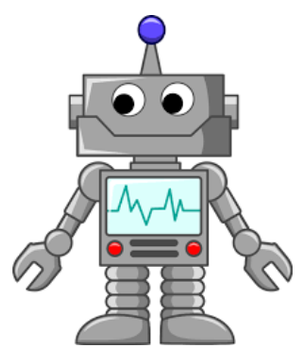
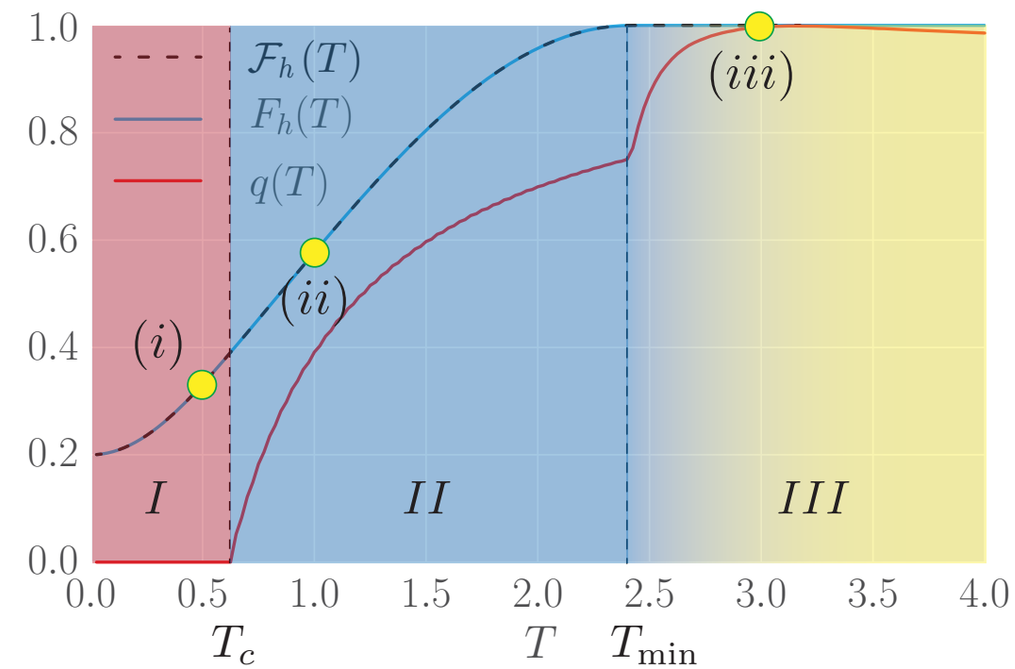
Reinforcement Learning in Different Phases of Quantum Control



nonintegrable
many-body spin chain





phase transitions
in the control landscape

PRX 8 031086 (2018)
PRA 97, 052114 (2018)
PRL 122, 020601 (2019)

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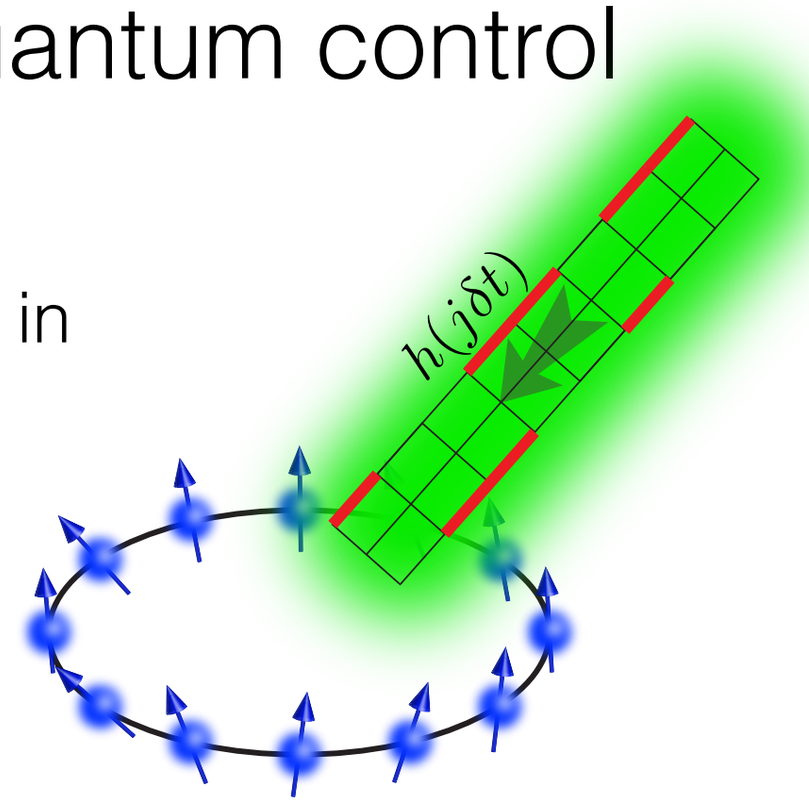
in this talk:
Reinforcement Learning (RL) for quantum control

→ **Example**: use RL to prepare many-body states in a nonintegrable spin chain

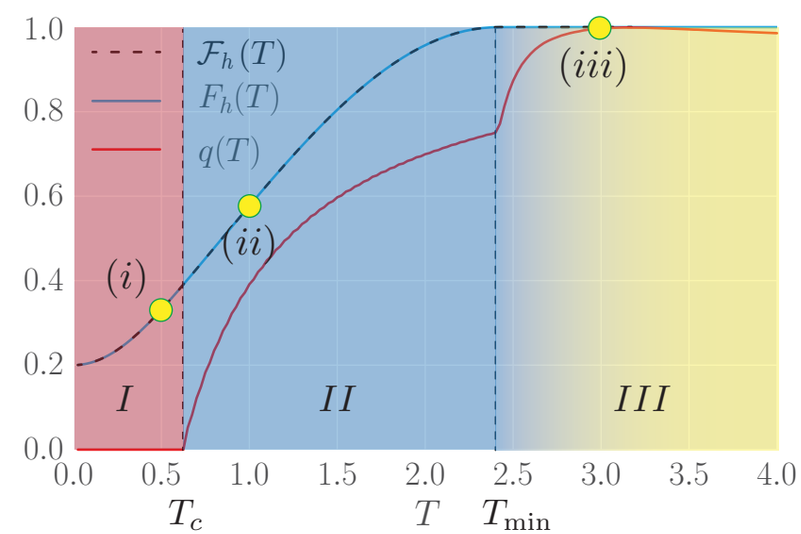
- RL and quantum control
- variational theory for optimal protocols



MB et al, PRX 8 031086 (2018)

→ **Phase transitions in the control landscape:**

- how "hard" is it for the RL agent to Learn?



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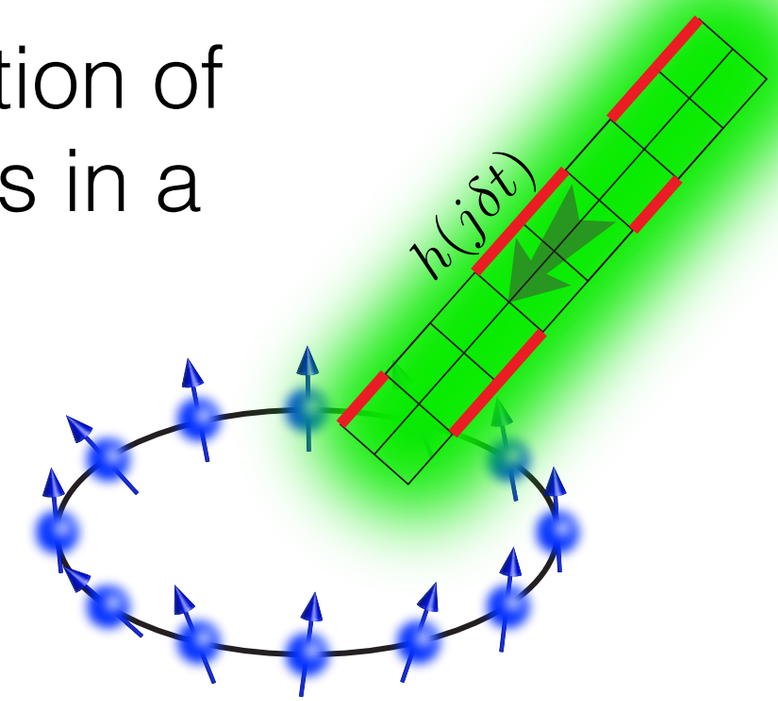
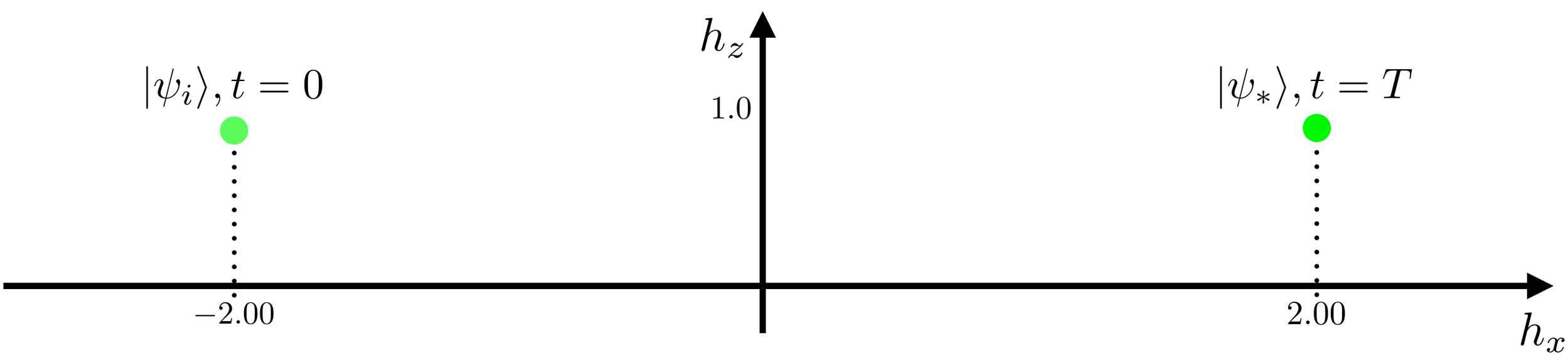
Example:

use RL for autonomous preparation of paramagnetic many-body states in a **nonintegrable spin chain**



$$H(t) = -\sum_{j=1}^{L} S^z_{j+1} S^z_j + \underbrace{h_z}_{=1} S^z_j + h_x(t) S^x_j$$

→ initial $|\psi_i\rangle$ and target $|\psi_*\rangle$ states are (paramagnetic) GS at:



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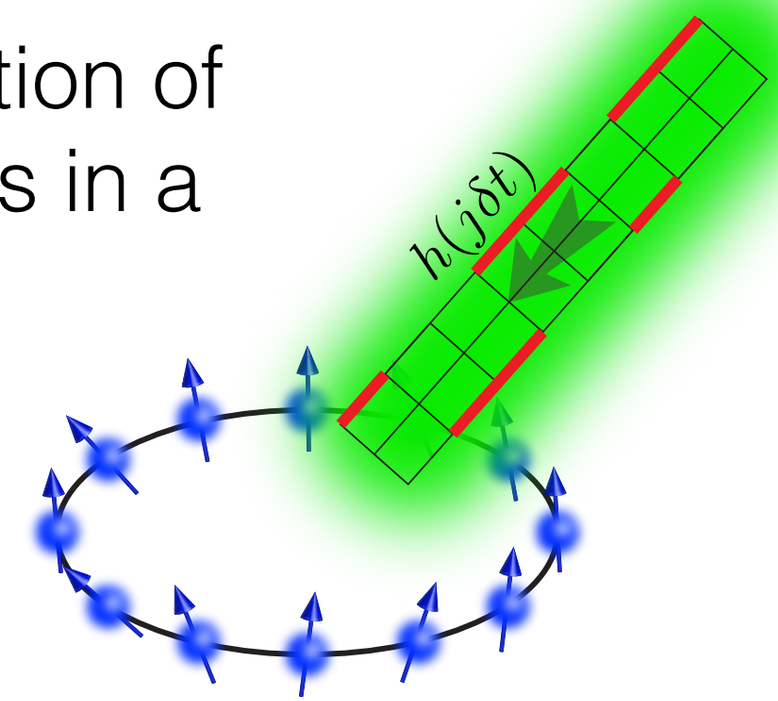

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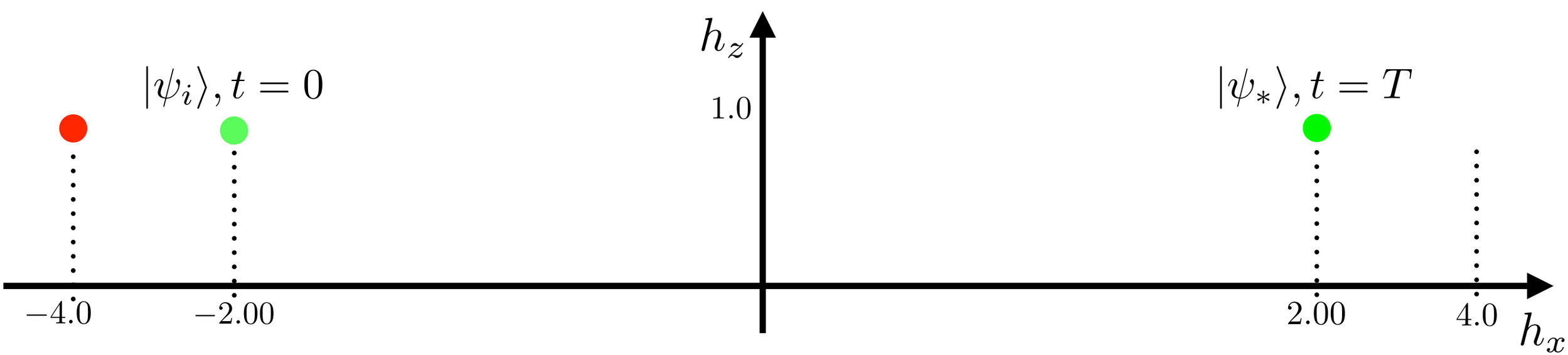

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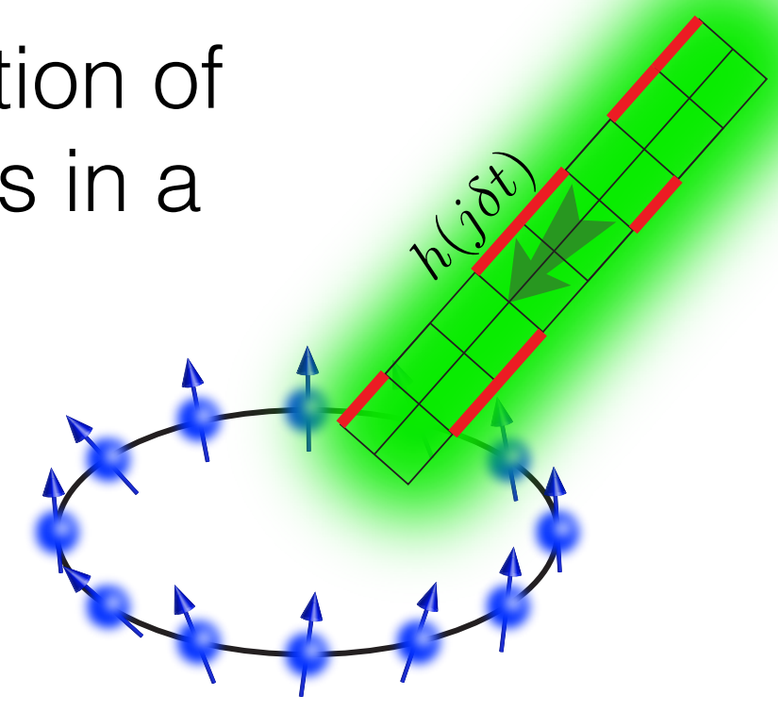

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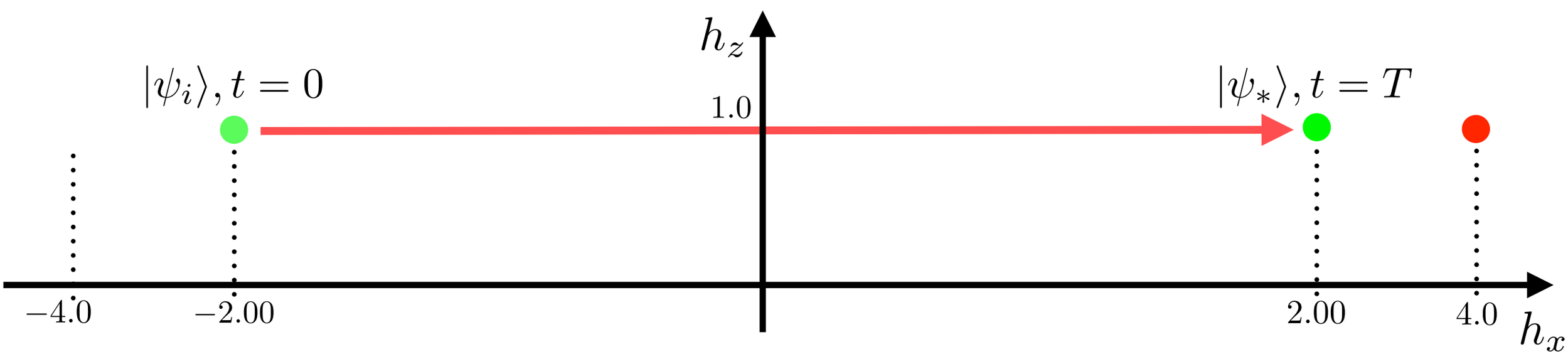

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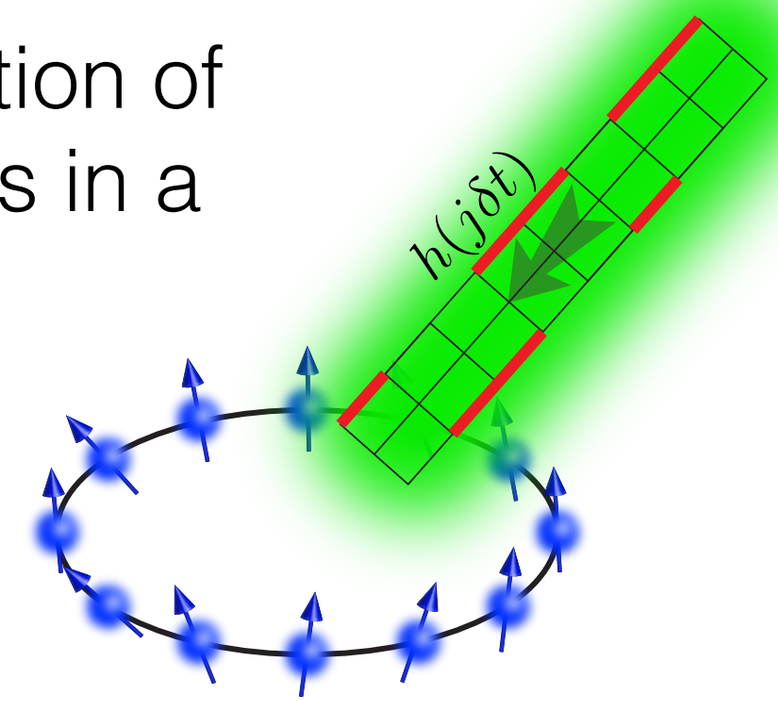

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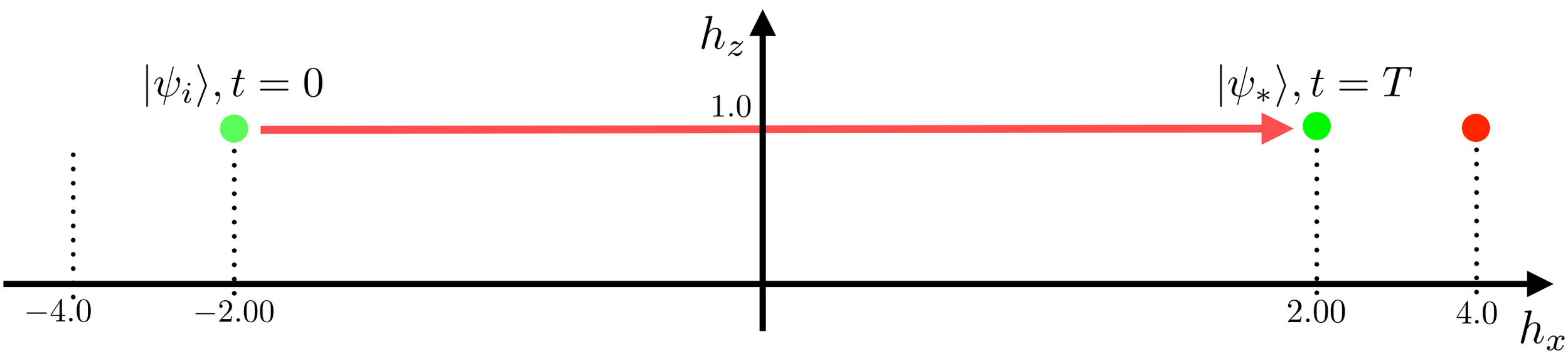

for example: $h_x(t) = [+4, +4, -4, +4, -4, -4, \ldots]$

$$\text{fixed \# of bangs, i.e. fixed total time } T$$

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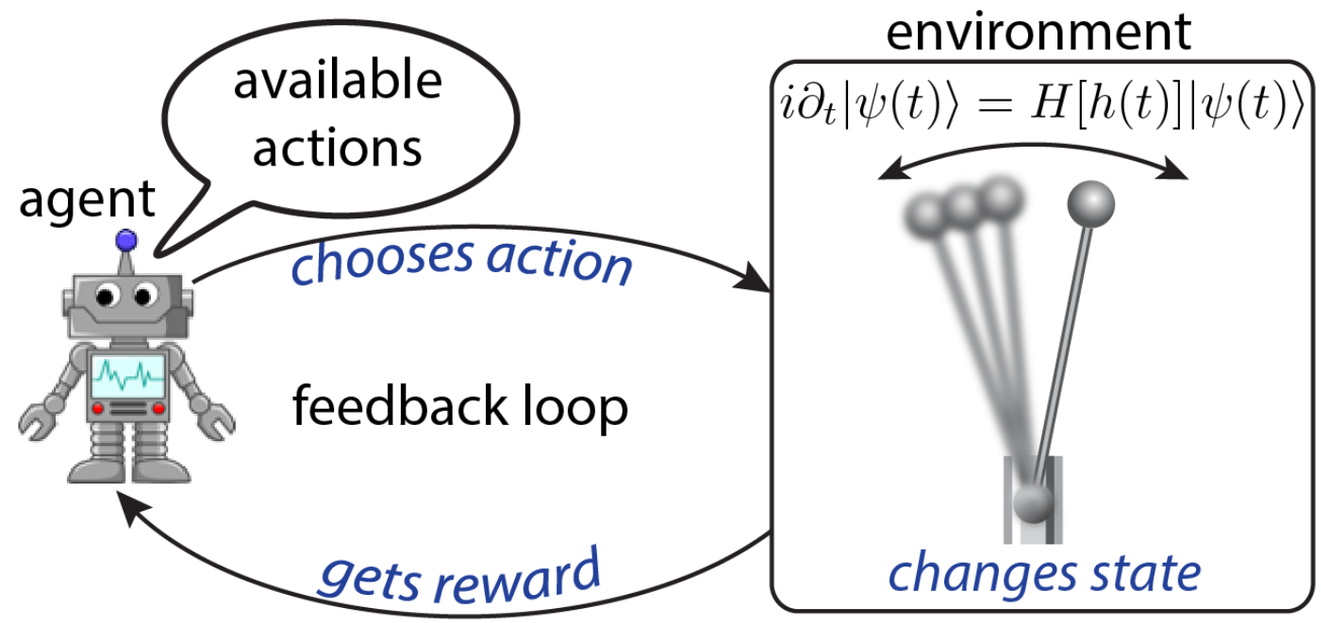
Quantum Control as an RL Problem

→ RL formalism



Sutton and Barto, *Reinforcement Learning: an Introduction*, MIT press

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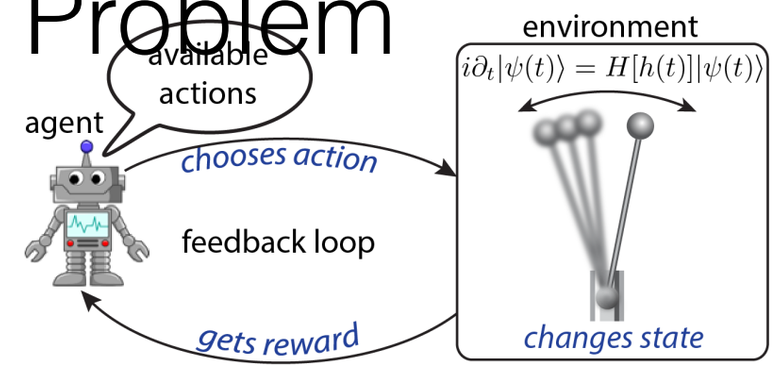

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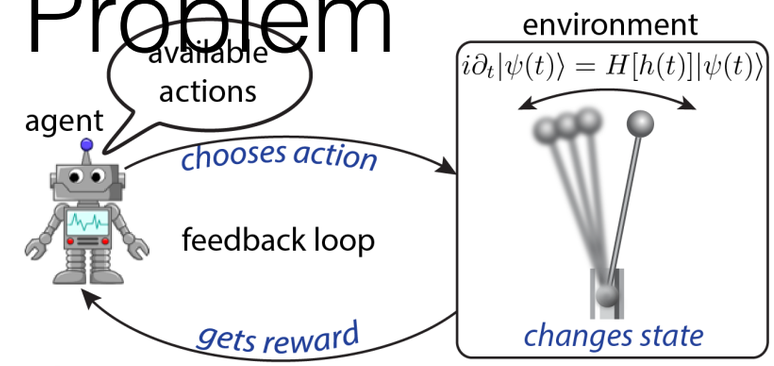

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$$s = h_x(t) = [+4, +4, -4, +4, -4, -4, \ldots]$$

$$\{|\psi(t)\rangle : |\psi(t)\rangle = U_h(t,0)|\psi_i\rangle\} \hat{=} \{h(t) : |\psi_i\rangle\}$$

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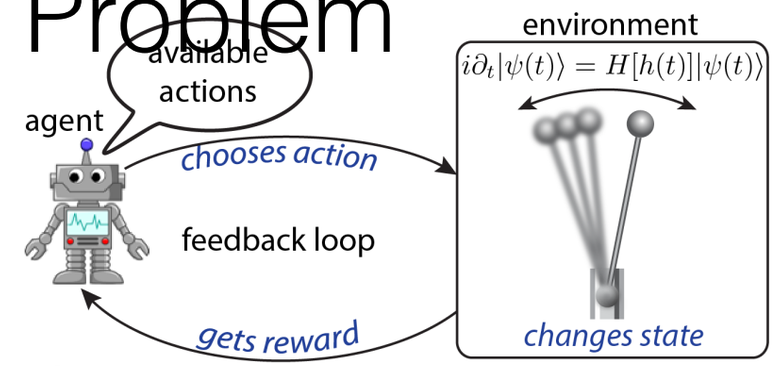

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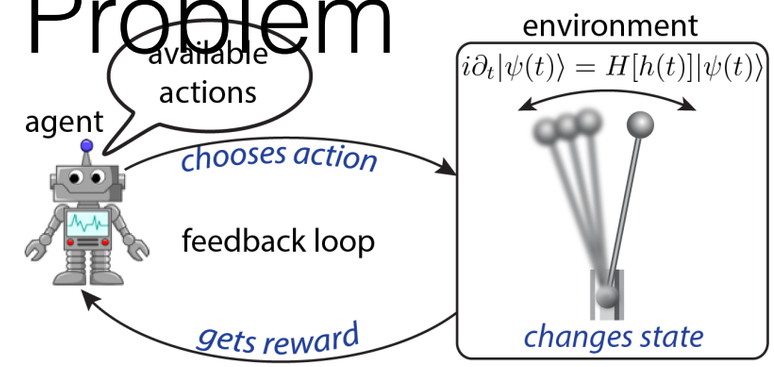

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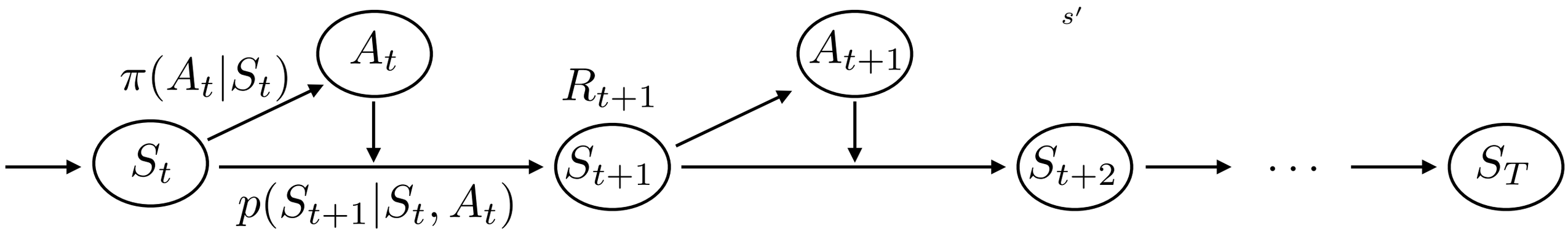
→ RL as Markov decision process

$$R_{t+1} = \sum_{s'} p(s'|S_t, A_t) r(s', S_t, A_t)$$



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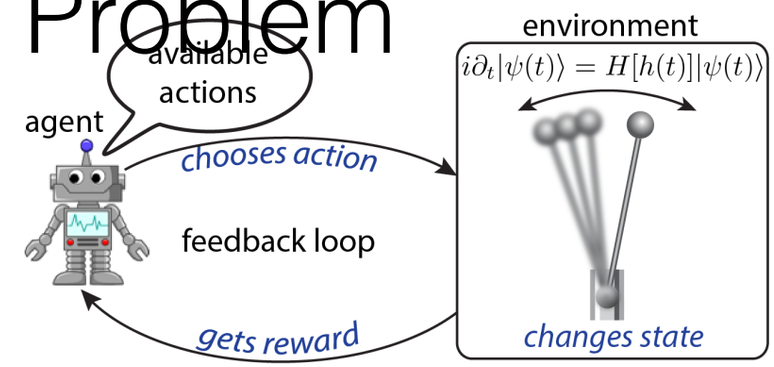

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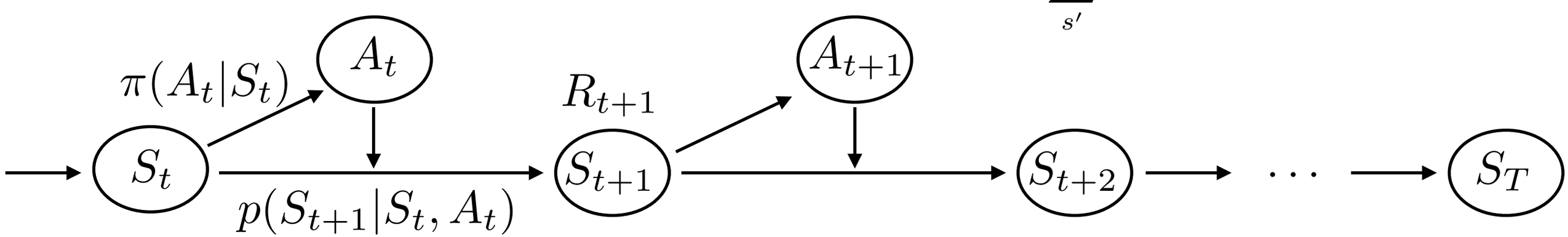

→ RL **objective**: maximize total *expected return* from step t onwards

$$Q(s,a) = \mathbb{E}_{a\sim\pi(a|s)}[R_{t+1} + \cdots + R_{t_f}|S_t = s, A_t = a]$$

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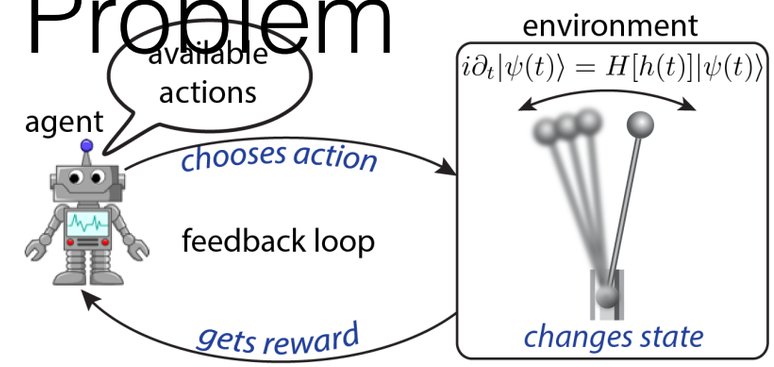

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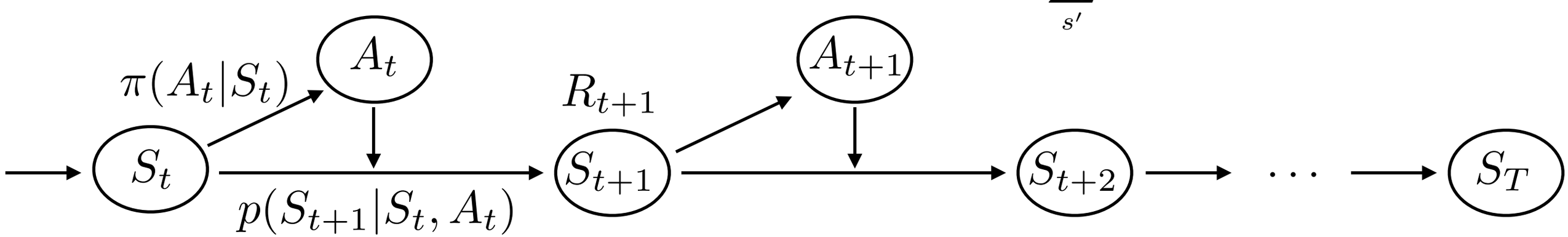

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→ Watkins Q-Learning (basic idea):

$$Q_{\text{new}}(S_t, A_t) = Q_{\text{old}}(S_t, A_t) + \alpha[R_{t+1} + \max_a Q_{\text{old}}(S_{t+1}, a) - Q_{\text{old}}(S_t, A_t)]$$

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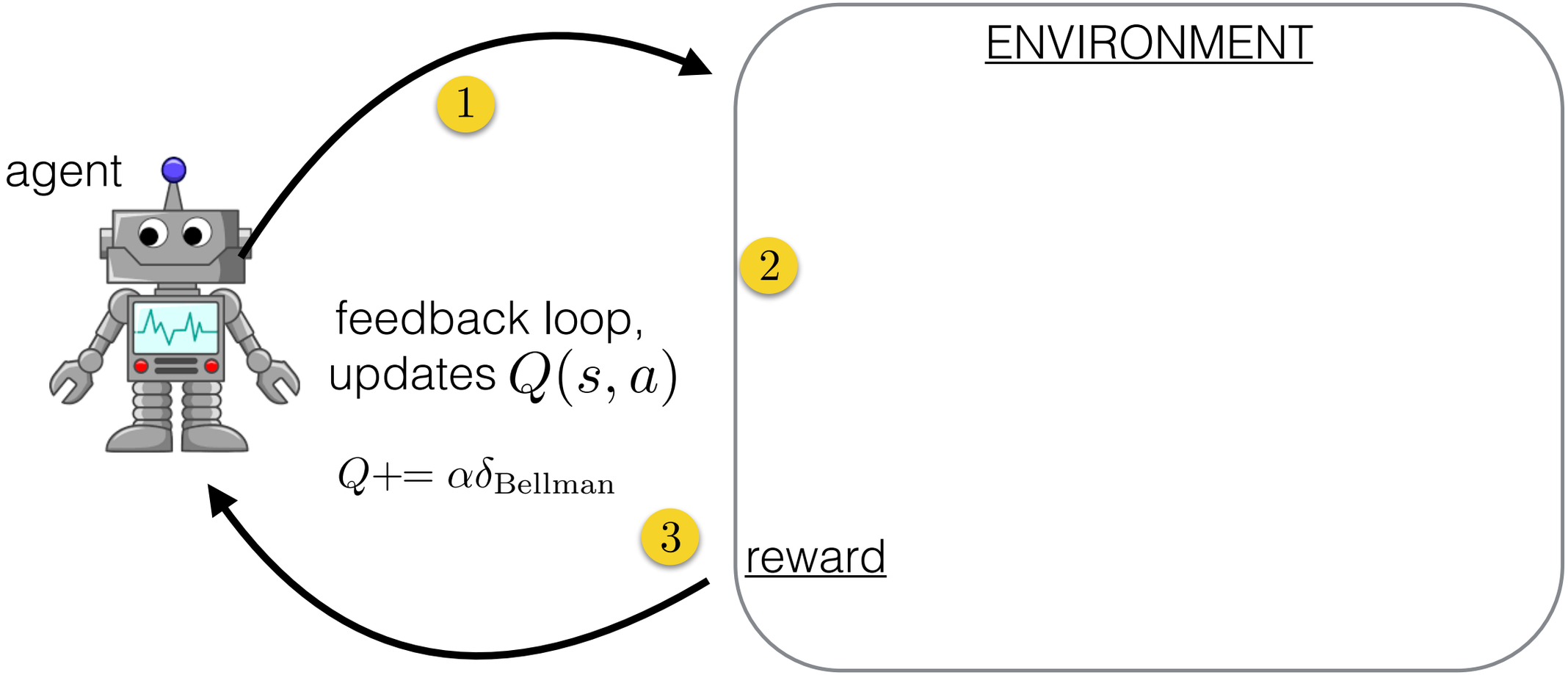
RL Applied to Quantum State Preparation



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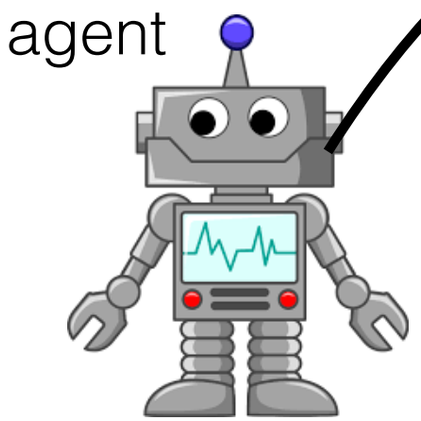

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feedback loop,
updates $Q(s,a)$

$Q \mathrel{+}= \alpha \delta_{\text{Bellman}}$

3

ENVIRONMENT

2

$$H(t) = H_0 + H_{\text{ctrl}}(t)$$

$|\psi_i\rangle$: GS of H_0

$|\psi_*\rangle$: target state

$$i\partial_t |\psi(t)\rangle = H(t)|\psi(t)\rangle \; t \in [0, T]$$

reward

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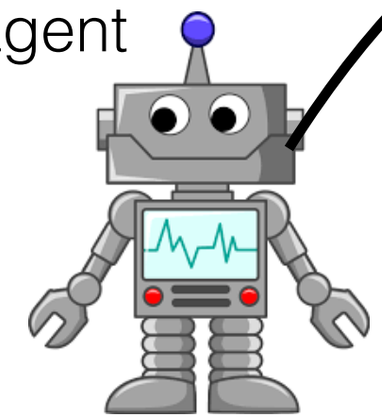
RL Applied to Quantum State Preparation

agent



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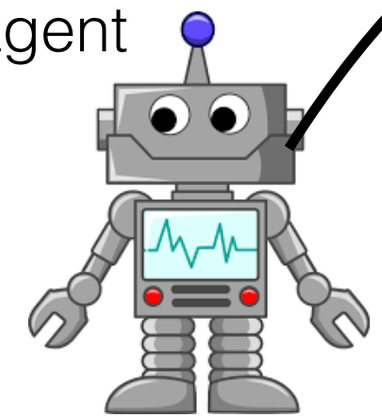

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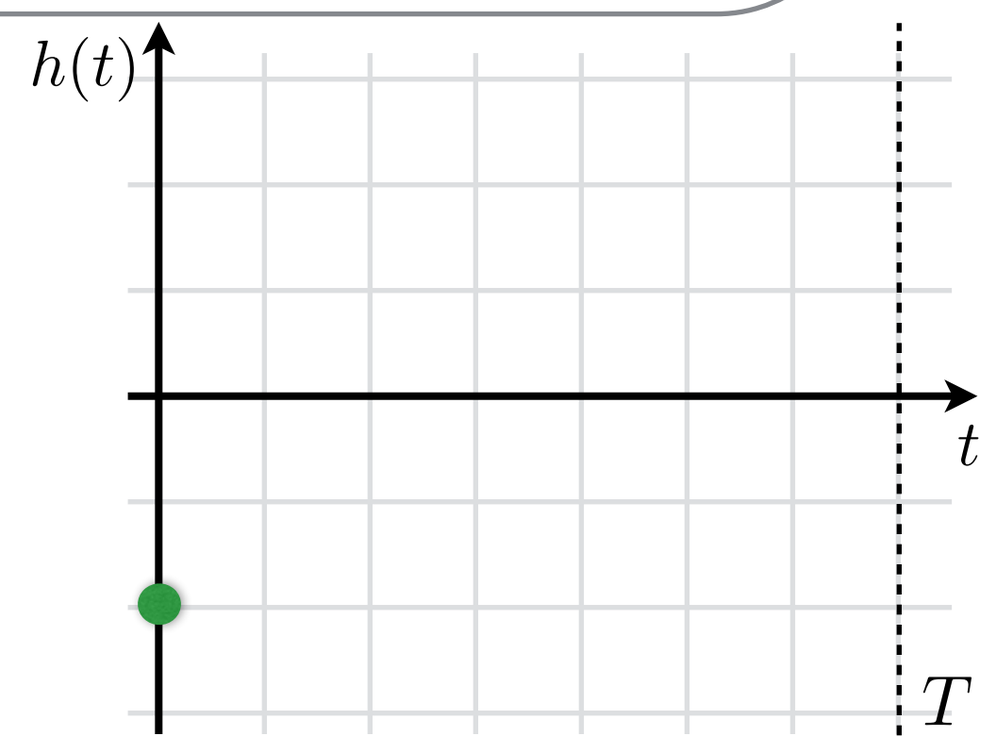

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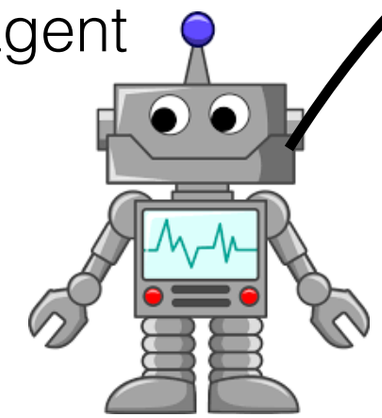

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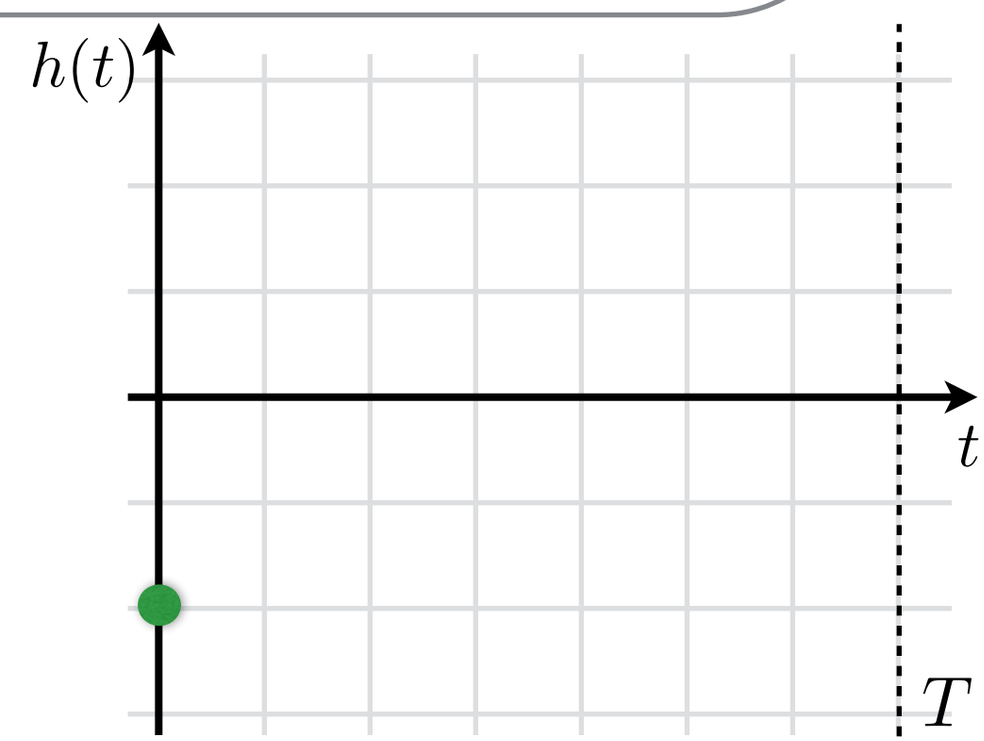

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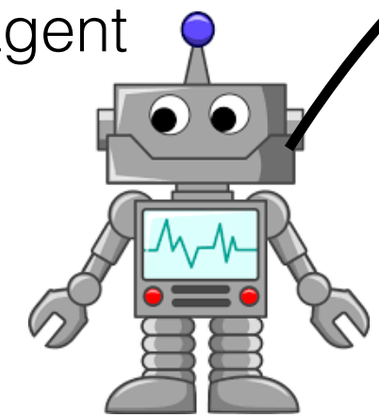

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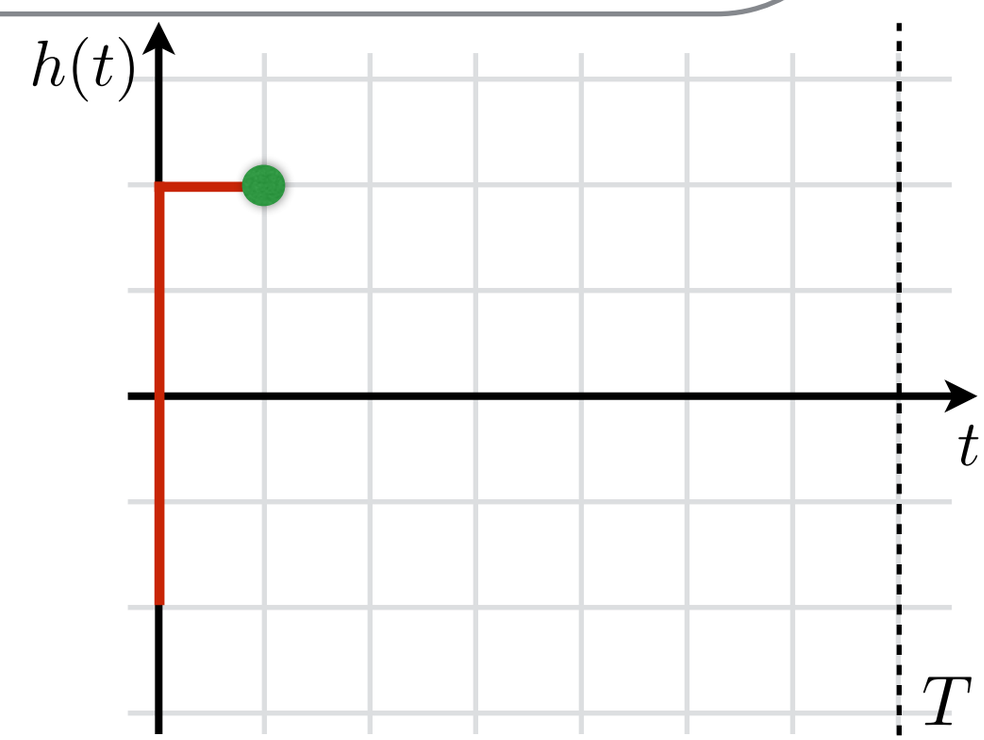

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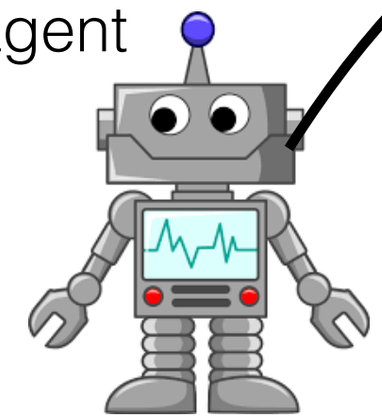

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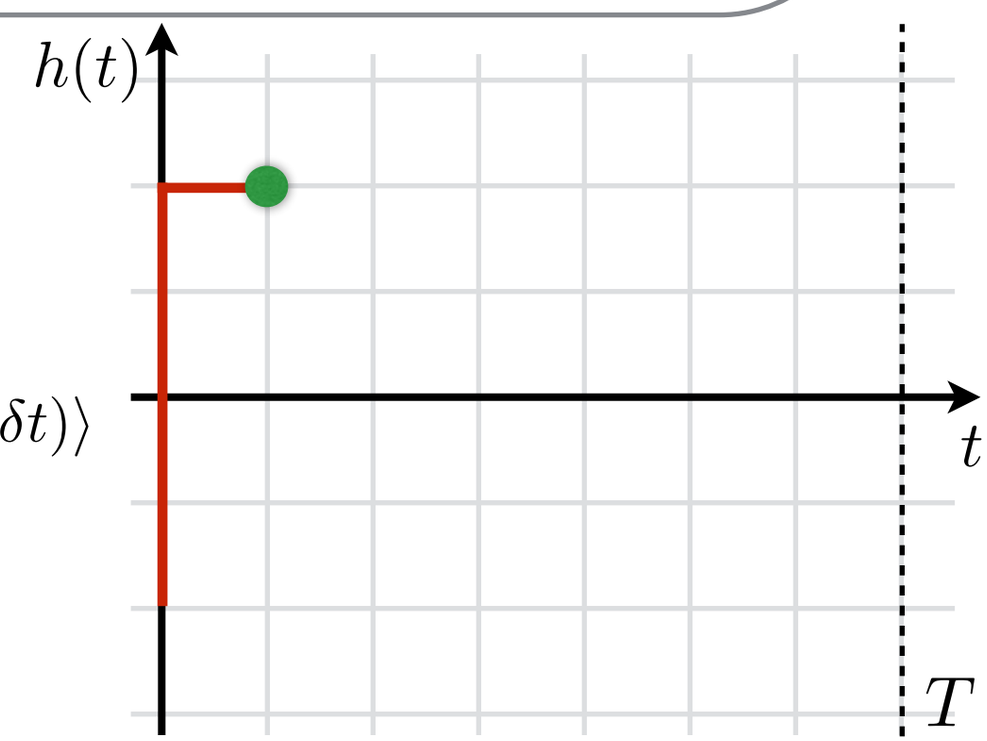

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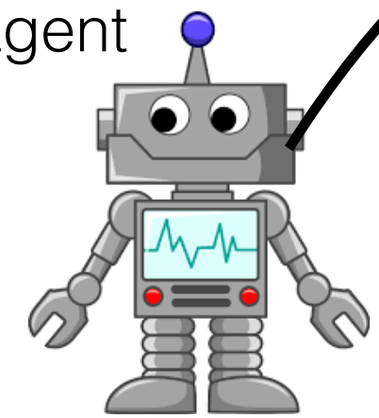

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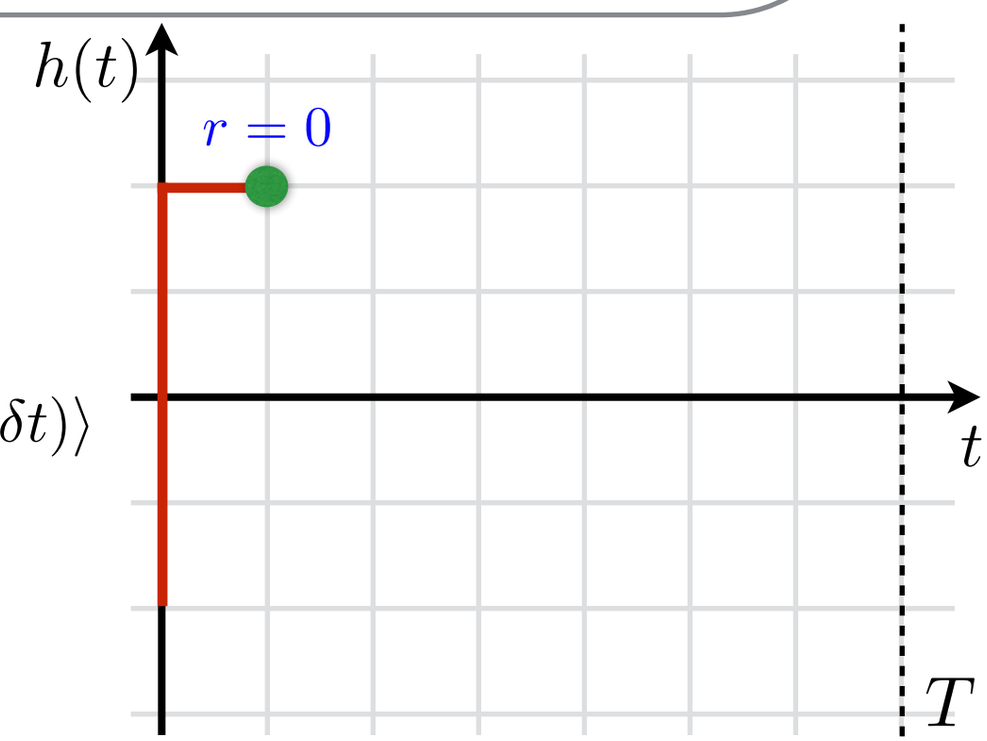
3 calculate reward r
and use it to update $Q(s,a)$
which in turn is used to choose subsequent actions



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RL Applied to Quantum State Preparation

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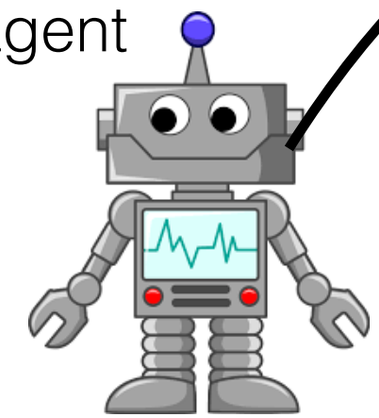

1

feedback loop,
updates $Q(s,a)$

$Q \mathrel{+}= \alpha \delta_{\text{Bellman}}$

3

ENVIRONMENT

2

$$H(t) = H_0 + H_{\text{ctrl}}(t)$$

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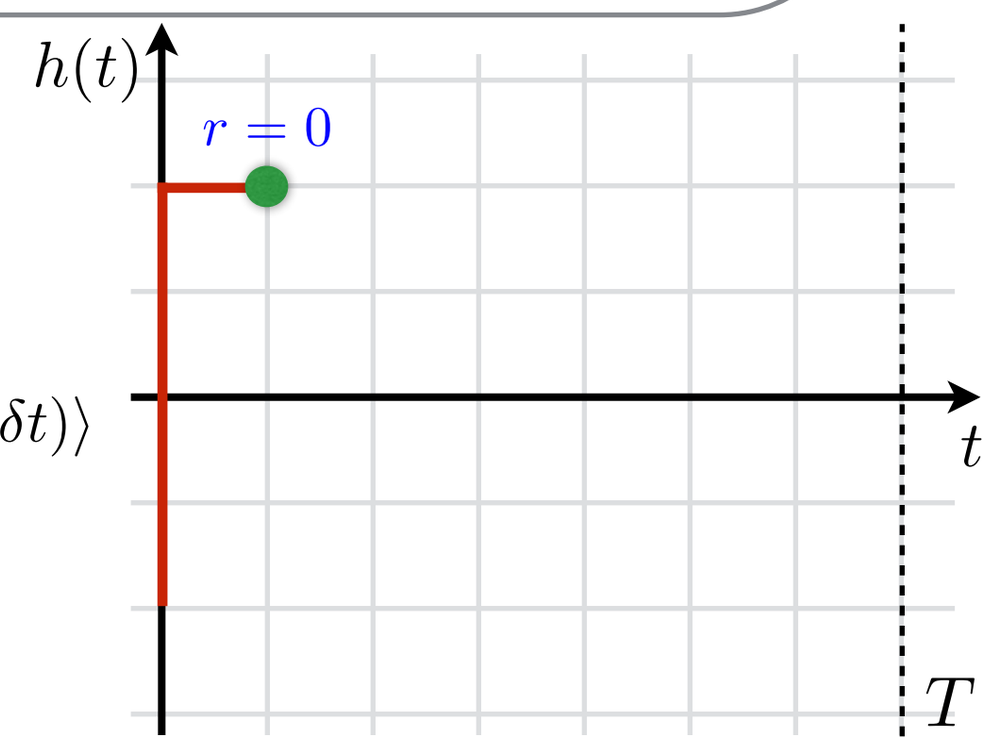

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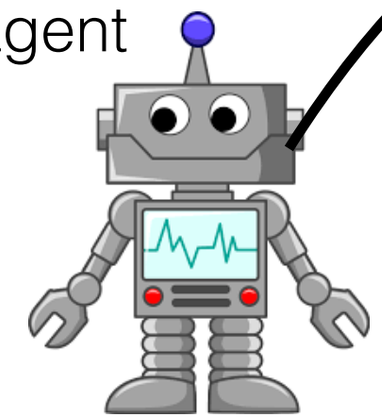

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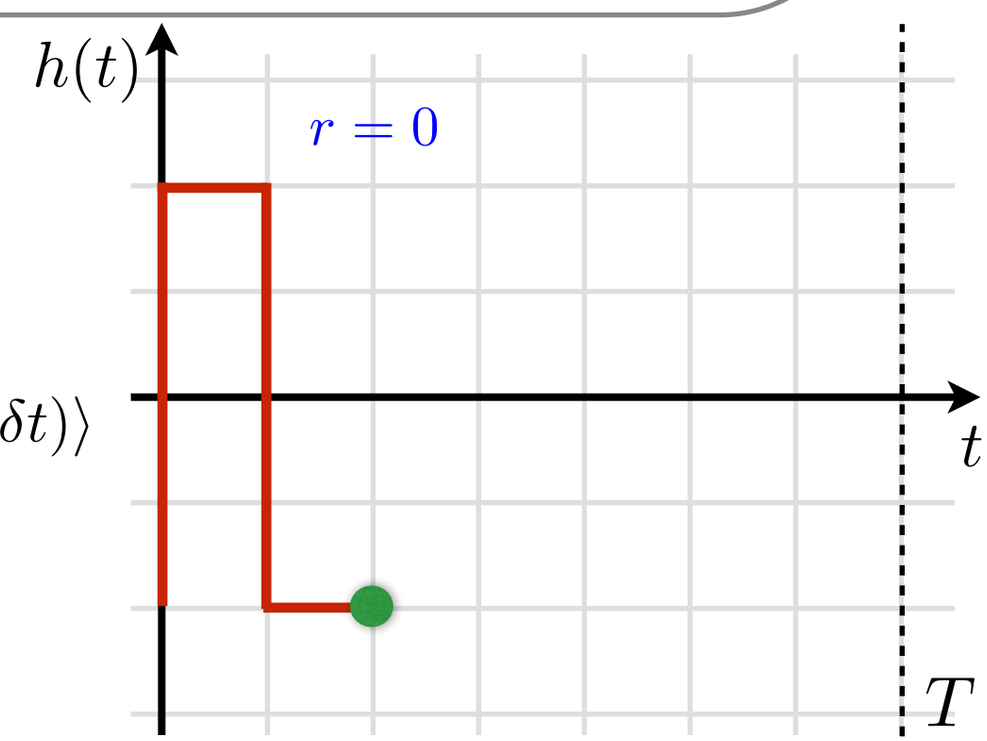

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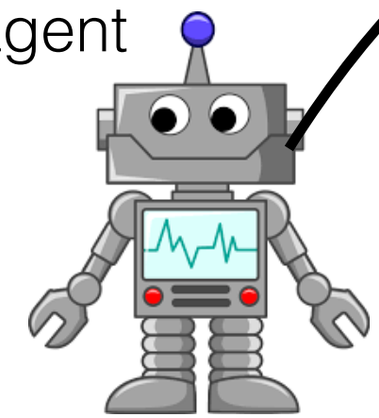

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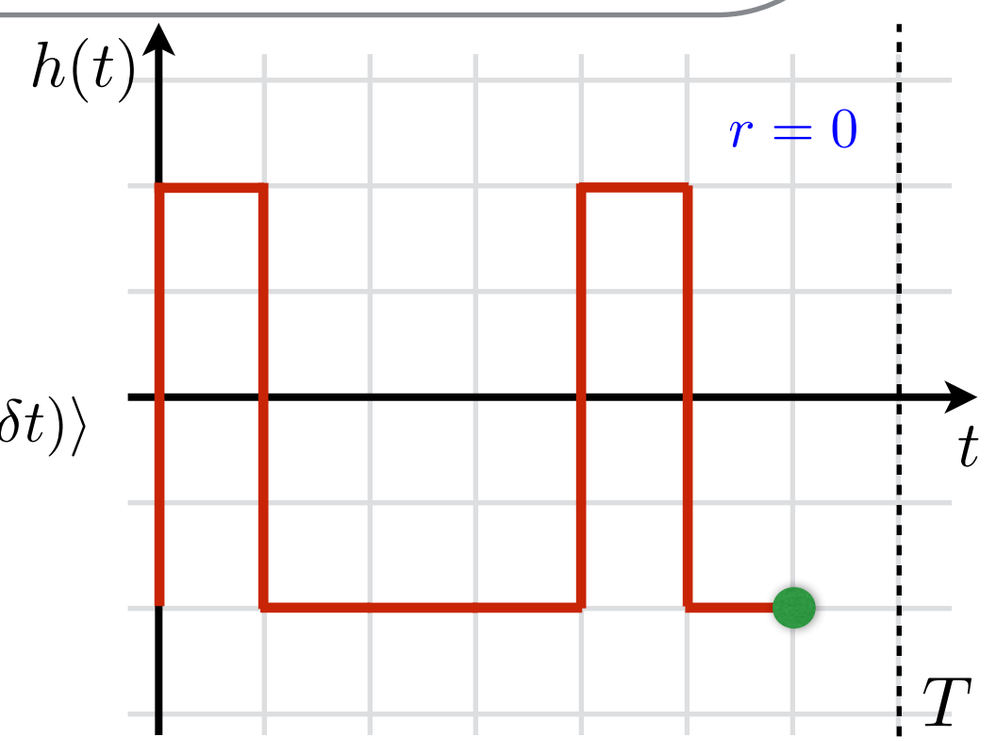

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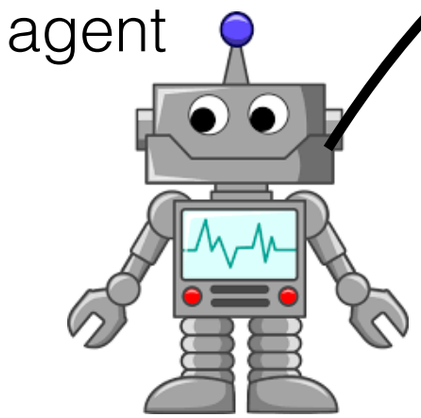
RL Applied to Quantum State Preparation



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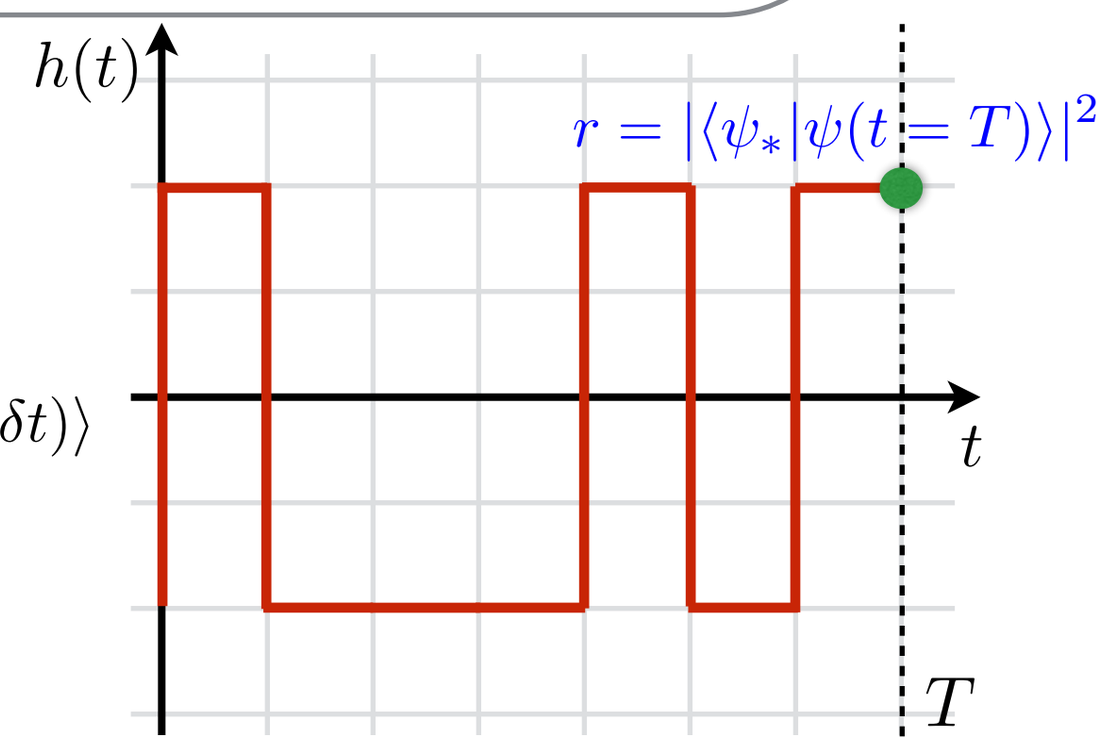

episode completed

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RL with Function Approximation

→ problem: state space has exponentially many configurations $|\mathcal{A}|^{N_T}$

- can we estimate values of not yet encountered states?

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$$Q(s,a) \to Q_\theta(s,a)$$

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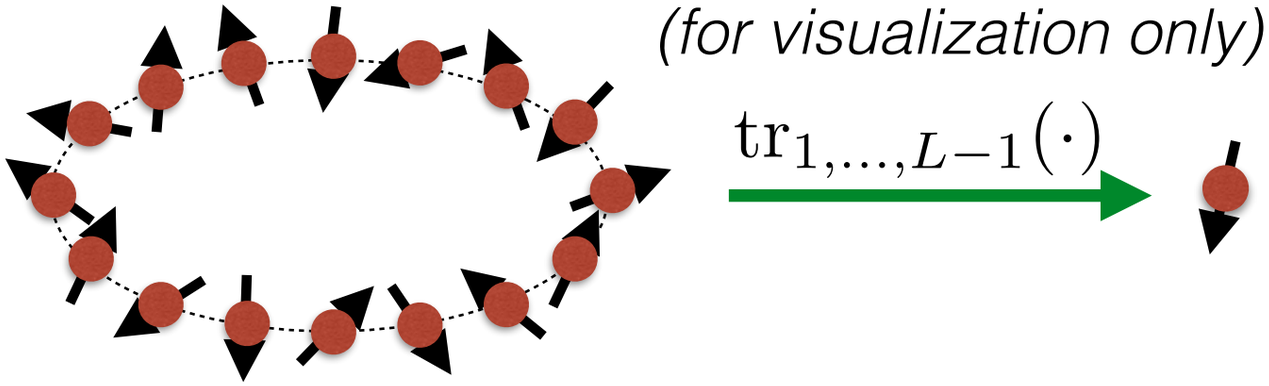
→ YES, via interpolation: parametrize the Q-function:

$$Q(s,a) \to Q_\theta(s,a)$$

- typical approach: use deep neural network (Deep RL)
- caveats:
  1) value-iteration RL algorithms have convergence guarantees only for *linear function approximators*
  2) may learn wrong Q-values (even if convergent)
- lots of empirical tricks to combine Deep Learning and RL

Learning Many-Body Quantum Control

$$H(t) = -\sum_{j=1}^{L} S^z_{j+1} S^z_j + S^z_j + h_x(t) S^x_j$$



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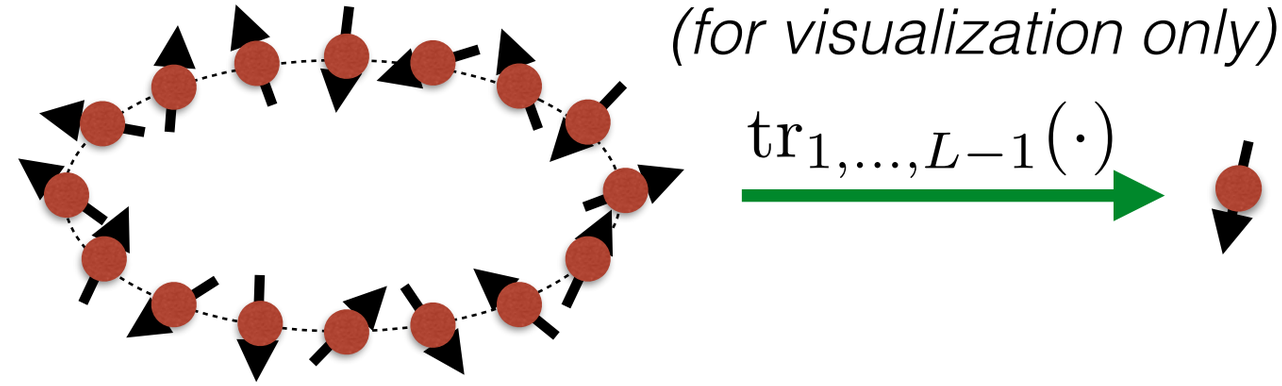
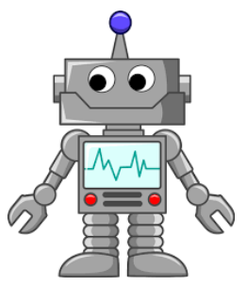
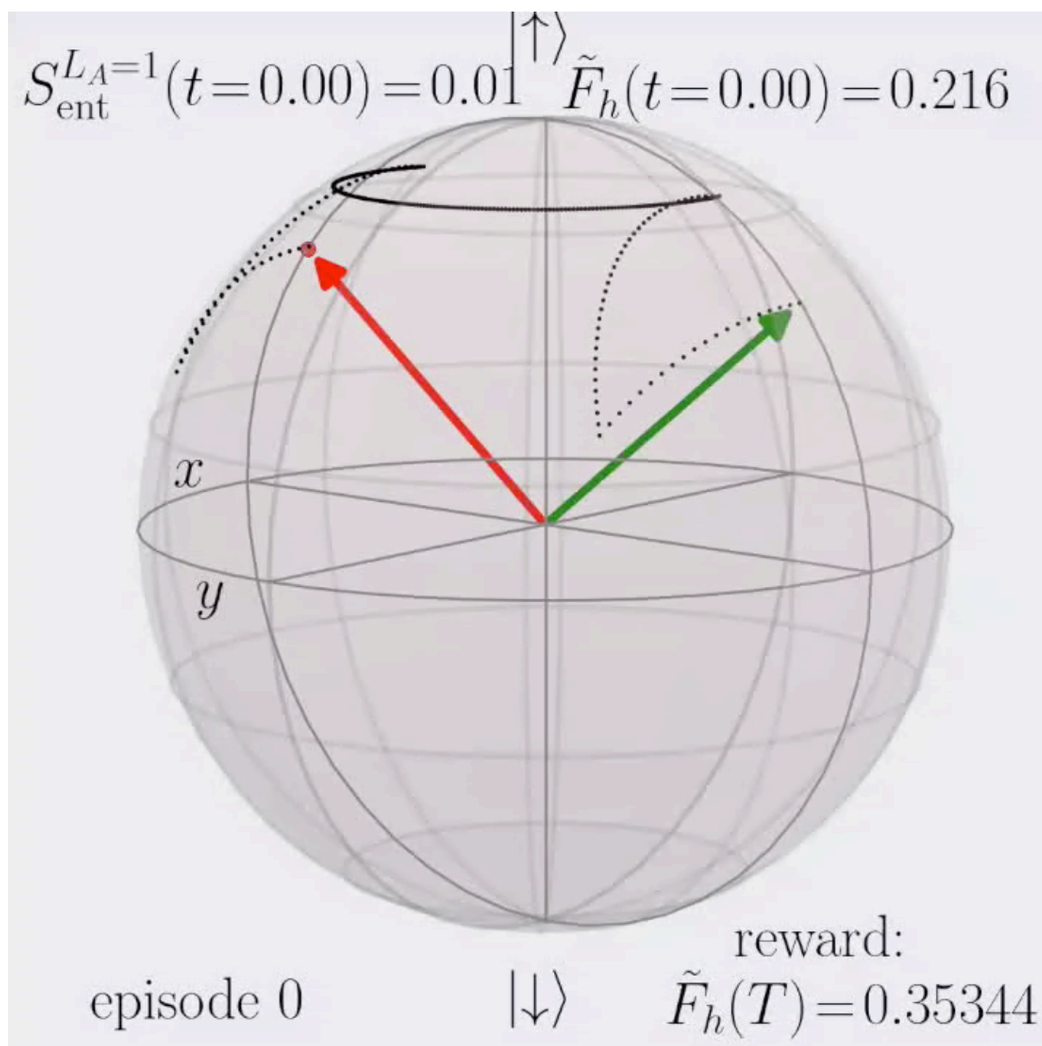


$h_x \in \{\pm 4\}$ bang-bang protocols





Bloch sphere

$$\tilde{F}_h = -\frac{1}{L} \log F_h$$

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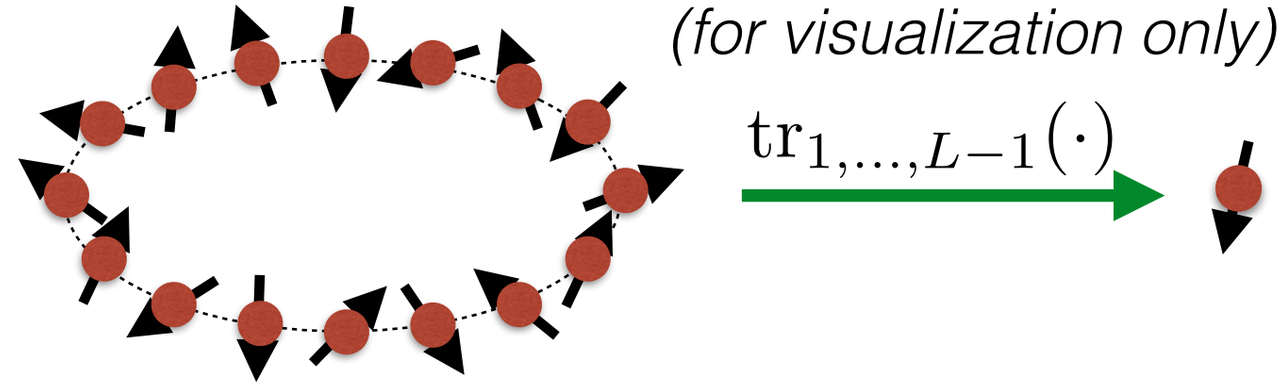

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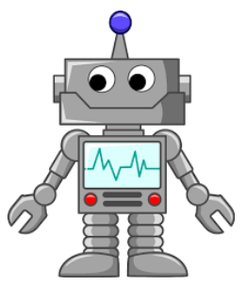

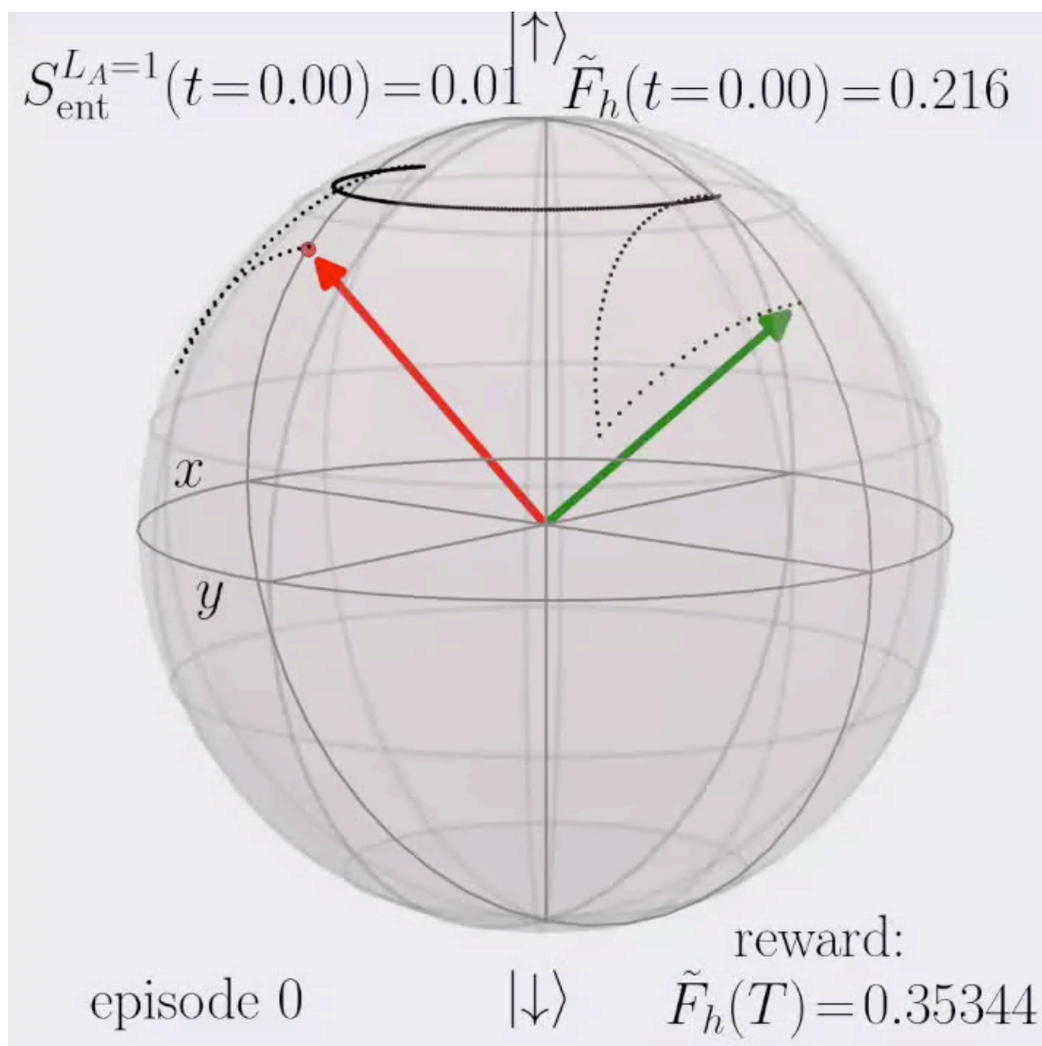

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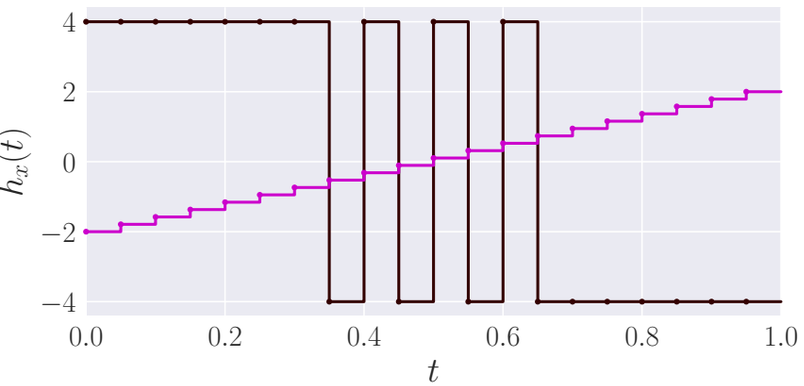
What do we Learn from the RL Agent?



$$H = \sum_j -S^z_{j+1}S^z_j - h_z S^z_j - h_x(t) S^x_j$$

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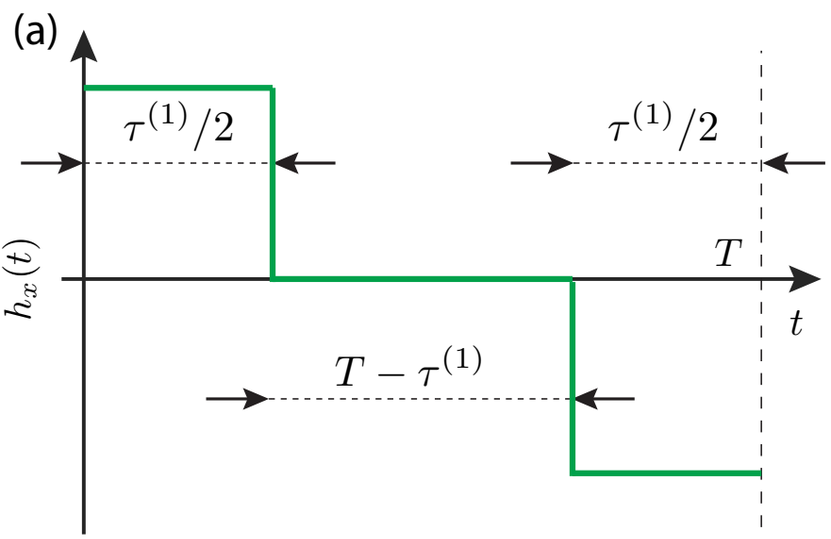

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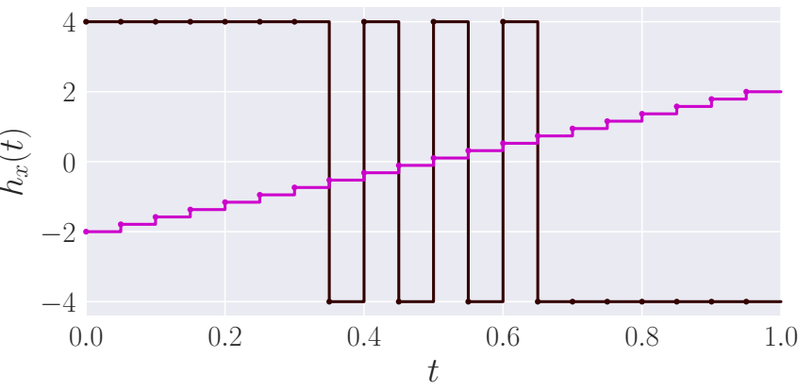

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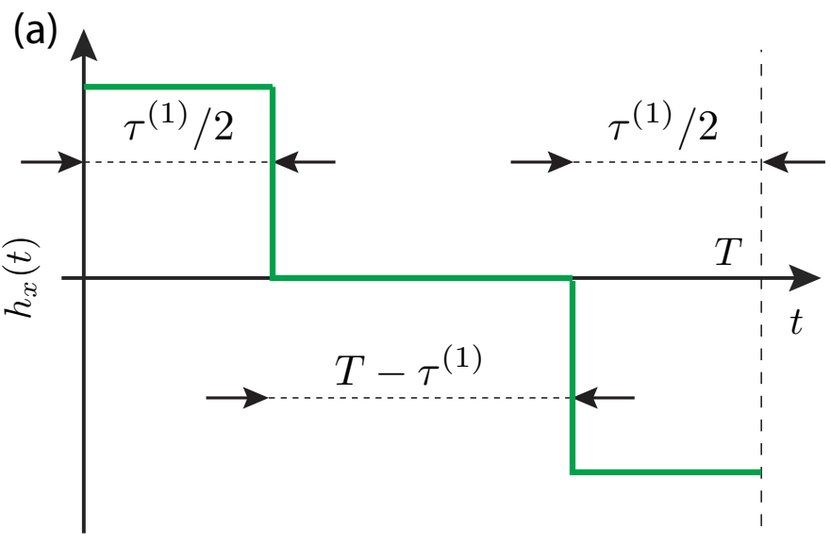

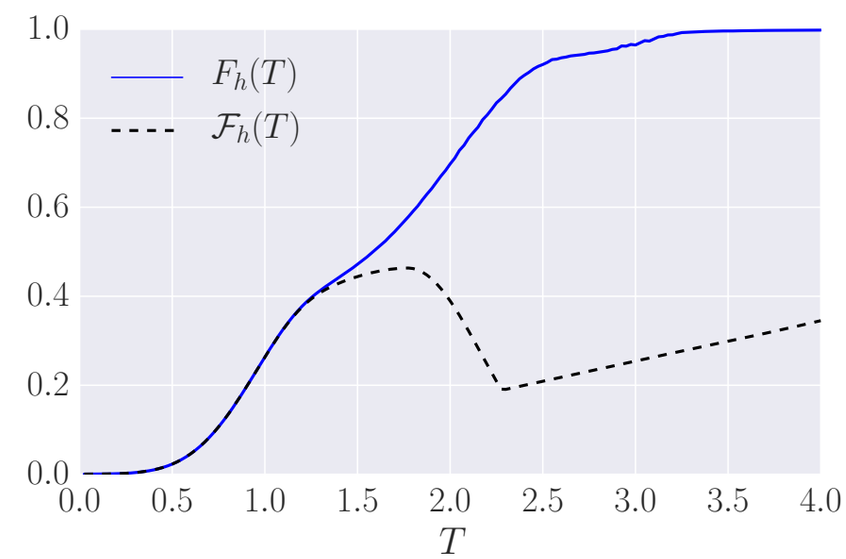

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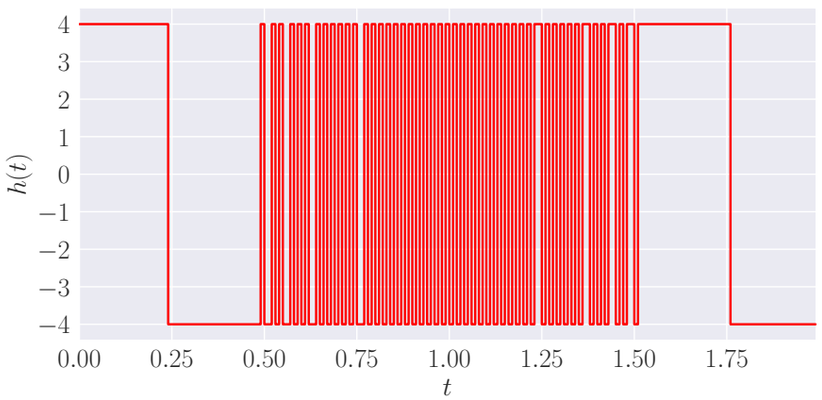

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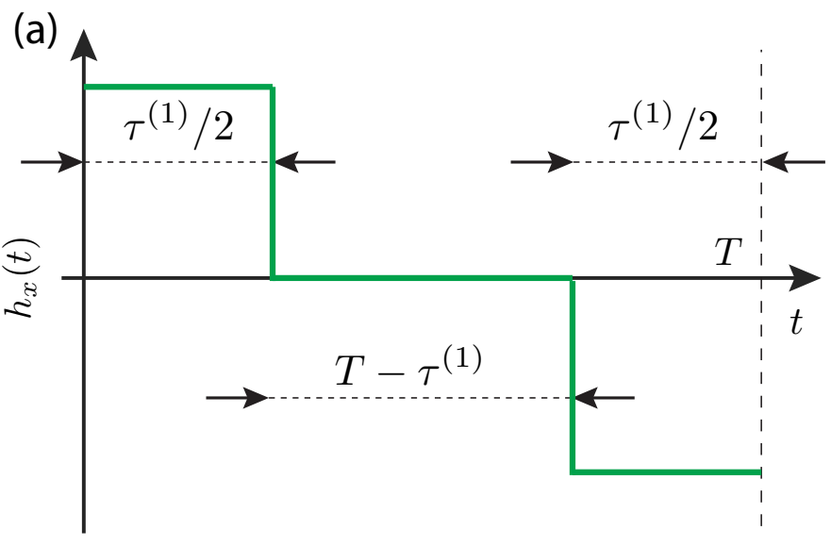

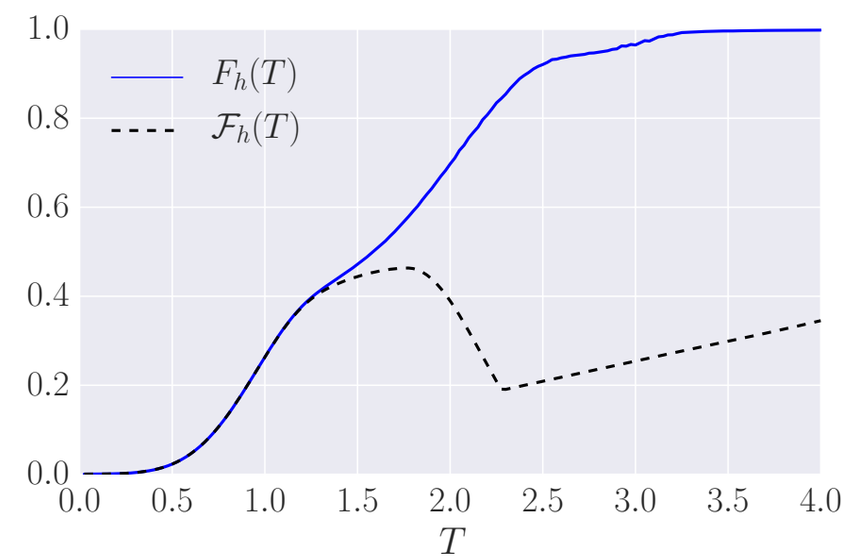

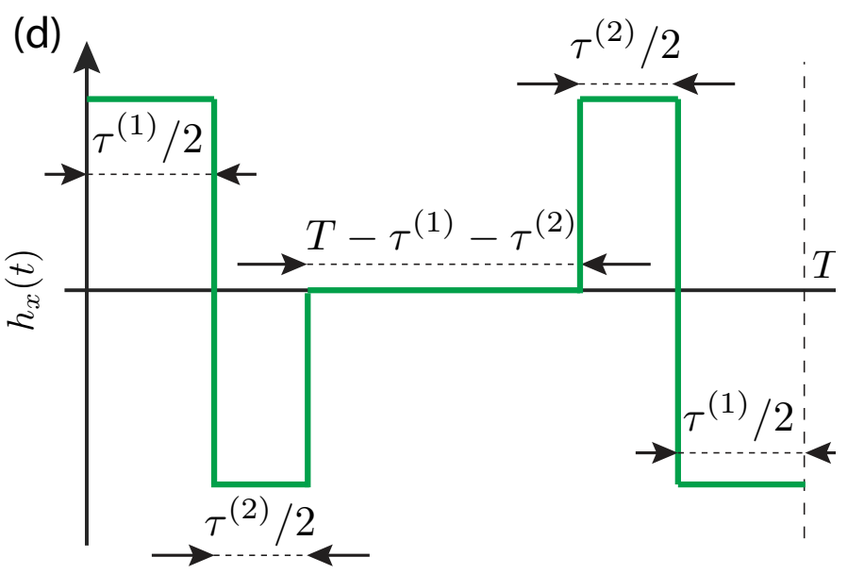

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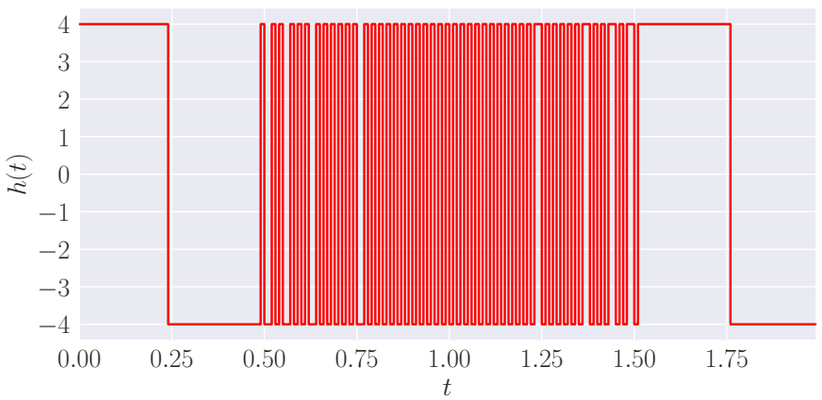

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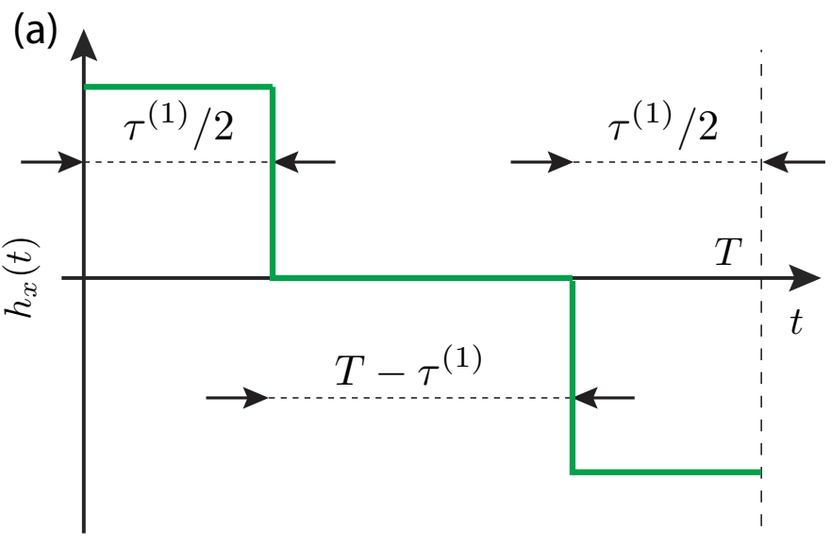

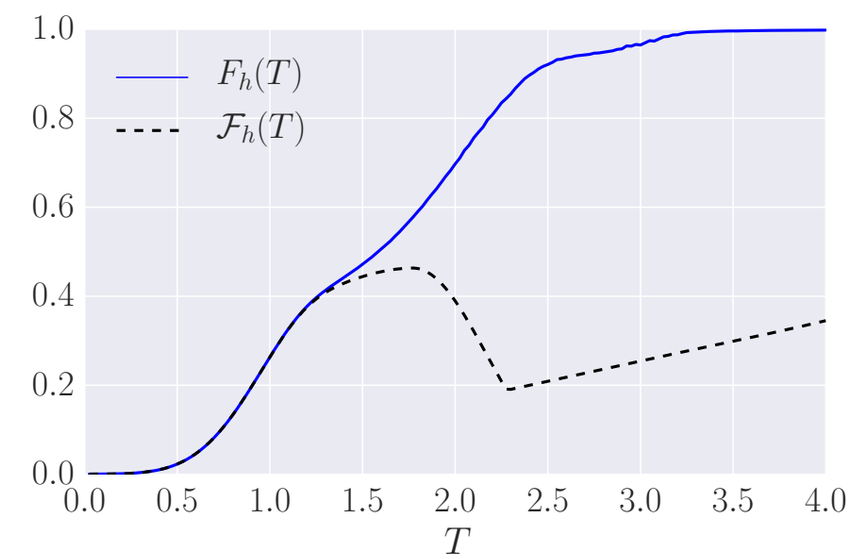

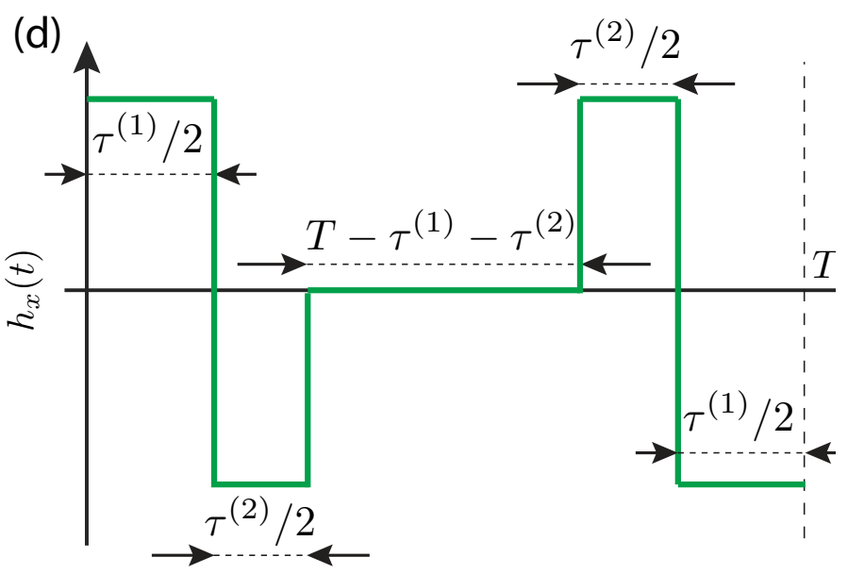

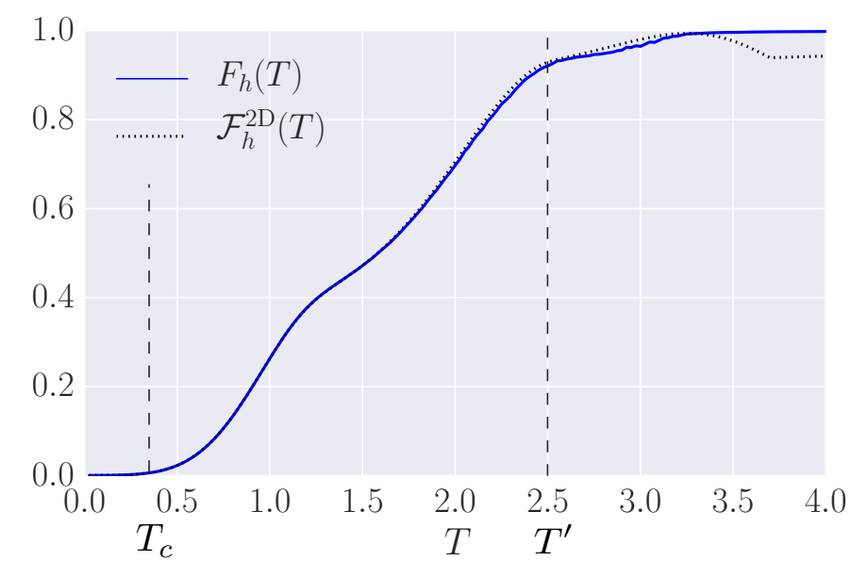

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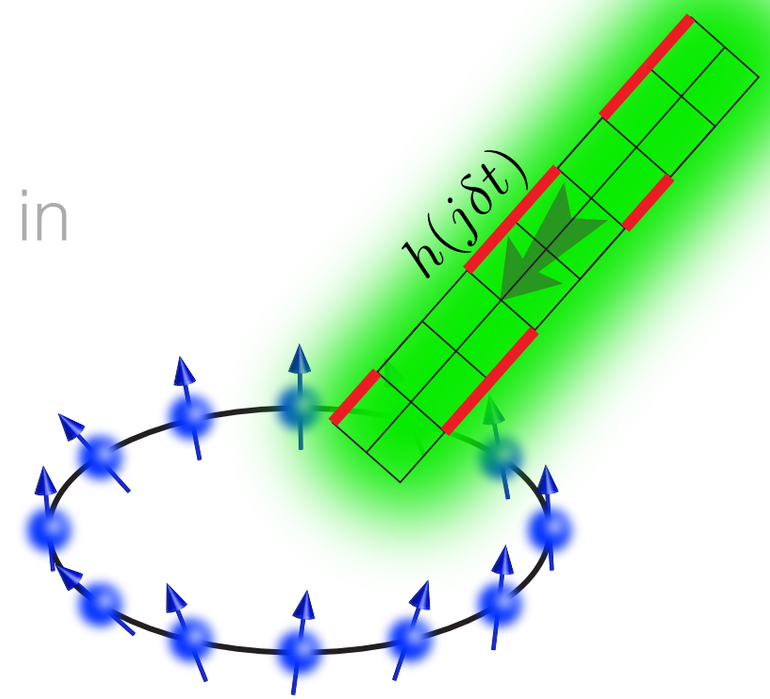
Berkeley
UNIVERSITY OF CALIFORNIA

Reinforcement Learning (RL) for quantum control

→ **Example**: use RL to prepare many-body states in a nonintegrable spin chain

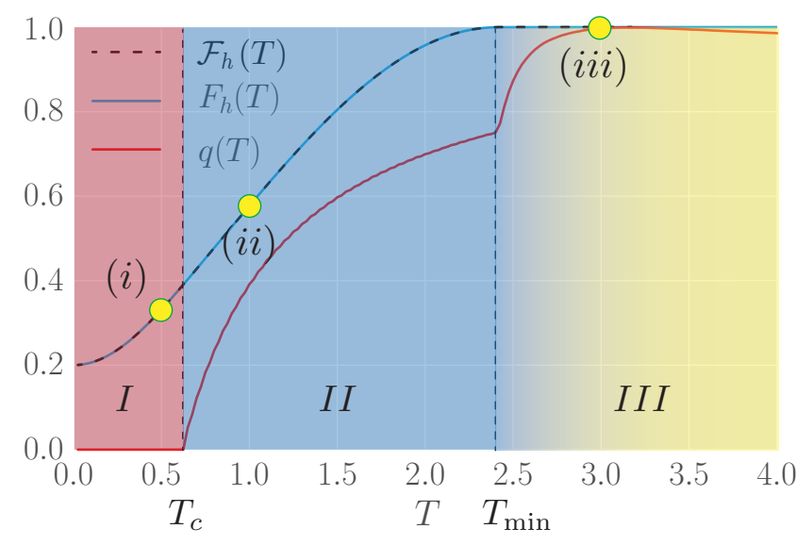
- RL and quantum control
- variational theory for optimal protocols



MB et al, PRX 8 031086 (2018)

→ **Phase transitions in the control landscape:**

- how "hard" is it for the RL agent to Learn?



PRA 97, 052114 (2018)
PRL 122, 020601 (2019)

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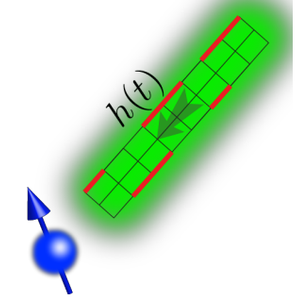
RL-inspired Discovery: Phase Diagram of Quantum Control



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bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

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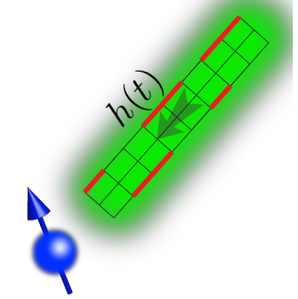
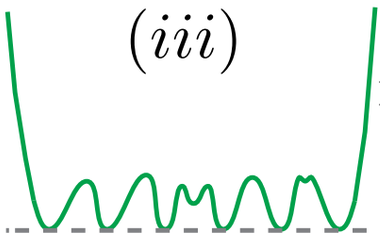


infidelity landscape (schematic)



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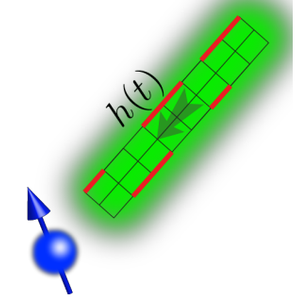

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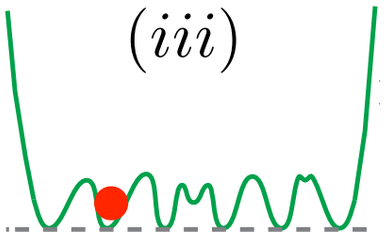

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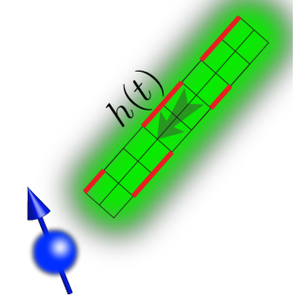

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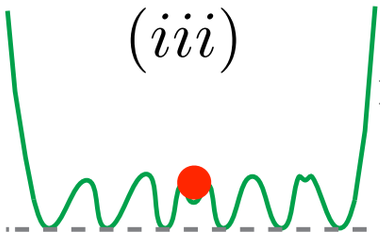

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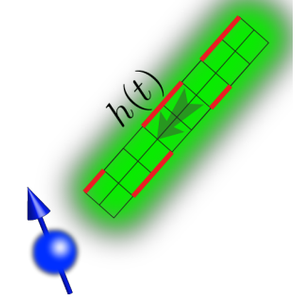

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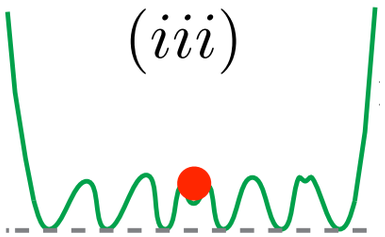

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bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

infidelity landscape minima: $\{h^\alpha\}$

$$\bar{h}(t) = \frac{1}{\#_{\text{real}}} \sum_\alpha h^\alpha(t)$$

Edwards-Anderson-like order parameter:

$$q(T) \sim \sum_{j=1}^{N_T} \overline{\{h(j\delta t) - \overline{h}(j\delta t)\}^2}$$

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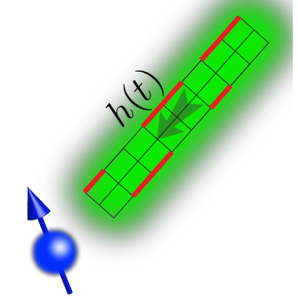

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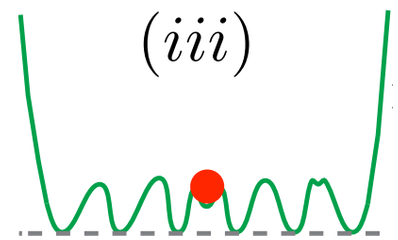
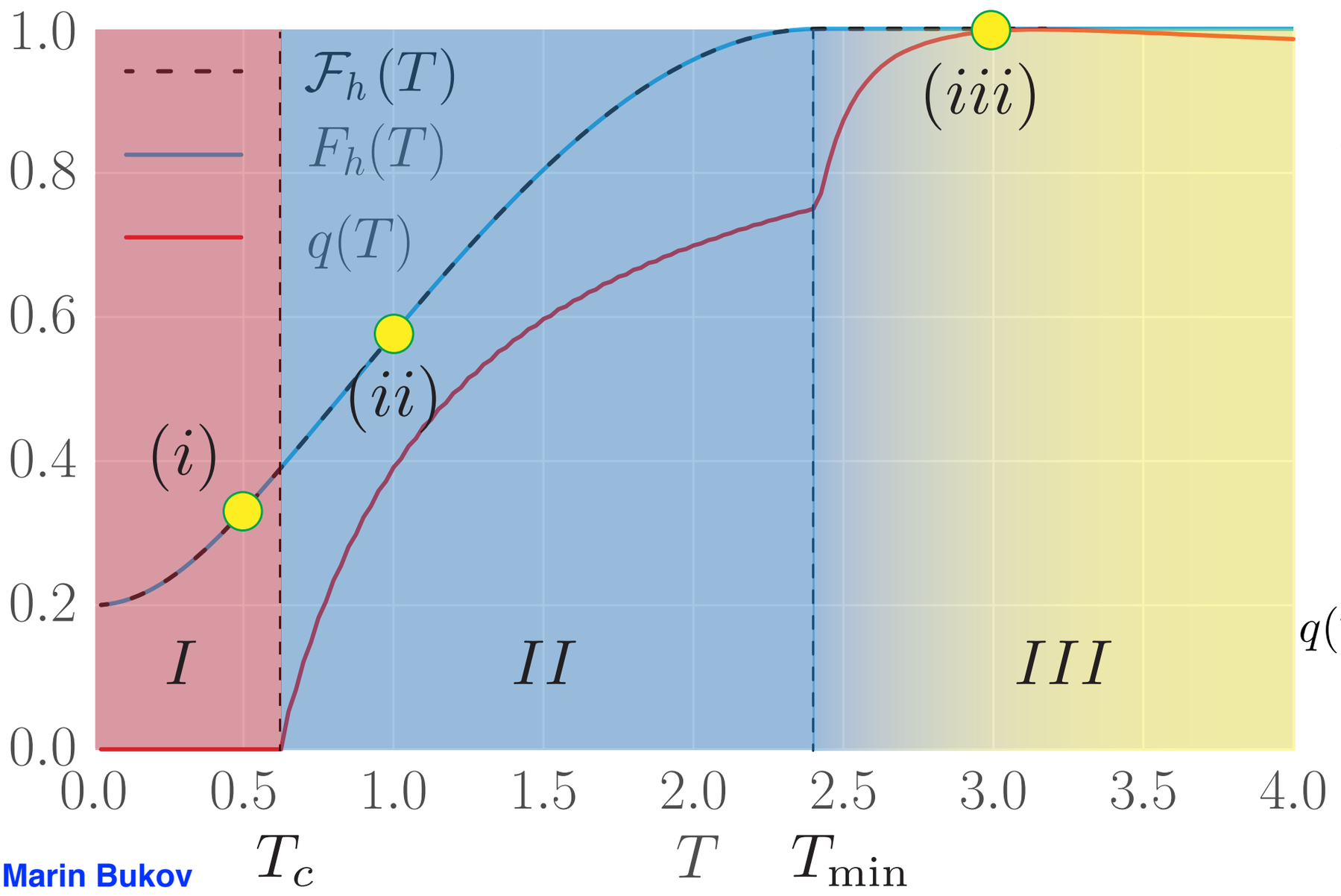
bang-bang protocols



$$h \mapsto 1 - F_h(T)$$



infidelity landscape minima: $\{h^\alpha\}$

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Edwards-Anderson-like order parameter:

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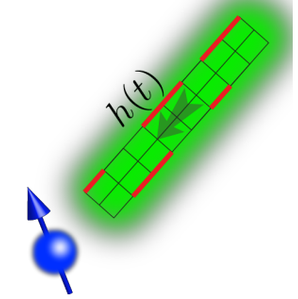
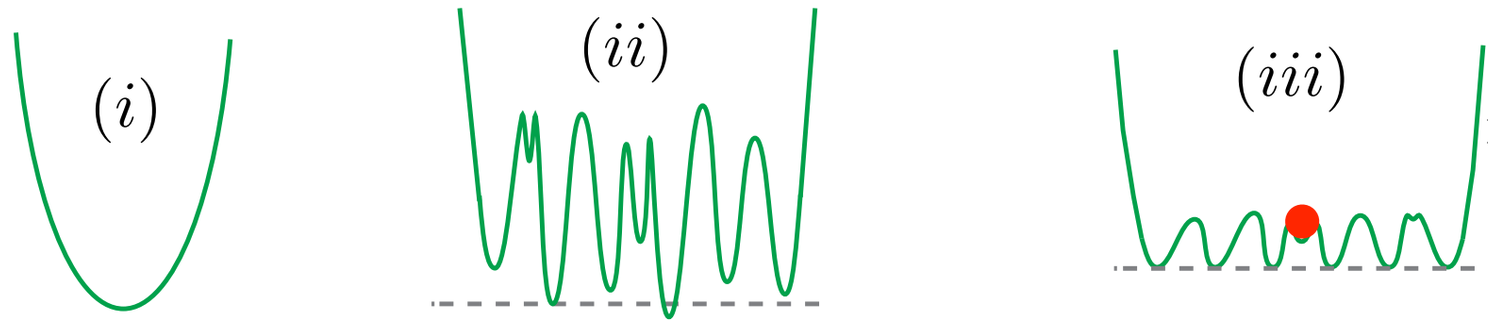
Marin Bukov

PRX 8 031086 (2018)

RL-inspired Discovery: Phase Diagram of Quantum Control



infidelity landscape (schematic)



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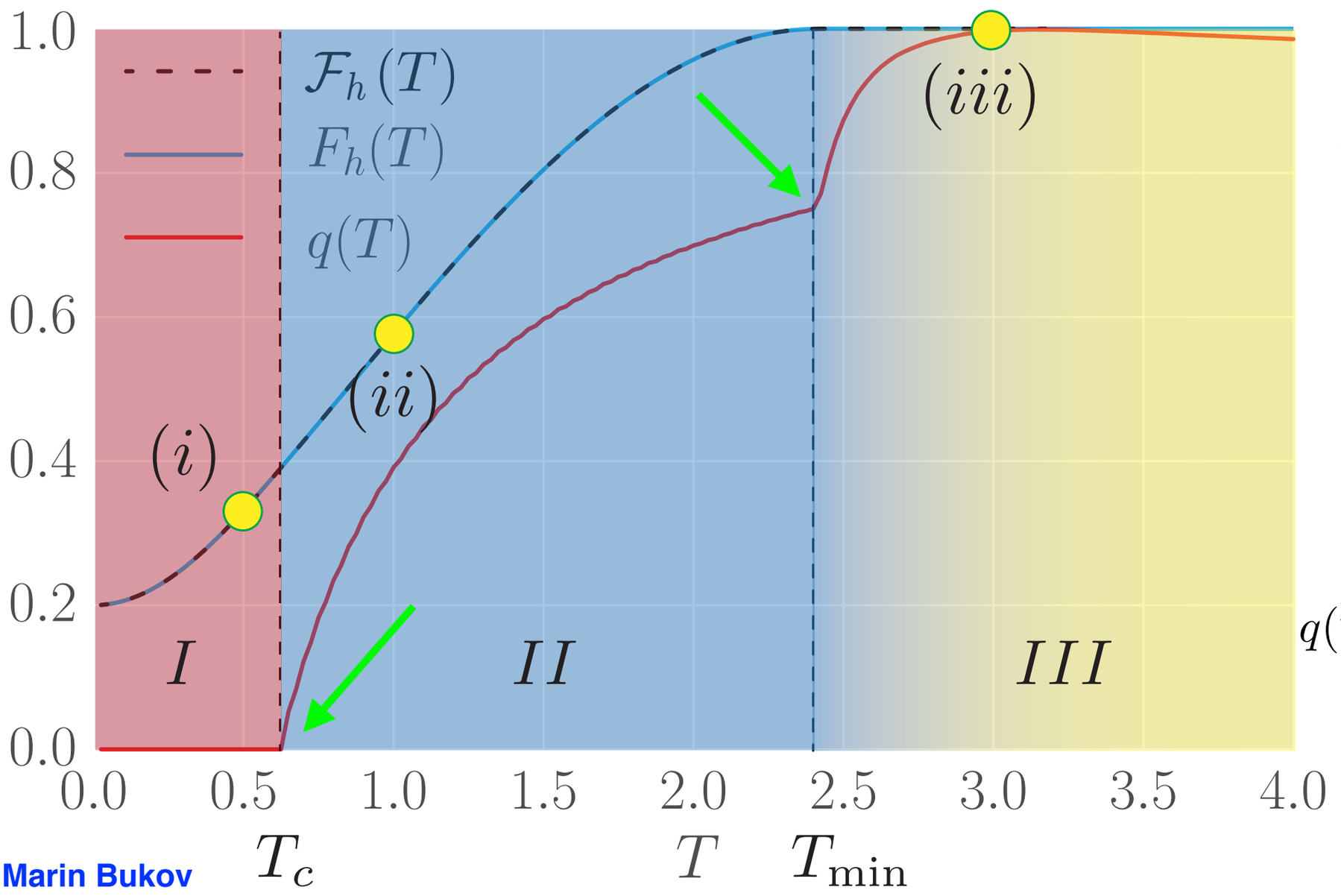

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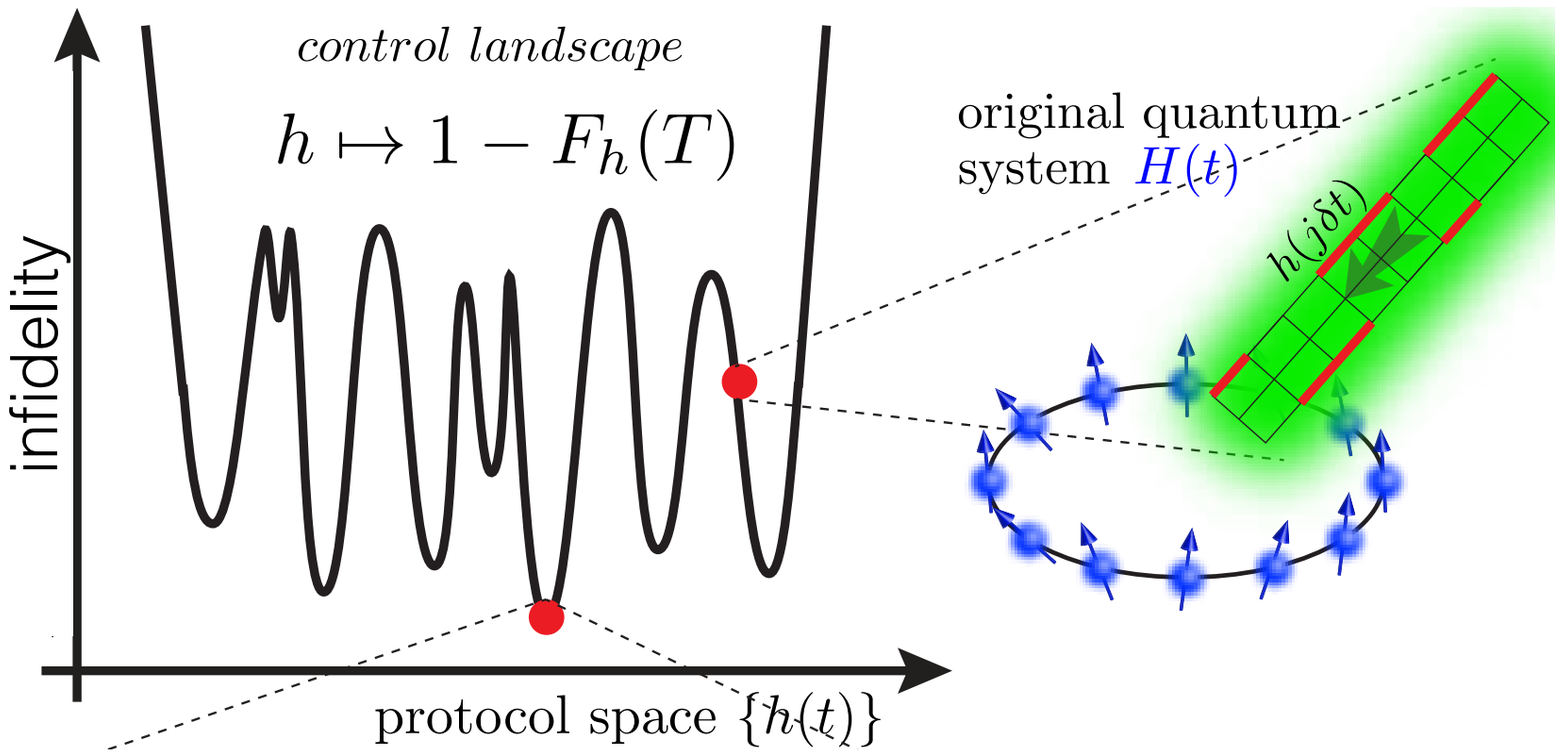
Effective Classical Energy Model

→ one-to-one correspondence:



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PRL 122, 020601 (2019)

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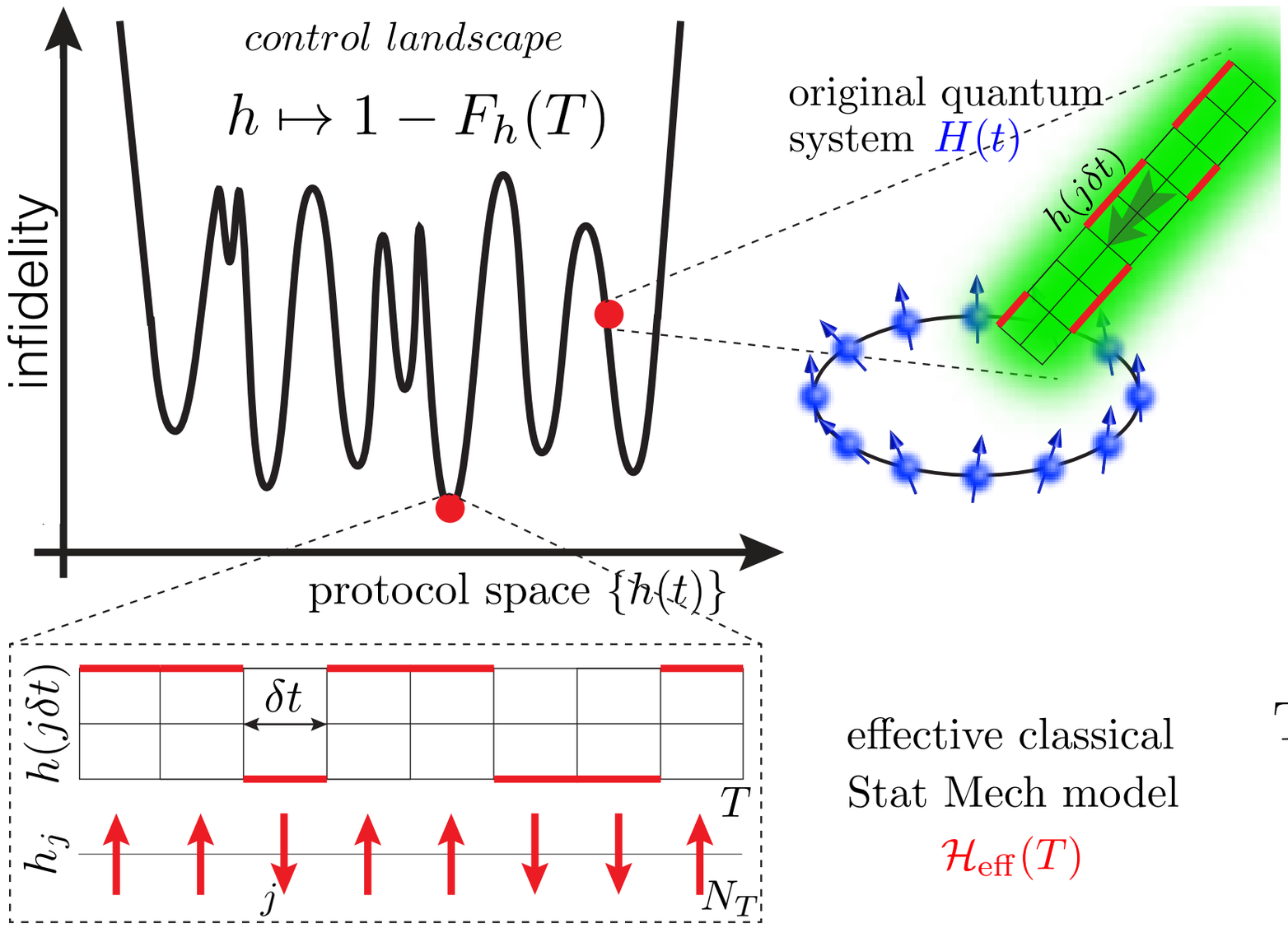

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effective classical Stat Mech model $\mathcal{H}_{\text{eff}}(T)$

$$\text{TD limit}: \; N_T \to \infty, \delta t \to 0$$
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PRL 122, 020601 (2019)

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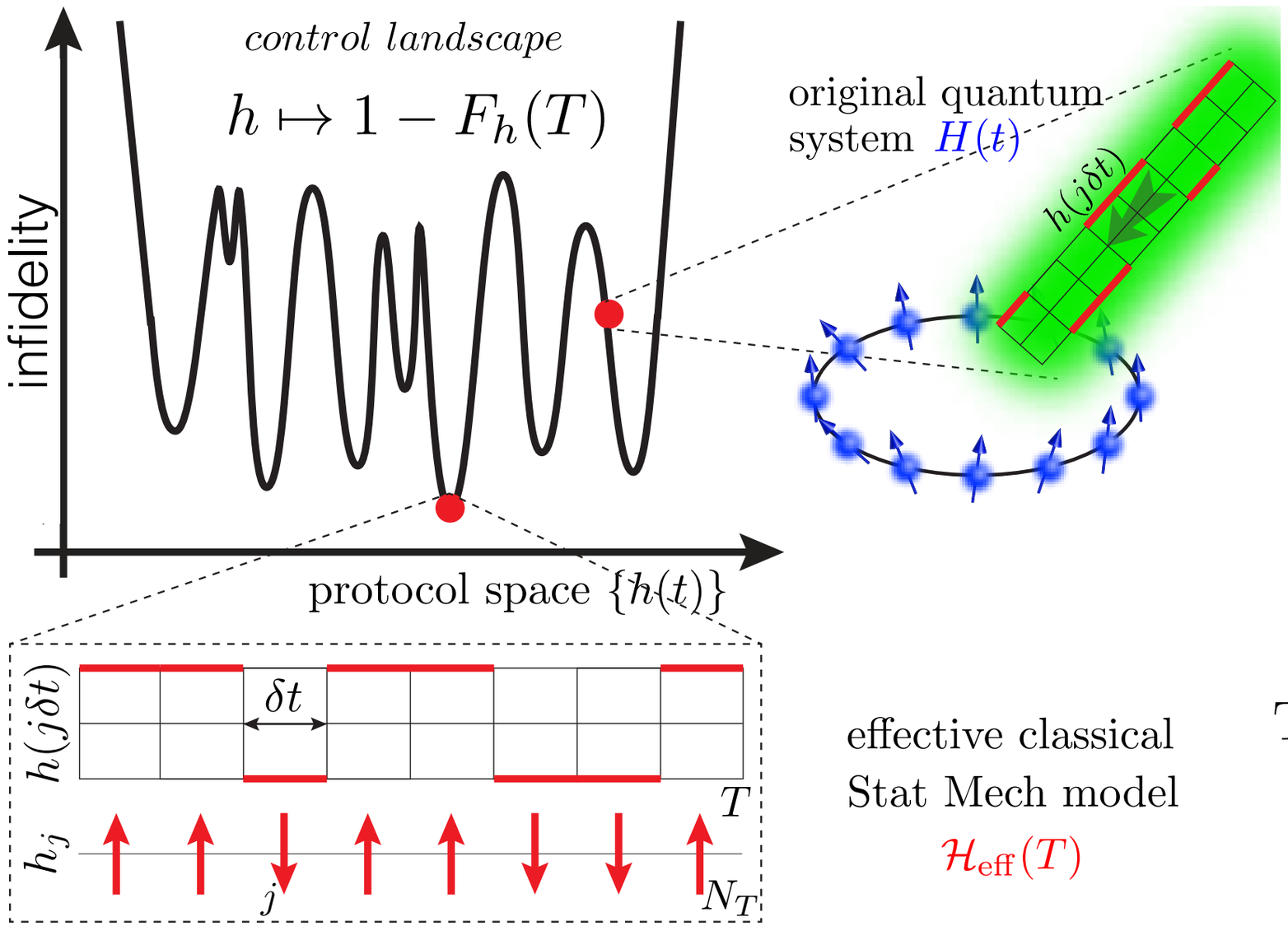

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→ effective *classical* spin energy describes control landscape

$$\mathcal{H}_{\text{eff}}(T) = I(T) + \sum_j G_j(T)h_j + \sum_{ij} J_{ij}(T)h_i h_j + \sum_{ijk} K_{ijk}(T)h_i h_j h_k + \ldots$$

j : sites on time lattice

PRL 122, 020601 (2019)

Symmetry Breaking in the Control Landscape

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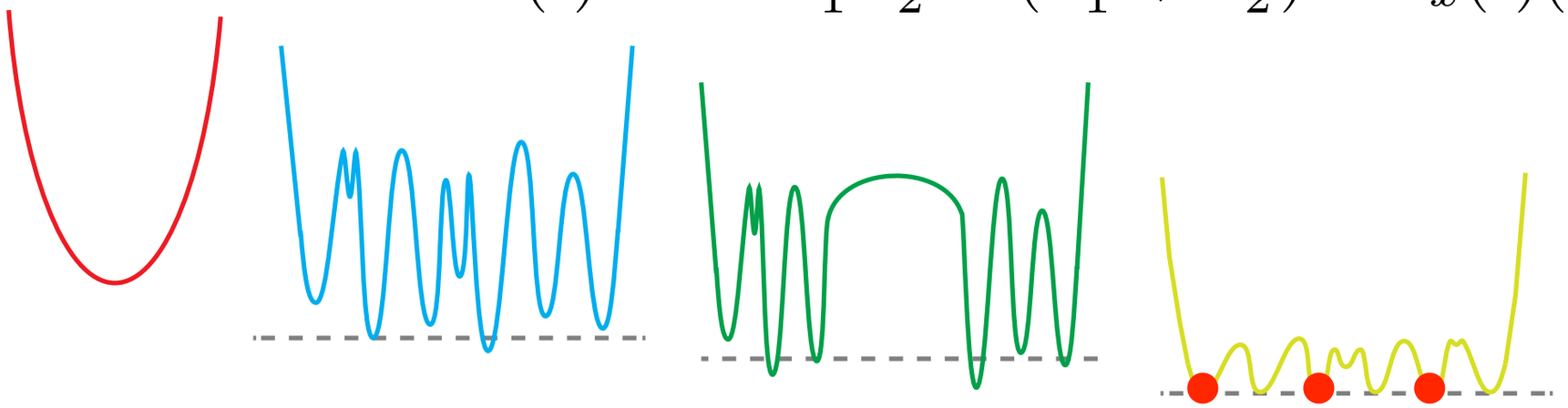

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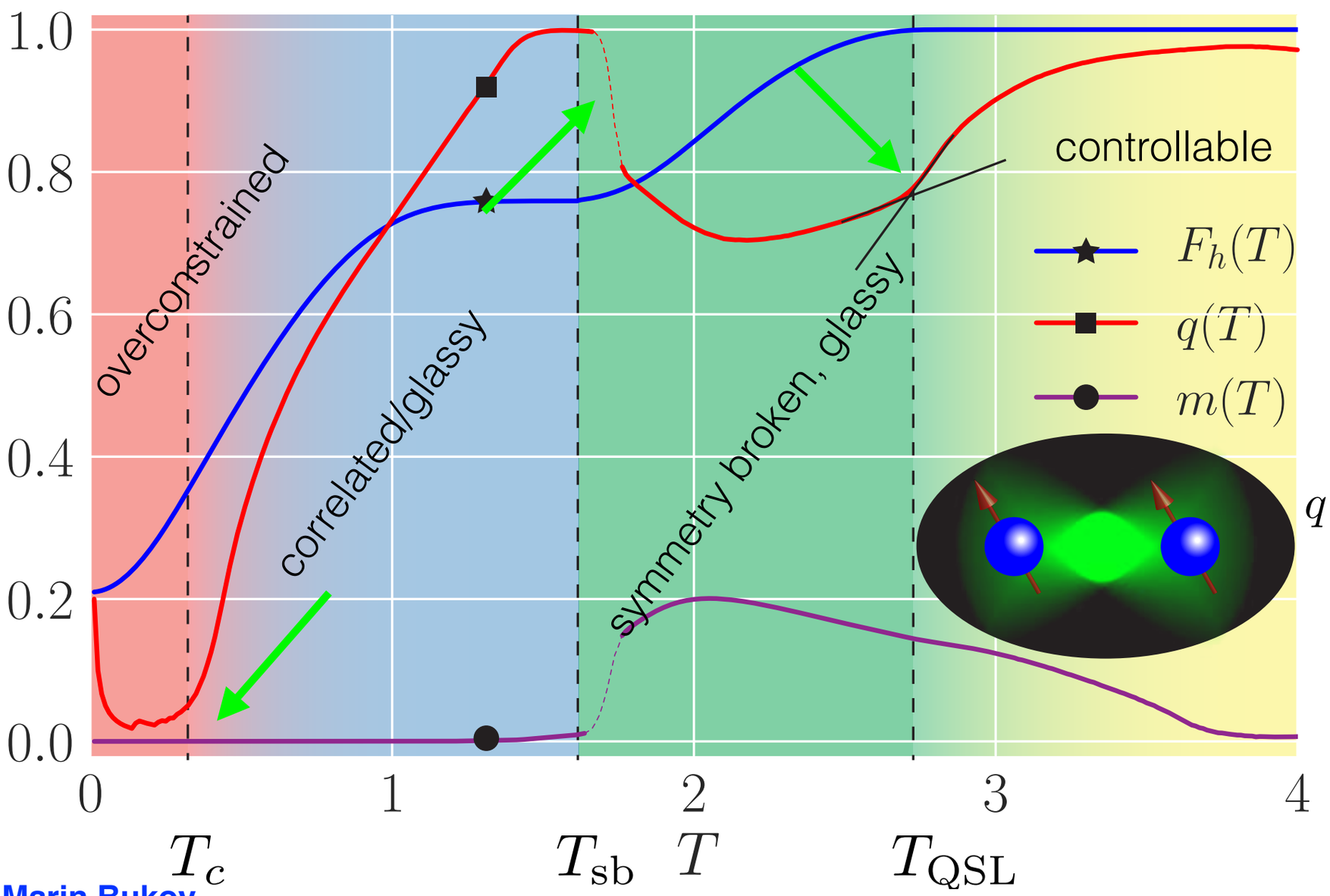

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PRL 95, 200201 (2005)
PRA 97, 052114 (2018)

Visualizing the Glassy Transition with t-SNE

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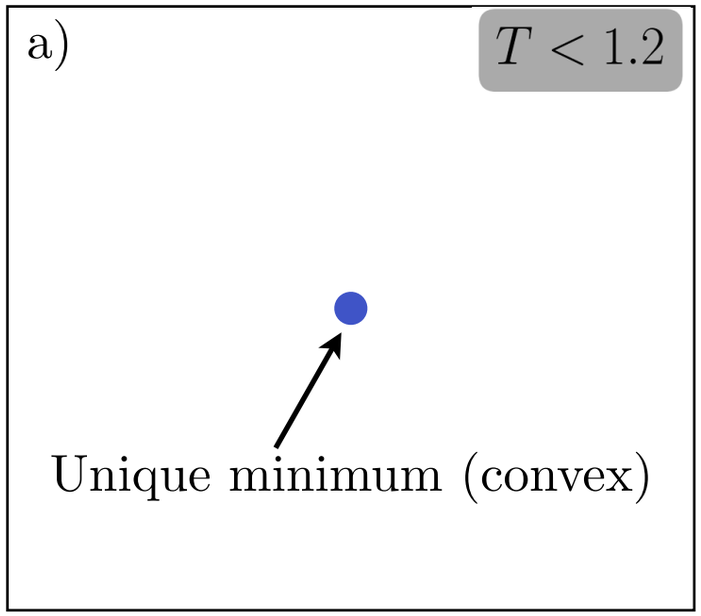

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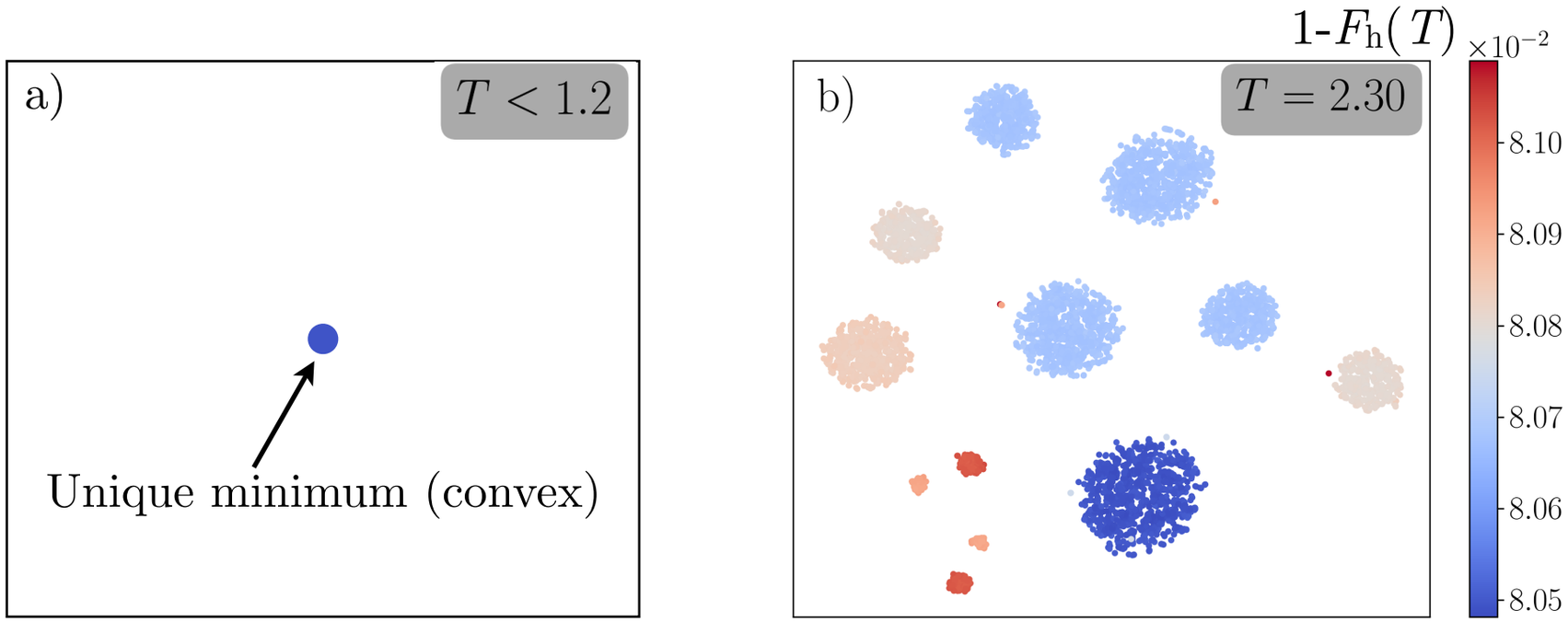

PRL 122, 020601 (2019)

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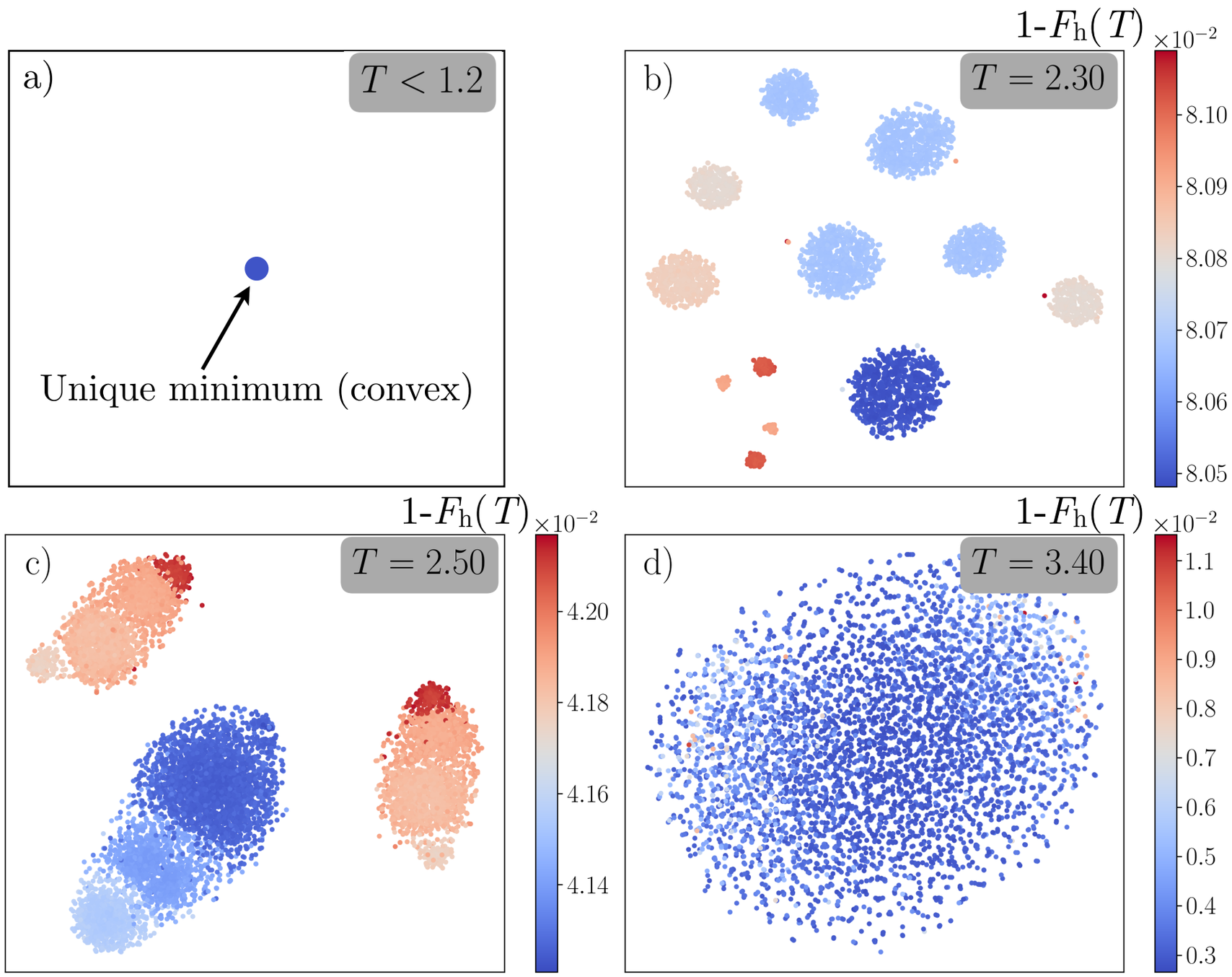

PRL 122, 020601 (2019)

Why RL in Nonequilibrium Dynamics?

→ **model-free:** find effective control degrees of freedom (dof)

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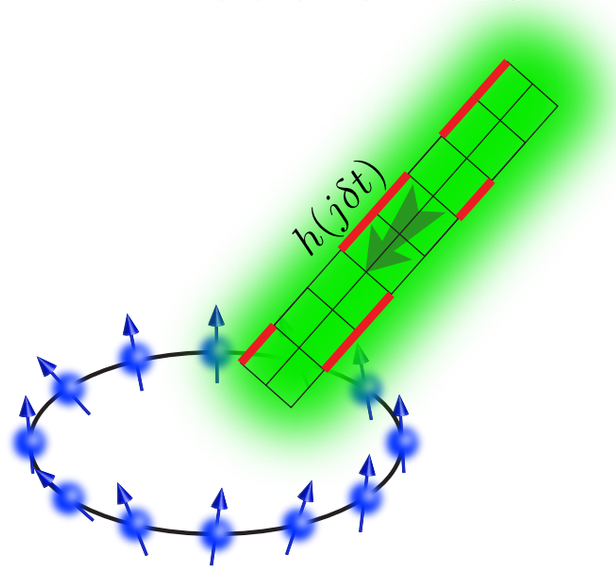
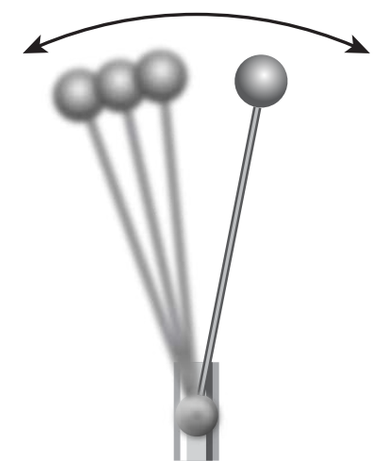
- can RL automate experimental setups?

Outlook

- Which problems can we study with RL that we can't do otherwise?
- How do we use RL to discover new physics?
- What are RL's most natural/appropriate applications in physics?





spin chain: PRX 8 031086 (2018)

control phases: PRL 122, 020601 (2019)

PRA 97, 052114 (2018)

Kapitza oscillator: PRB 98, 224305 (2018)

QuSpin: *http://weinbe58.github.io/QuSpin*

open source python package for **many-body** lattice systems

(with P. Weinberg, BU)

RL and Optimal Control (OC)

→ different sides of the same medal

- OC: appeared in optimization problems: variational calculus
- RL: appeared first in behavioral psychology: decision making

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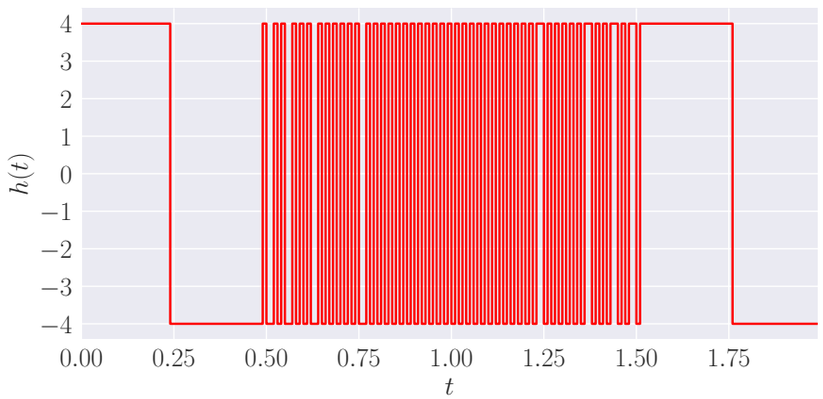
OC ← *closely related* → **RL**	
based on: ***variational calculus***	***Markov decision processes***
• needs model for environment to express cost function in.	• no model of controlled system, adaptive, autonomous.
• best suited for deterministic environments.	• stochastic/uncertain environments.
• differentiable cost function C_h uses gradient descent.	• reward function can be discontinuous, noisy.
• advantage: if we can compute analytically derivative of C_h.	• figures out effective degrees of freedom without a model.

What do we Learn from the RL Agent?



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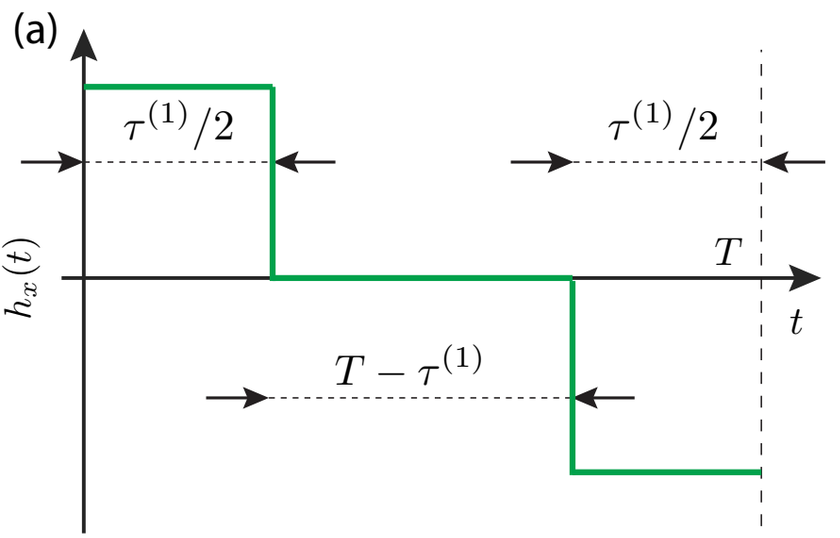

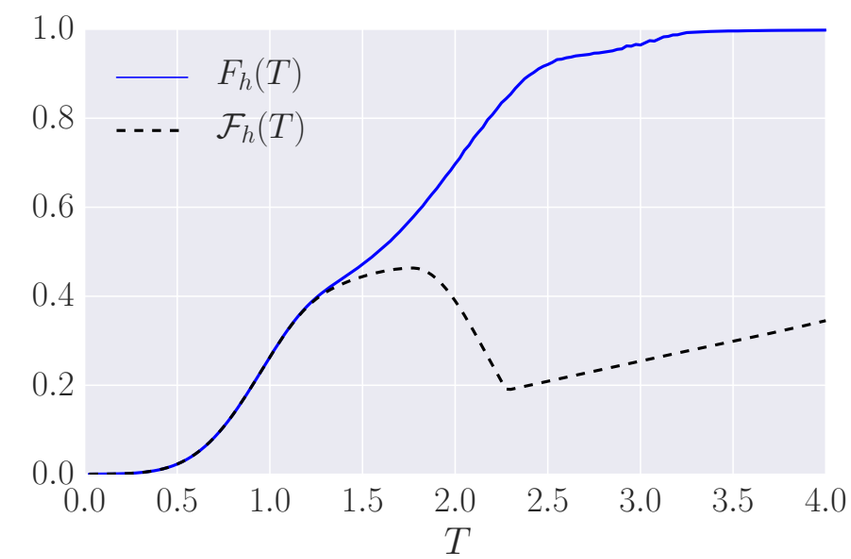

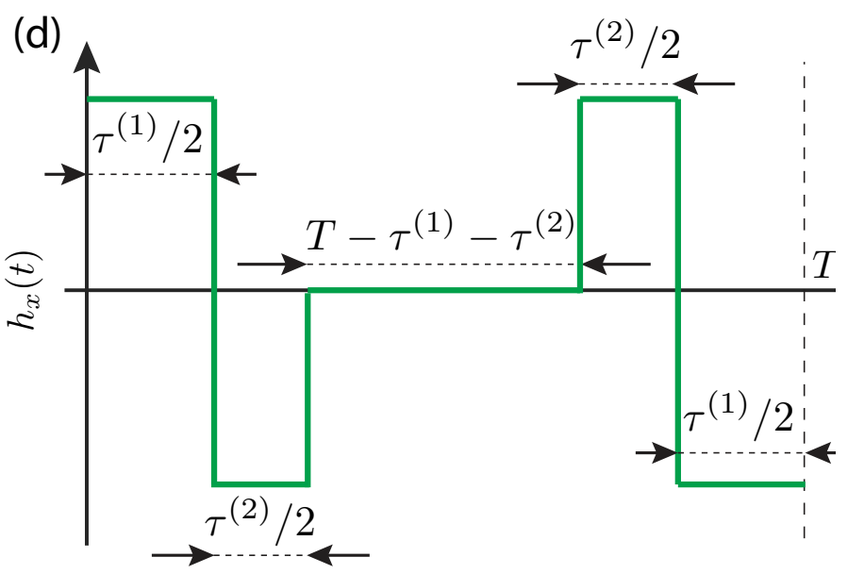

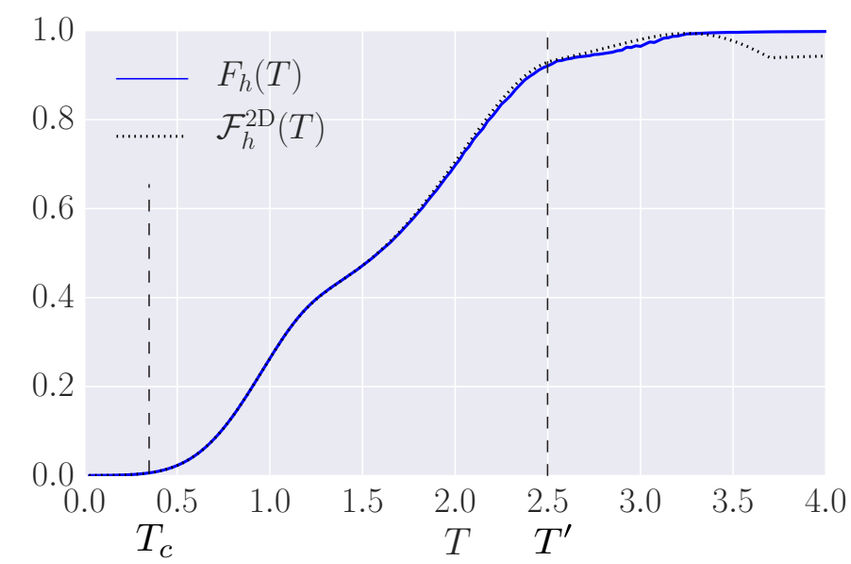

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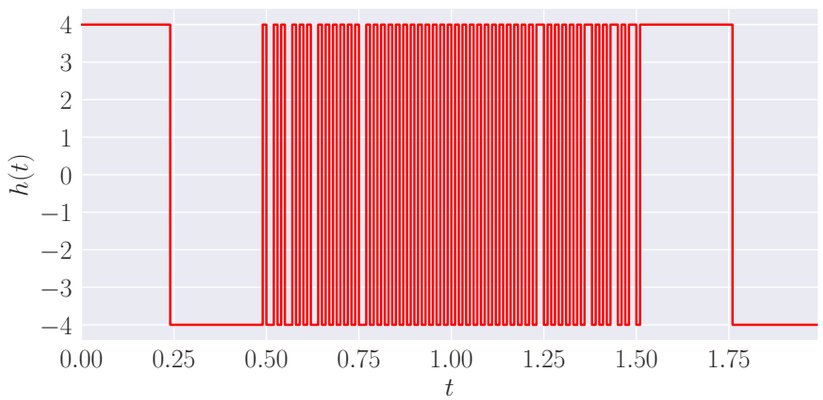

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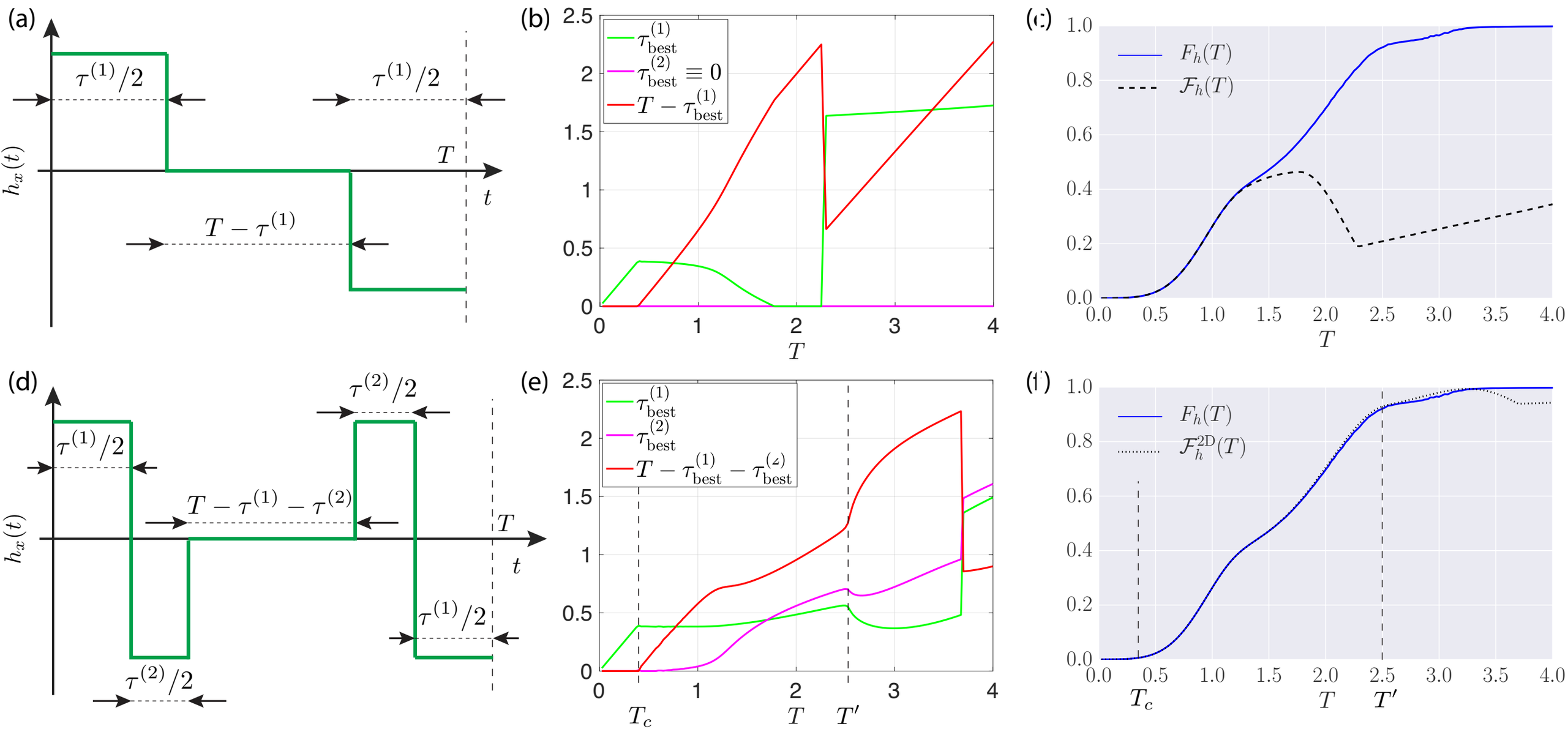

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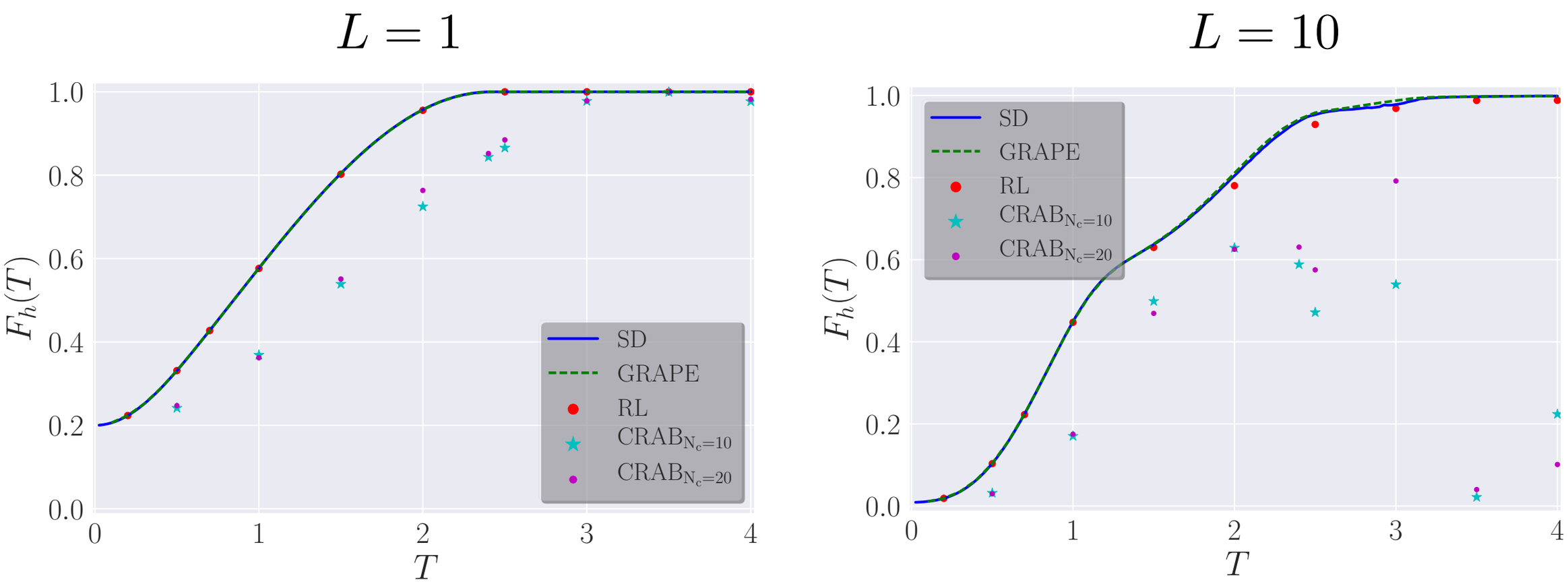
RL vs OC



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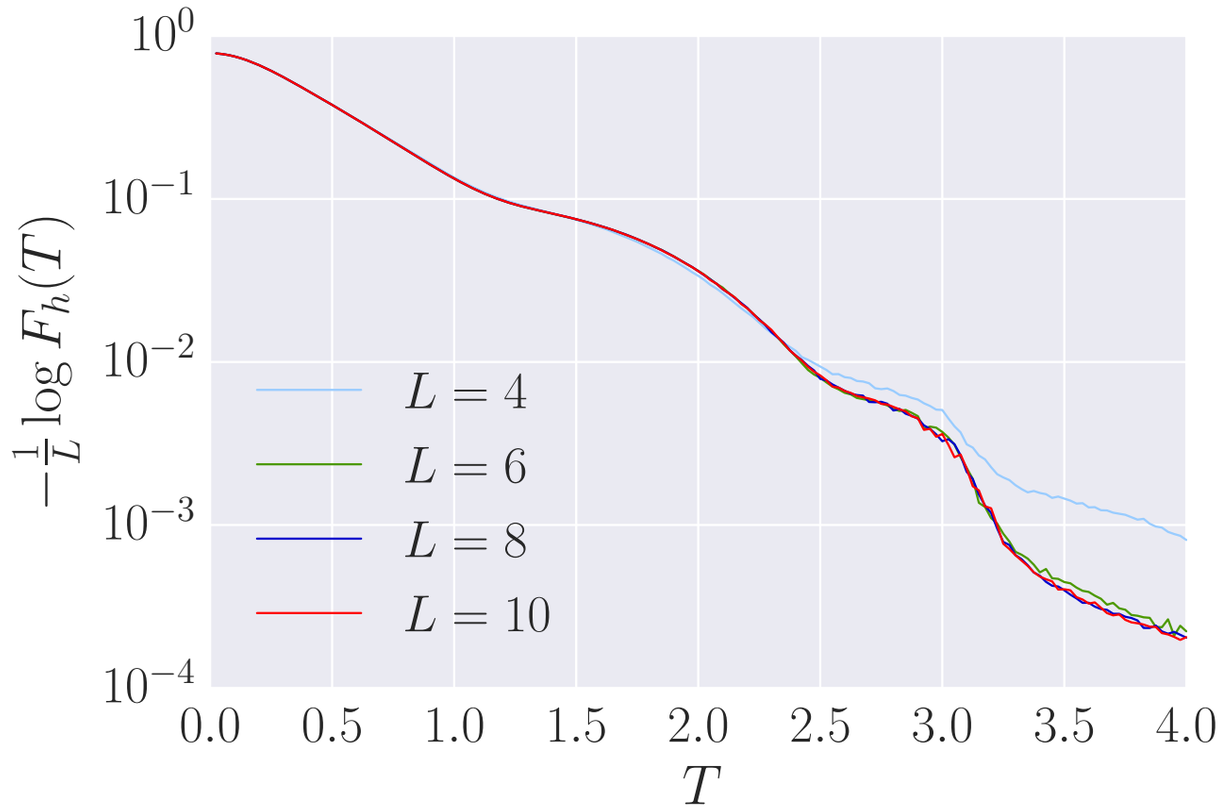
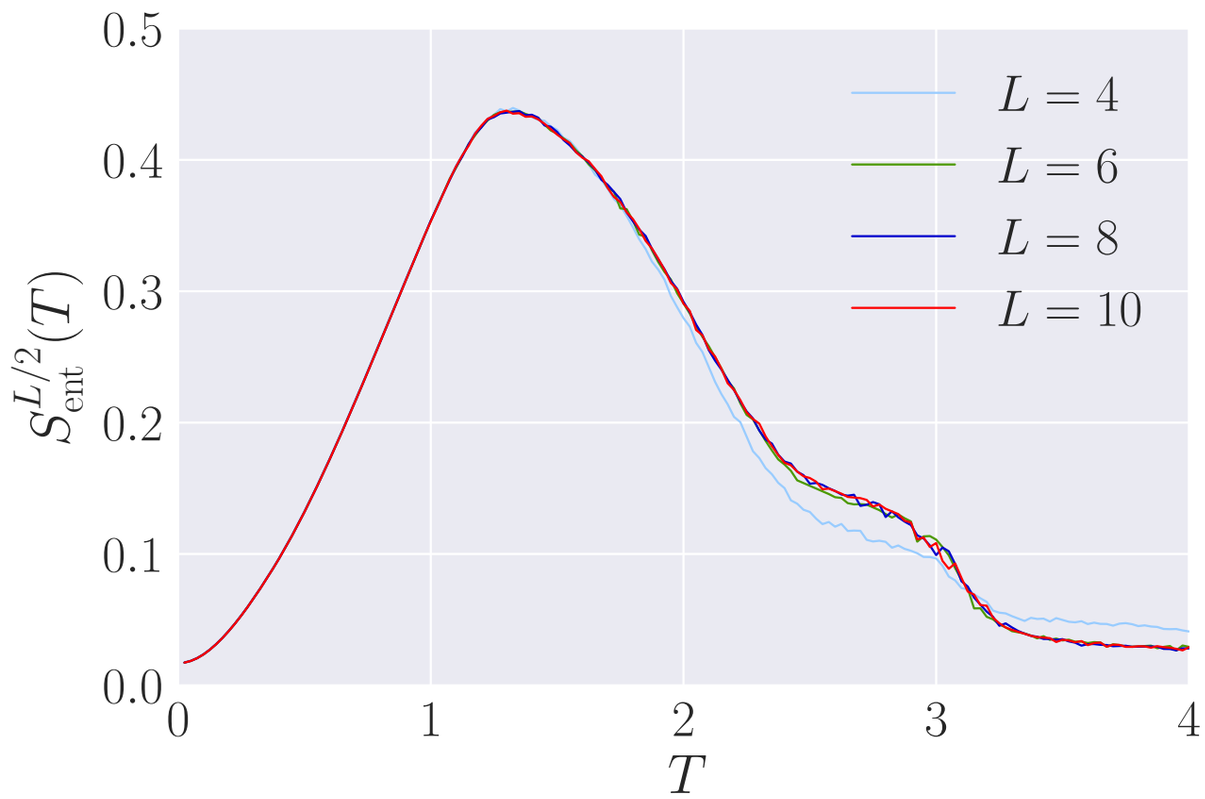
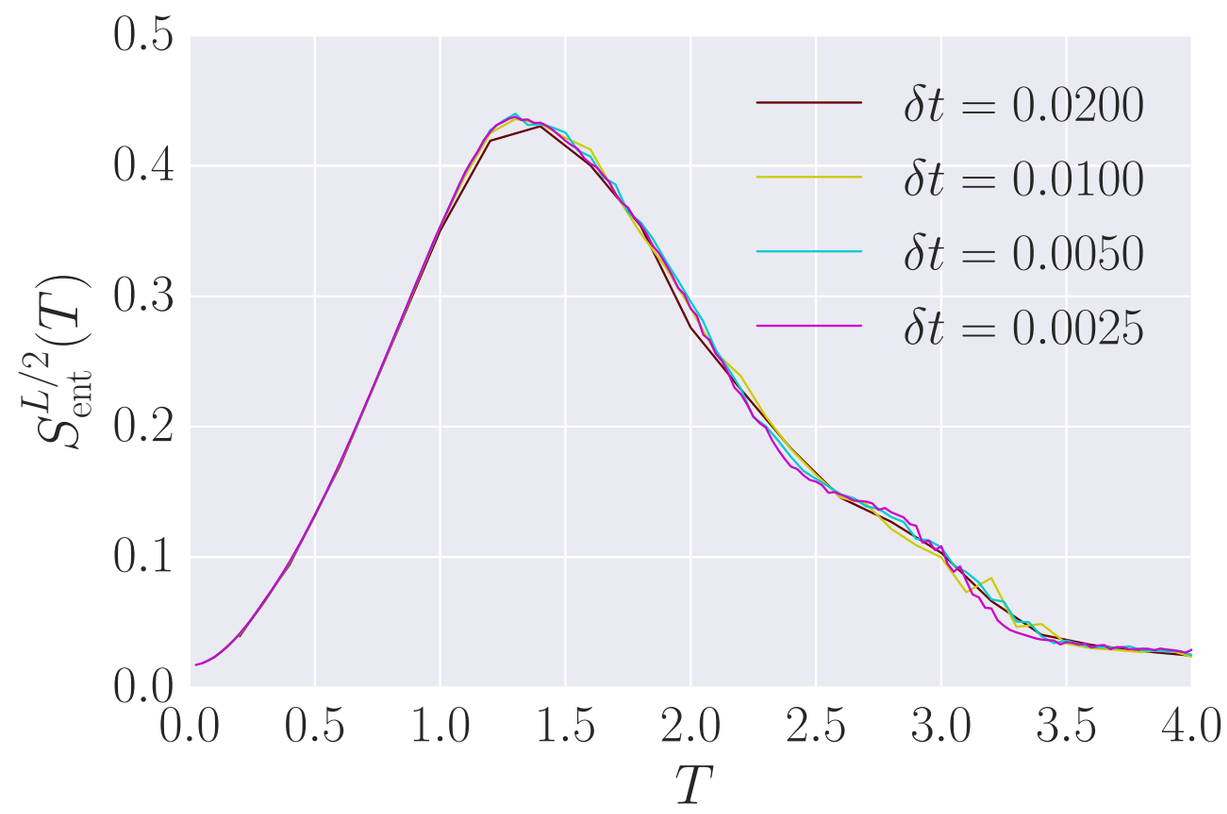
System Size Dependence







PRX 8 031086 (2018)

Properties of Effective Model $\mathcal{H}_{\text{eff}}(T)$

→ properties of effective coupling strengths: GS in frustrated!

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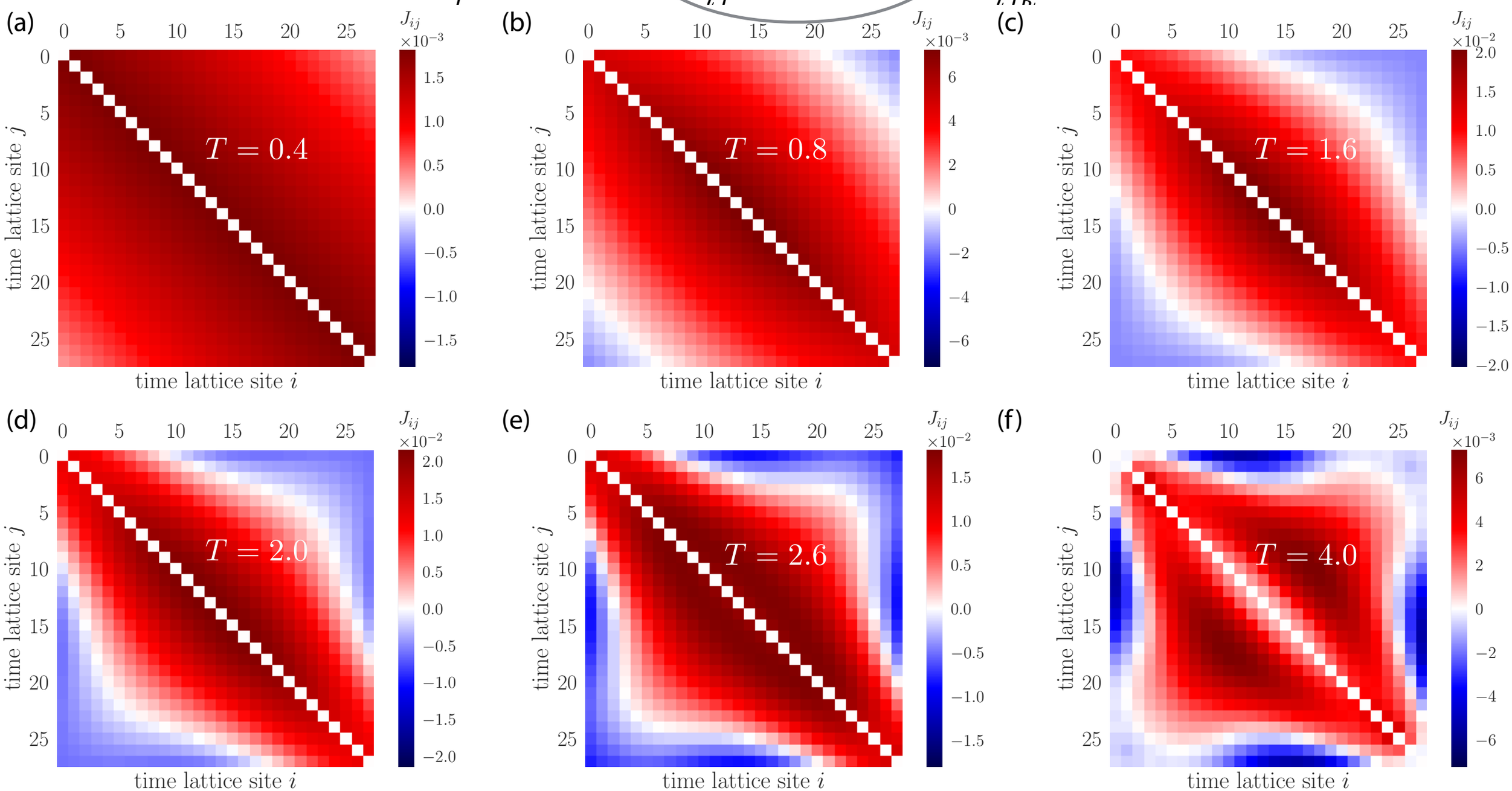

PRL 122, 020601 (2019)

Spontaneous Symmetry Breaking

$$H(t) = -2S_1^z S_2^z - (S_1^z + S_2^z) - h_x(t)(S_1^x + S_2^x)$$

$\mathbb{Z}_2$ *symmetry of protocols:*

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PRA 97, 052114 (2018)

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→ *either:* symmetric & unique

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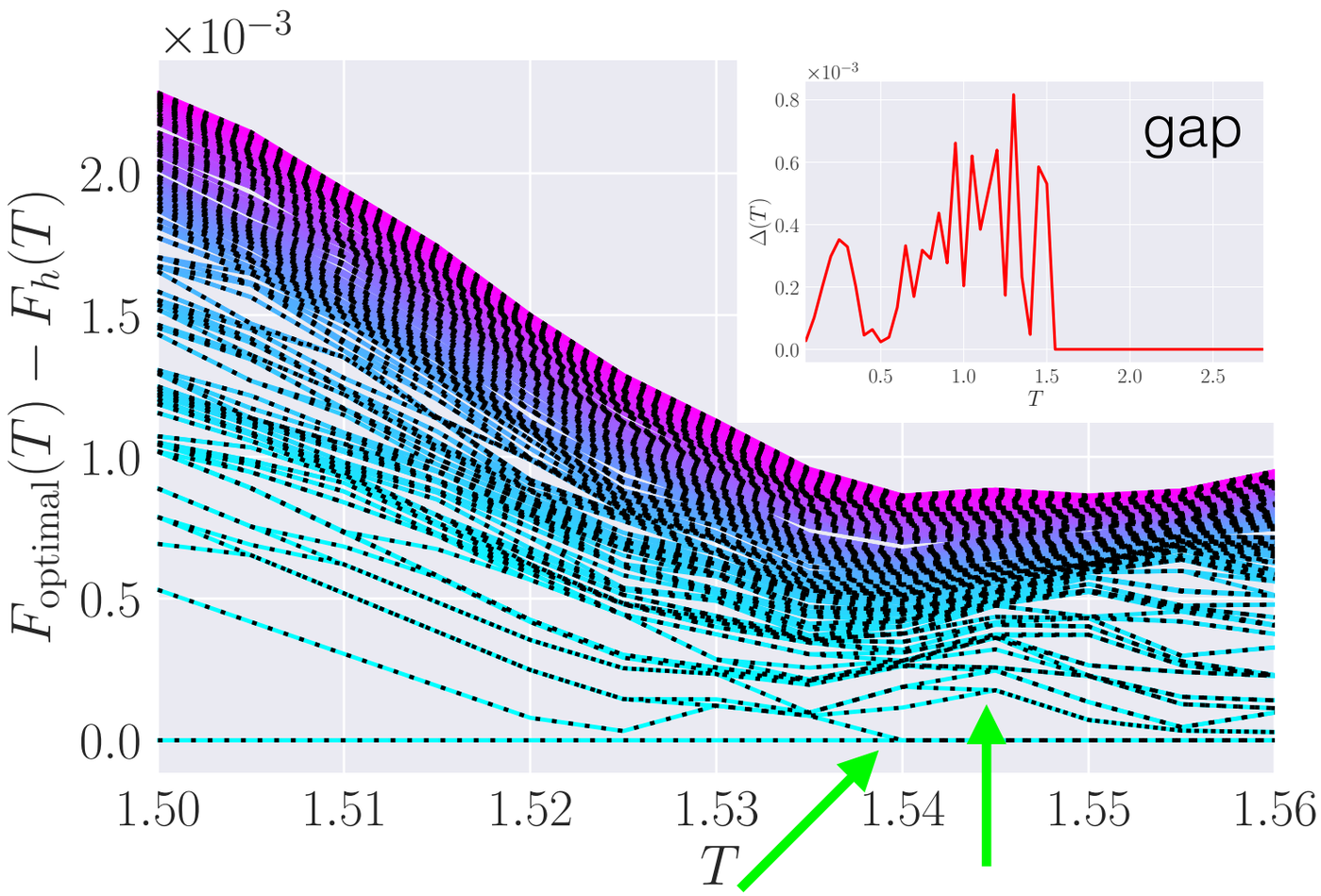
'spectrum' of effective classical model $\mathcal{H}_{\text{eff}}(T)$



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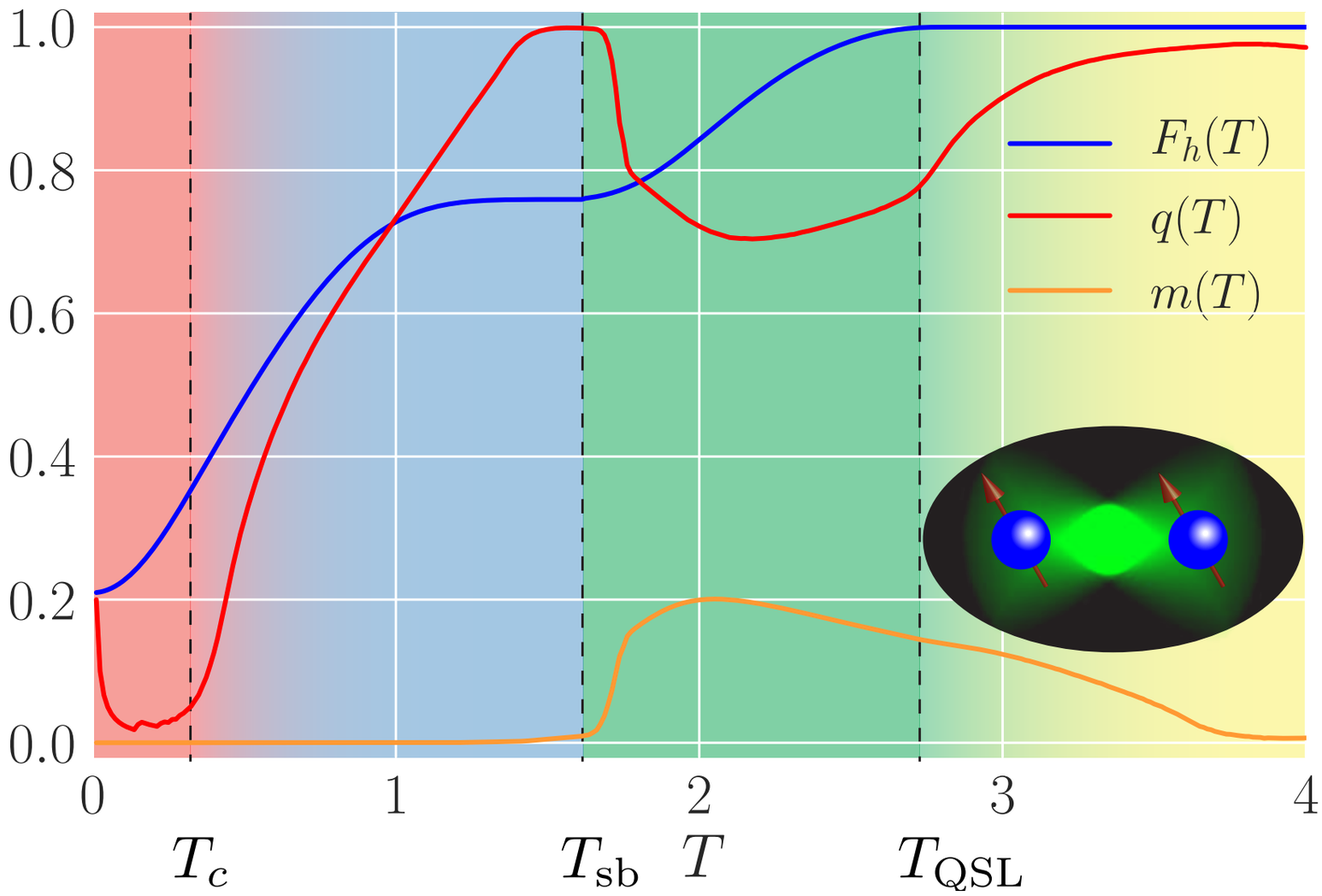

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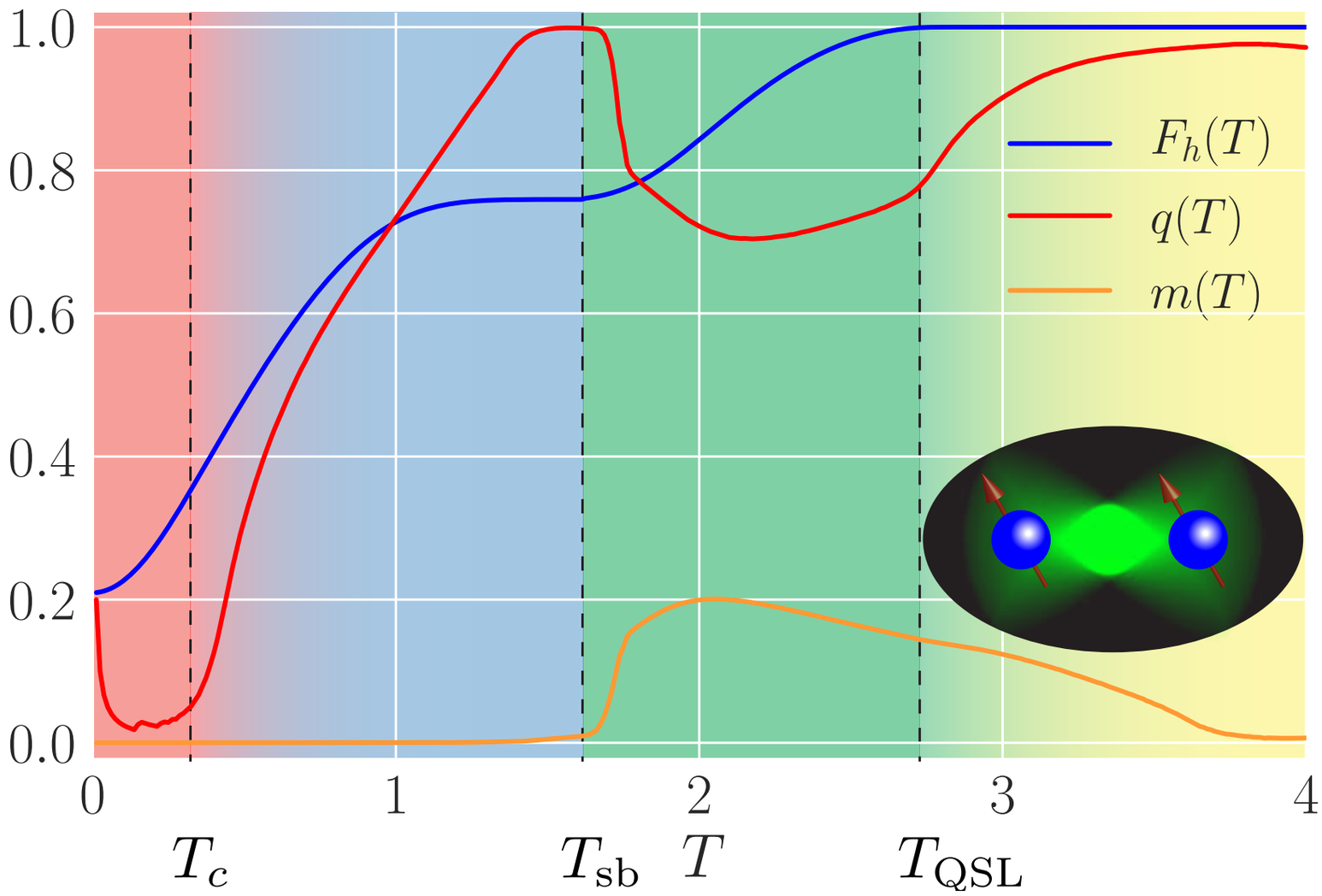

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→ "magnetisation" order parameter: $m(T) = \frac{1}{N_{\rm real}} \sum_{\alpha=1}^{N_{\rm real}} |m_{h^\alpha}(T)|$

PRA 97, 052114 (2018)

Implications for Physics

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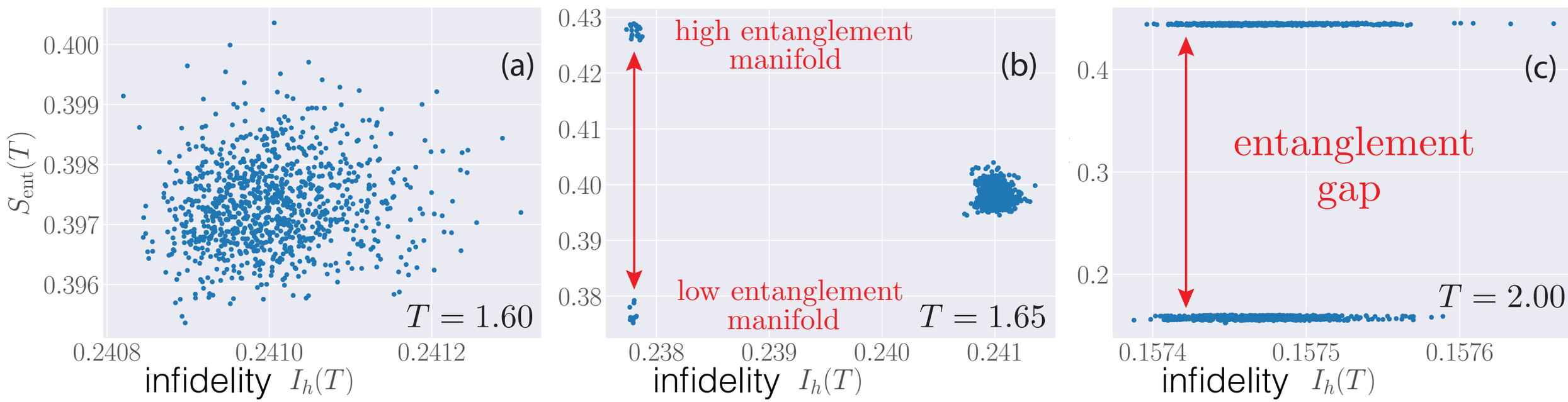

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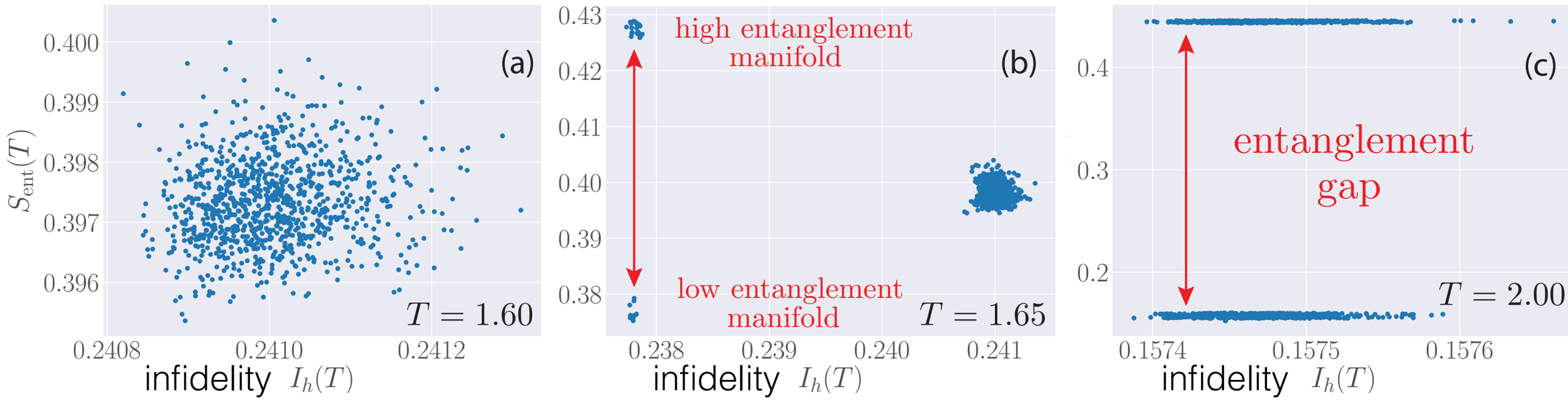

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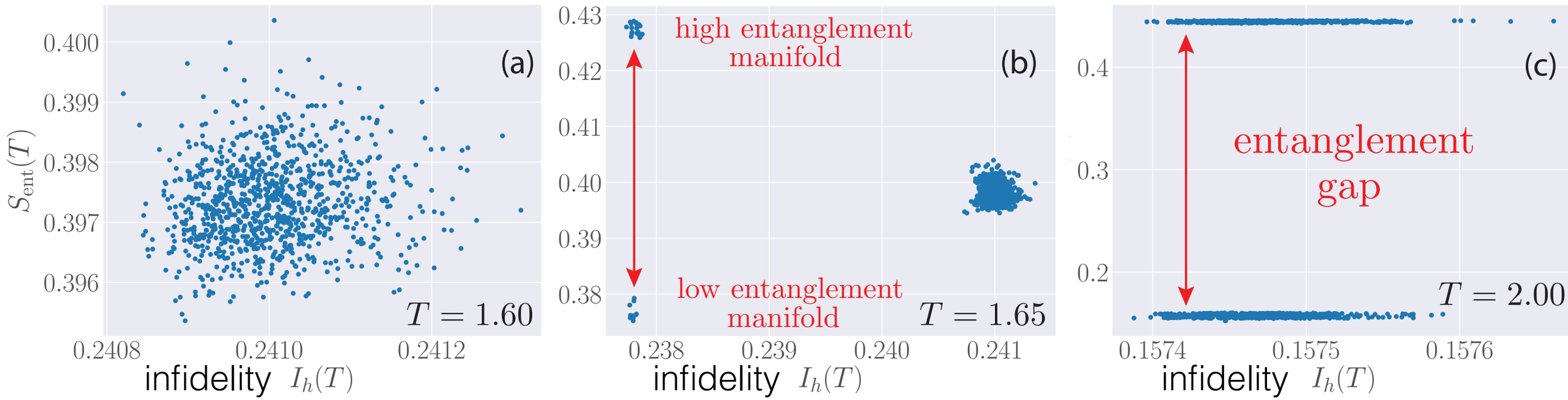
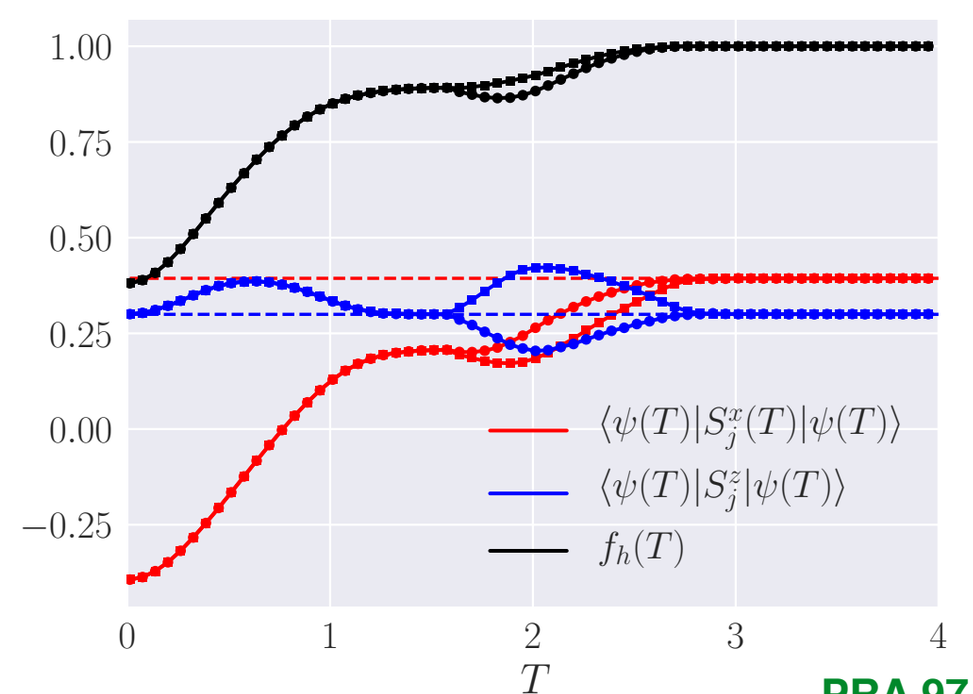


→ can be detected in expectations of local observables



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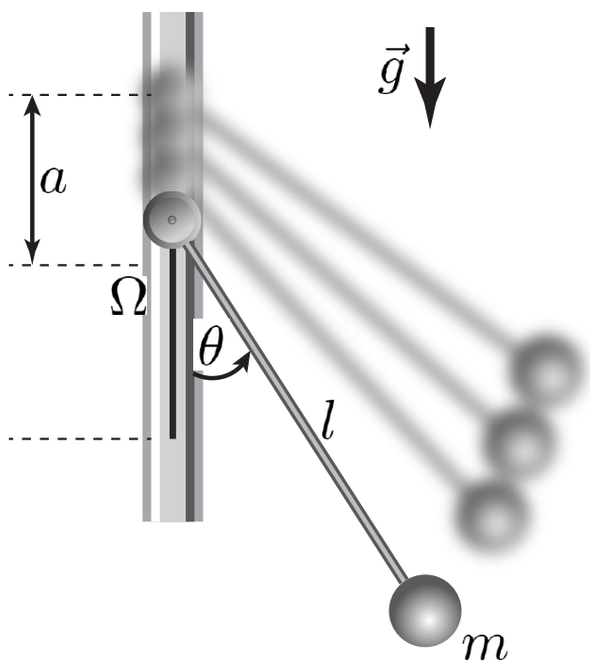
Example:

use RL for autonomous preparation
non-equilibrium states in a ***simulation of an "experiment"***

→ Kapitza oscillator



Marin Bukov

bluedwarf1127: YouTube (2009)

Berkeley
UNIVERSITY OF CALIFORNIA

Example:

use RL for autonomous preparation
non-equilibrium states in a ***simulation of an "experiment"***

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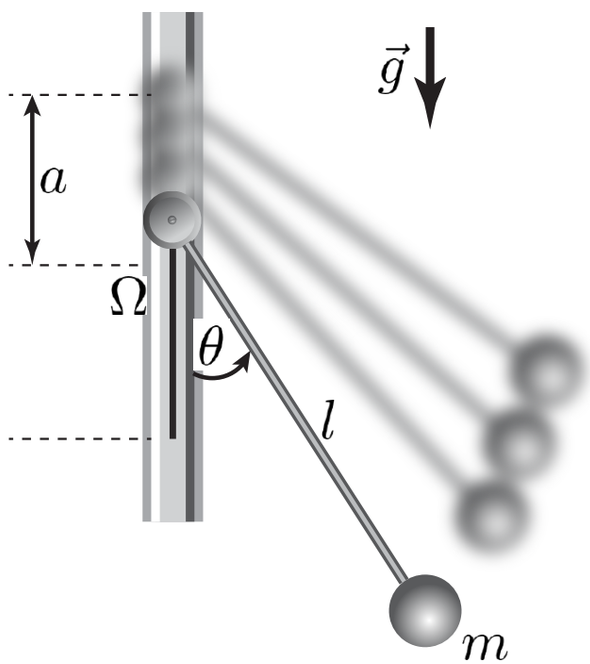

Marin Bukov

bluedwarf1127: YouTube (2009)

The quantum Kapitza oscillator

→ how do we understand ‘dynamical stabilization’?

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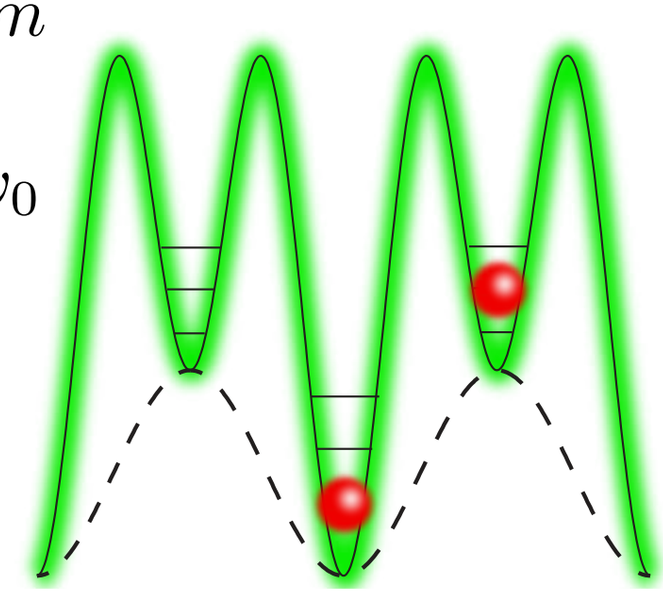

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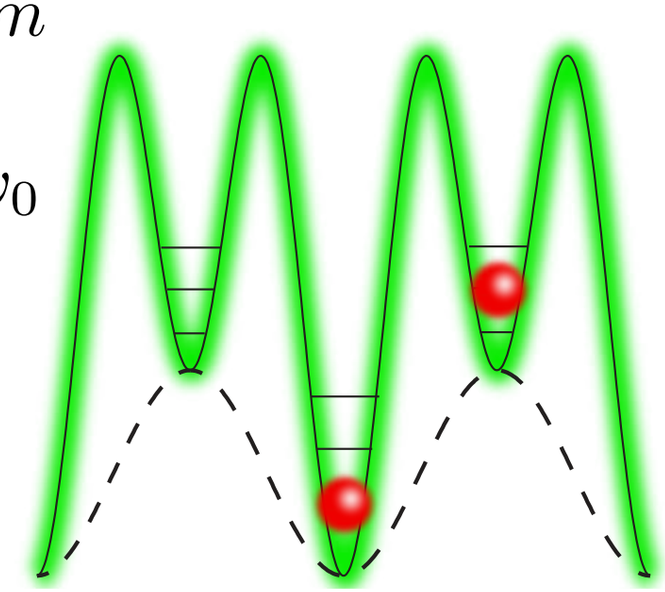

→ finite frequencies: Floquet Hamiltonian $H_F(\Omega)$

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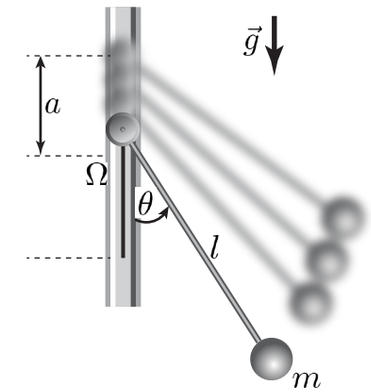
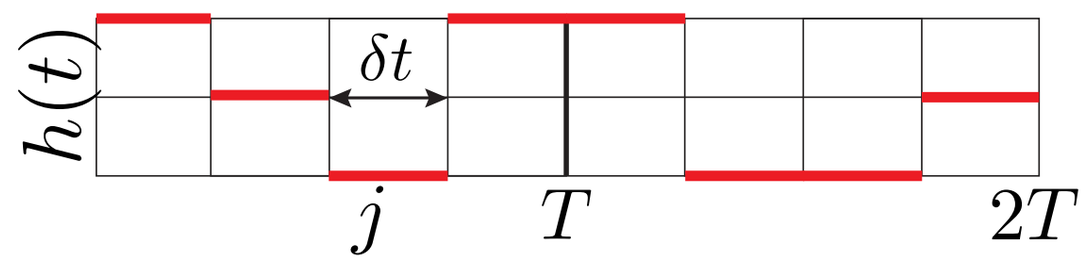
Floquet Control Problem



→ find optimal control field ***on top of periodic drive***

$$H_{\rm rot}(t) = H_0 + H_{\rm drive}(t) + H_{\rm control}(t)$$

$H_{\rm control}(t) = h(t)\sin\theta$ horizontal kicks



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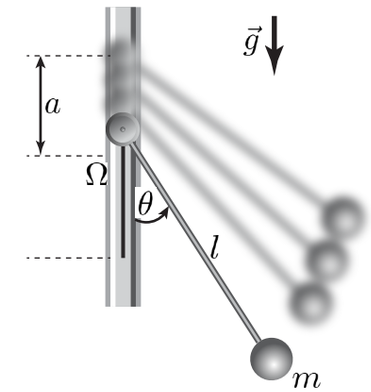

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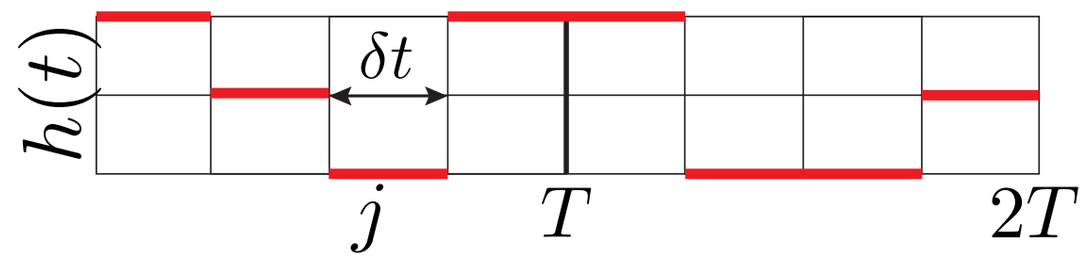
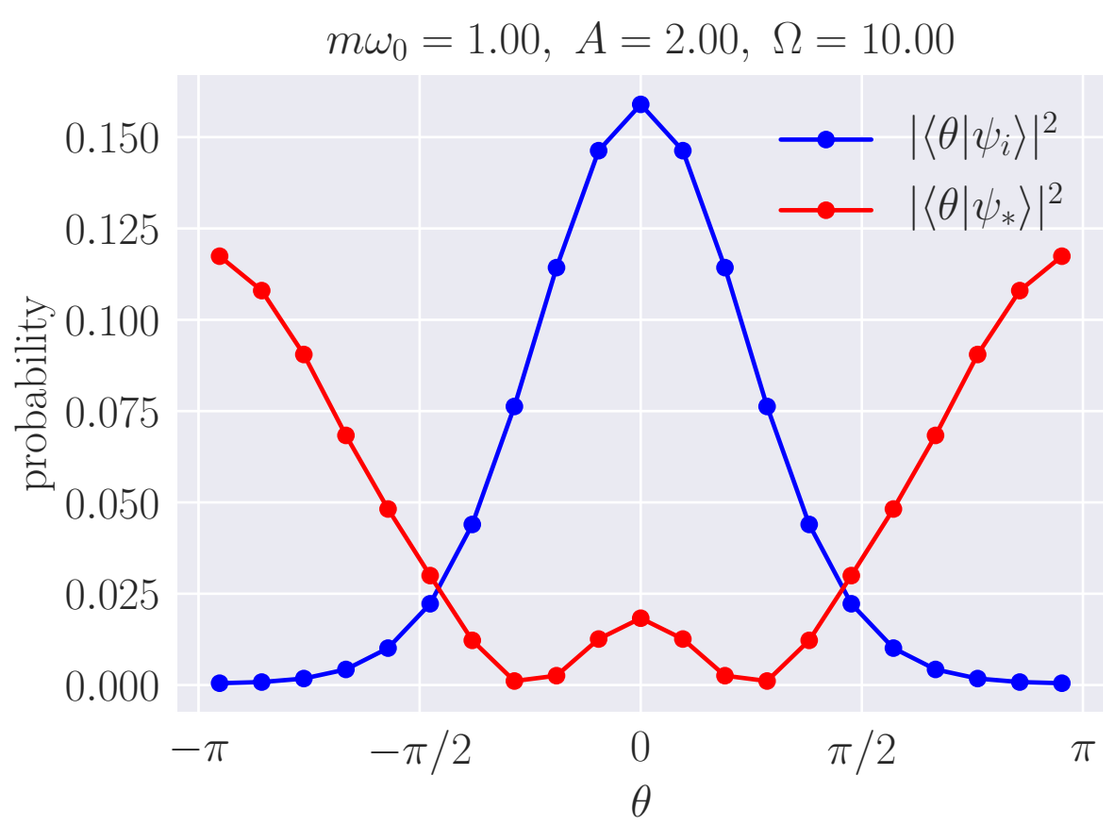


initial state: $|\psi_i\rangle$: GS of H_0

target state: $|\psi_*\rangle$ inverted position eigenstate of $H_F(\Omega)$



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Simulation of a Quantum Experiment

→ **no** direct **access** to quantum state:
"play game w/o looking at screen" (only know score)

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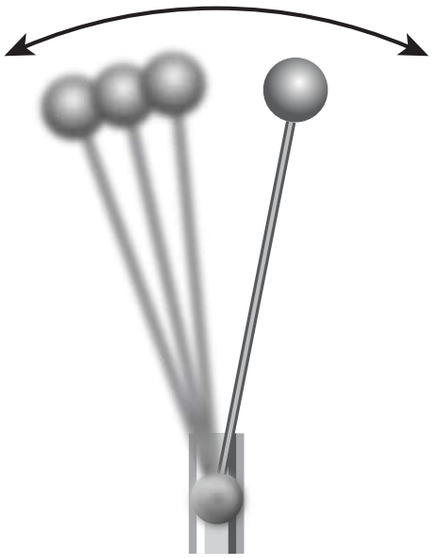

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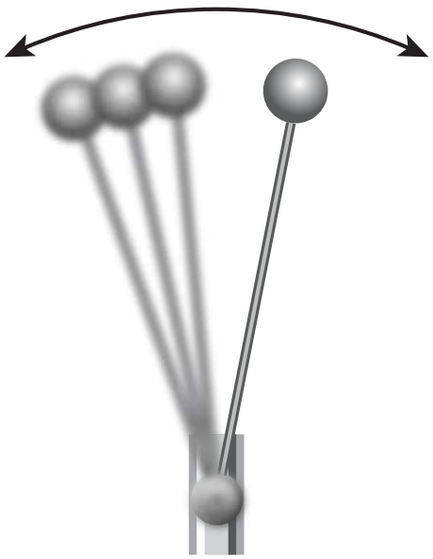

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→ **probabilistic** quantum measurements

$+1$ with probability $F_h(t_f) = |\langle\psi(t_f)|\psi_*\rangle|^2$
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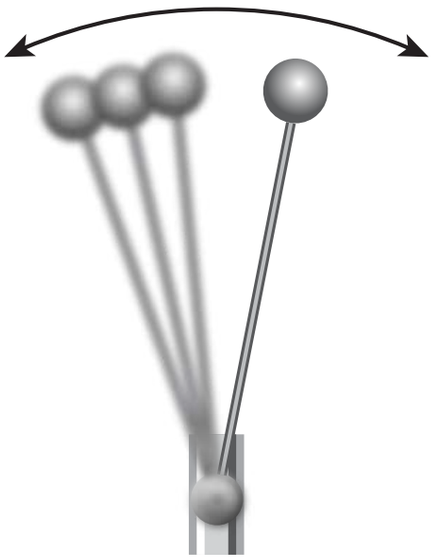

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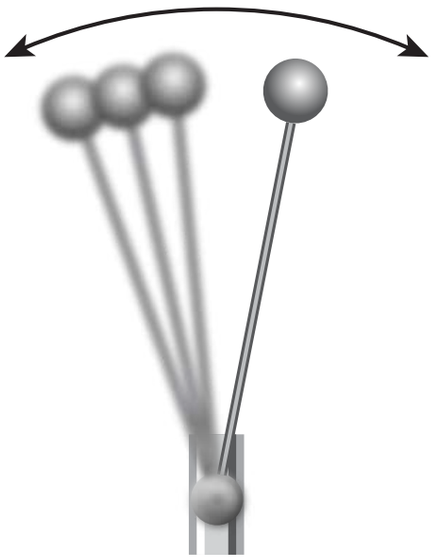

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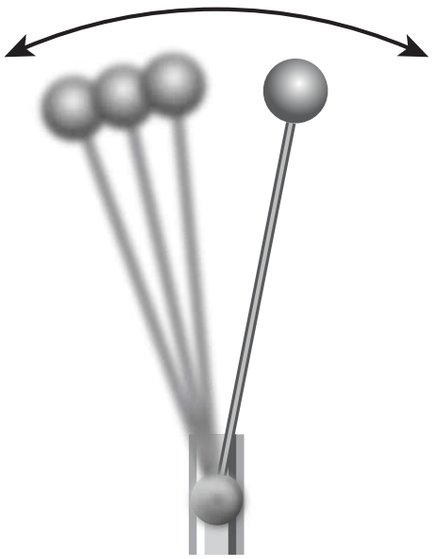

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→ *additionally:* all other problems of how to actually prepare
the state if the above were absent and no analytic solution is known

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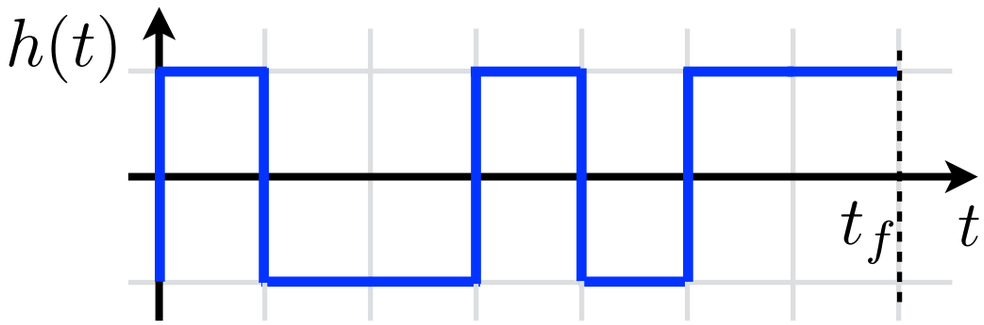

measurement: -1

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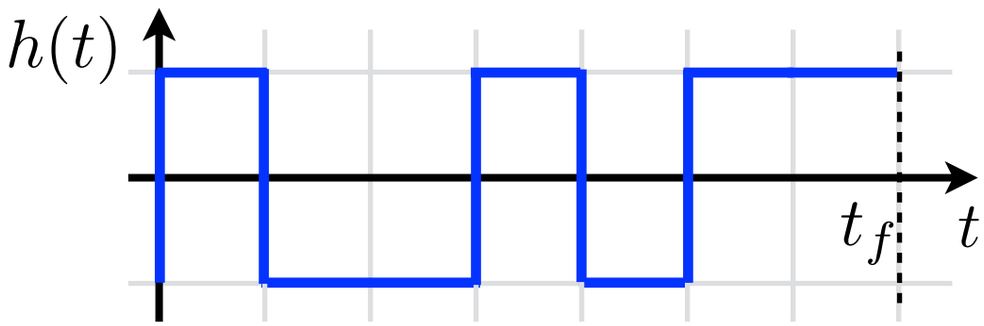

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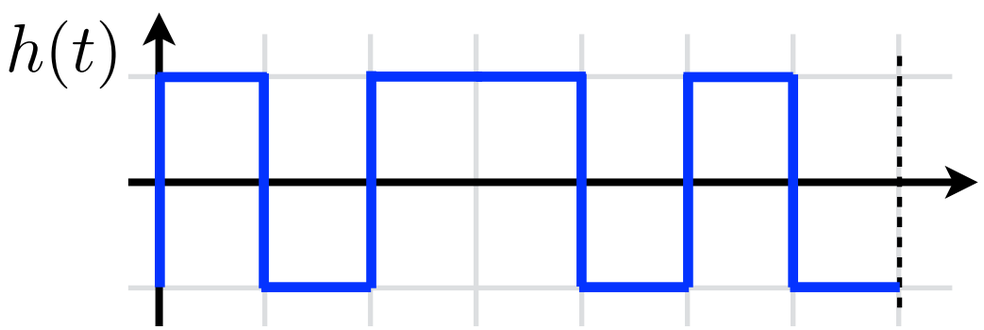

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(different final state: different probability to be in the target state)

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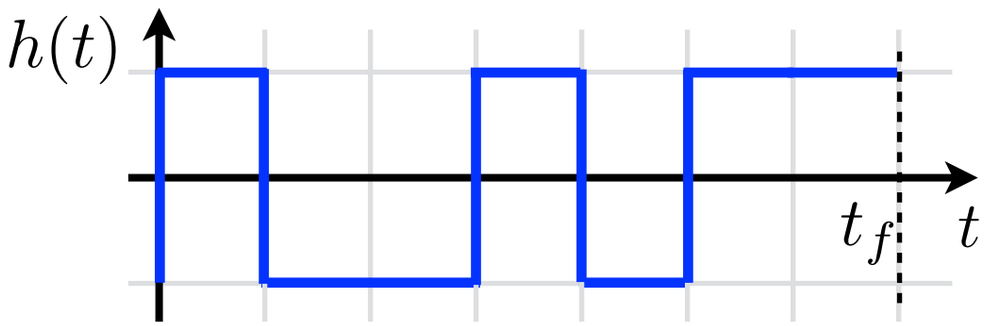

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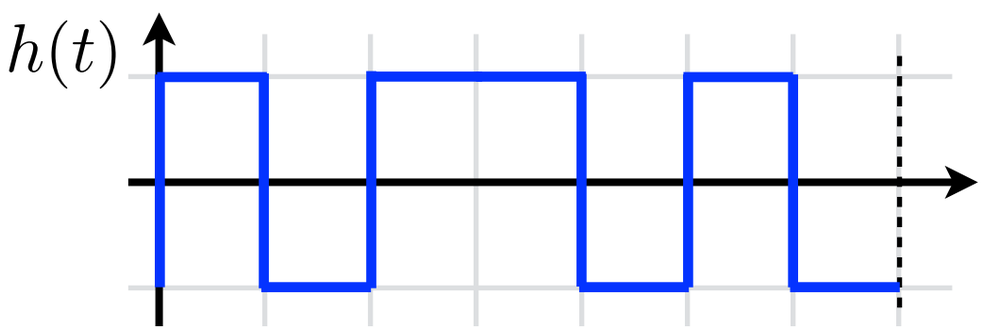

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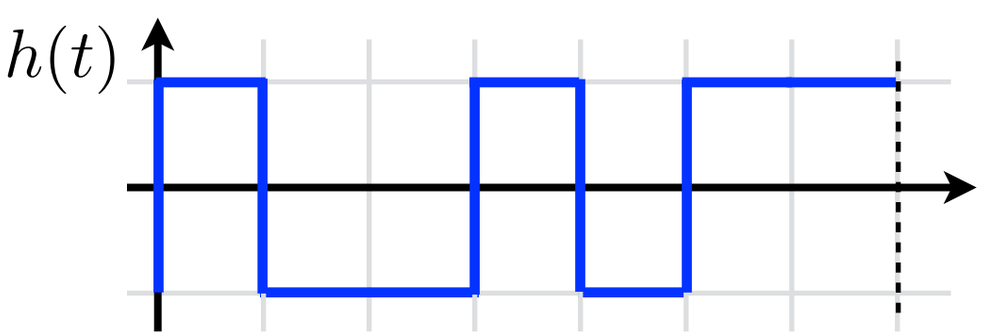

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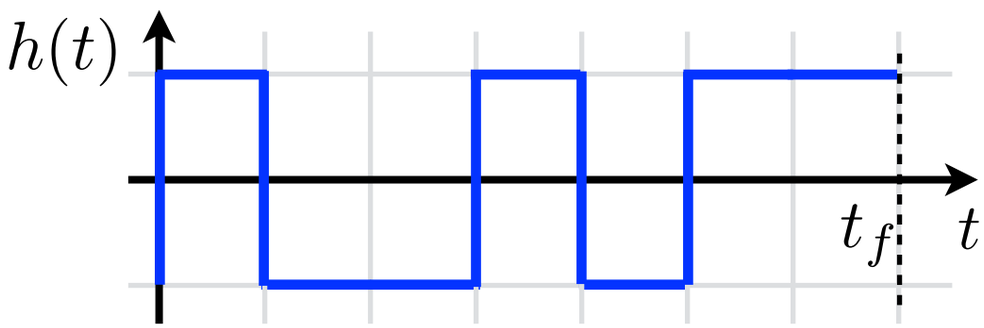

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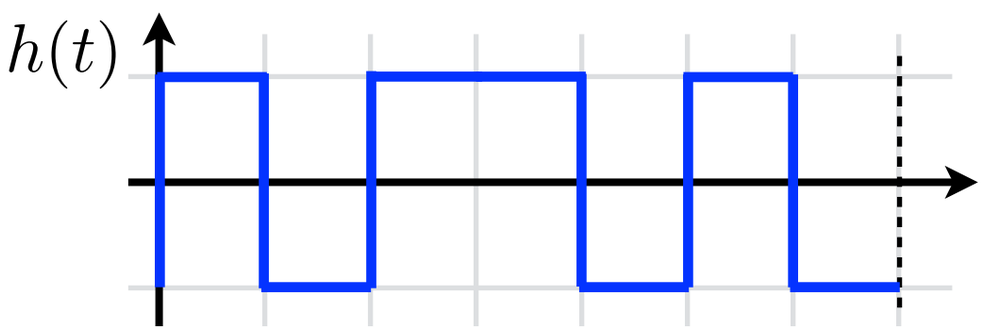

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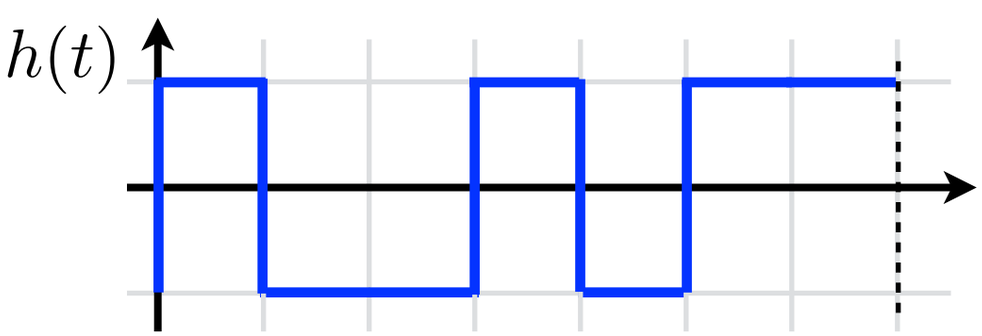

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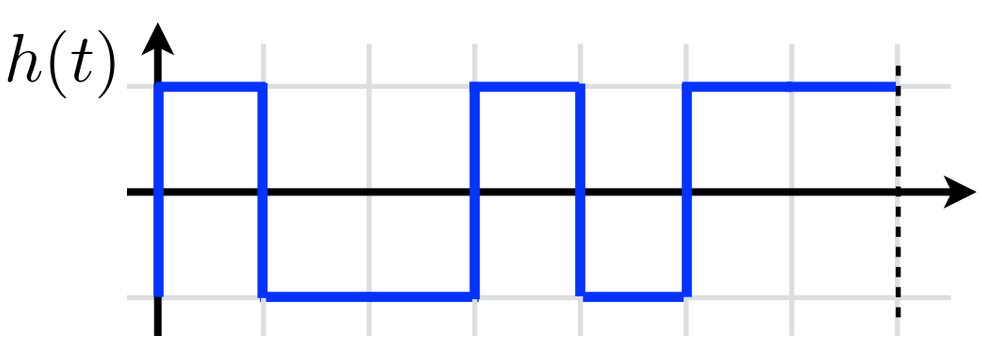

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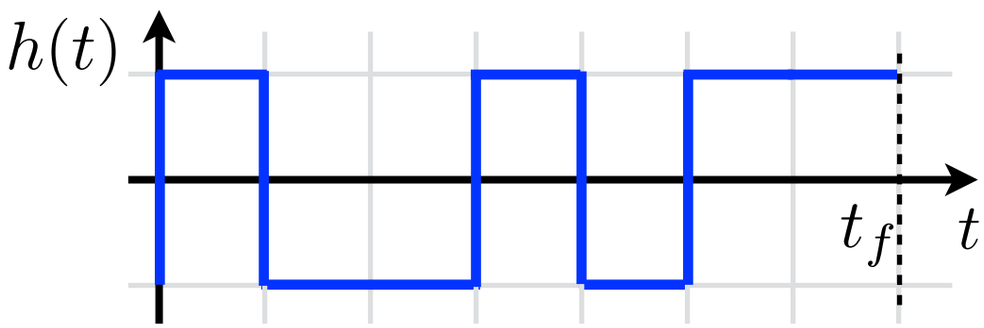

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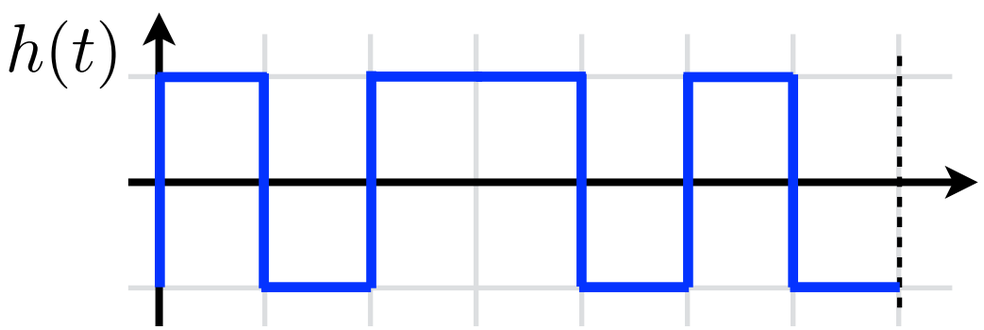

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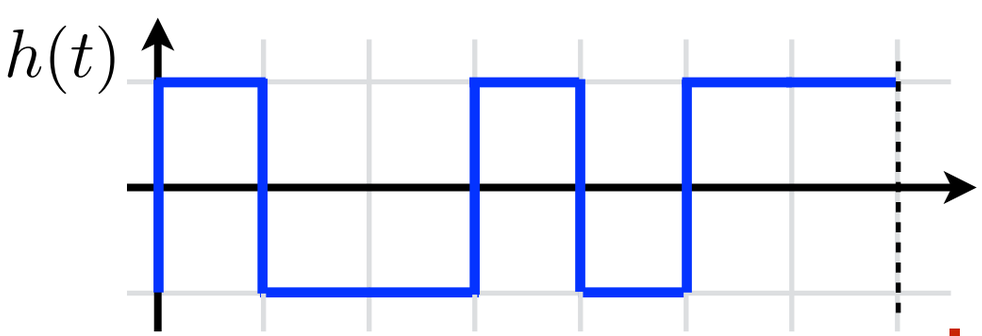
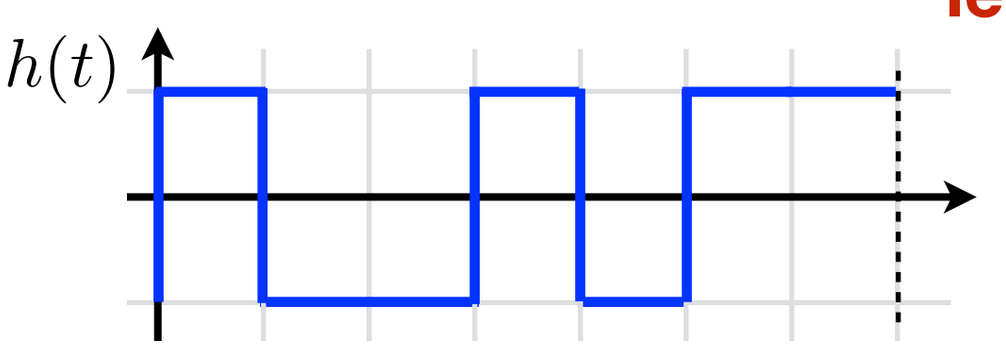


measurement: $+1$

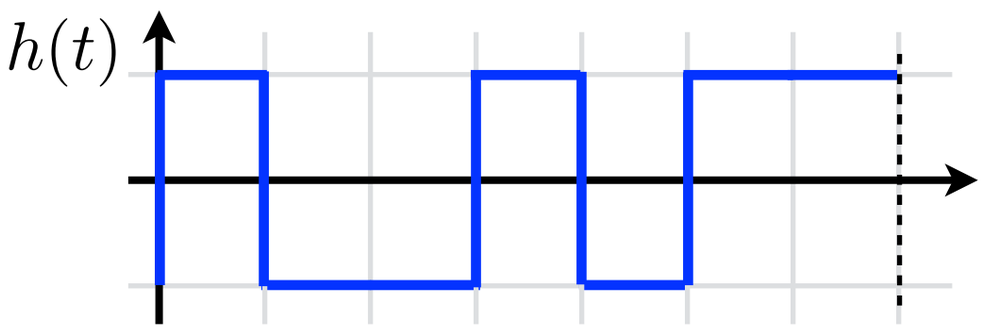
learn from noisy, nondeterministic rewards



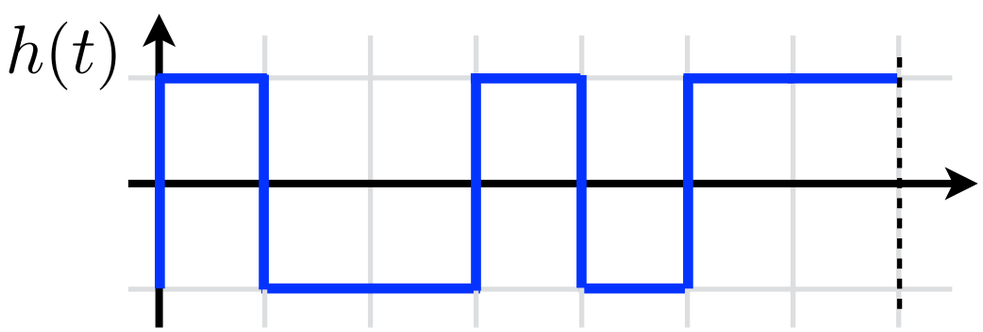
measurement: -1

Let's get rid of this 'quantumness' for a sec

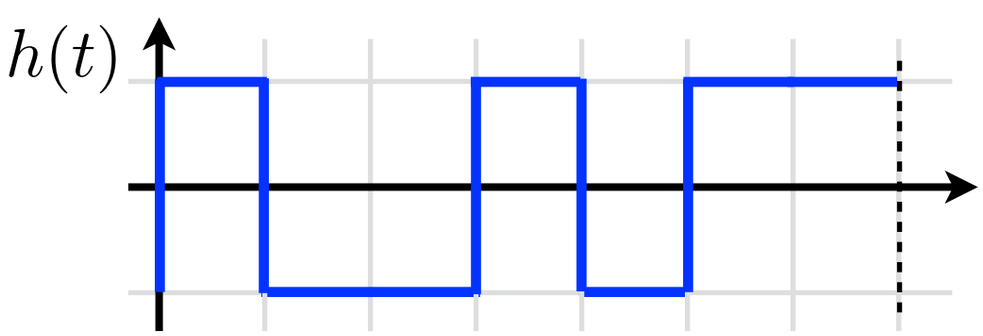
→ repeat protocol again!



measurement:
$F_h = |\langle\psi(T)|\psi_*\rangle|^2 = 0.632$



measurement: $F_h = 0.592$



measurement: $F_h = 0.668$

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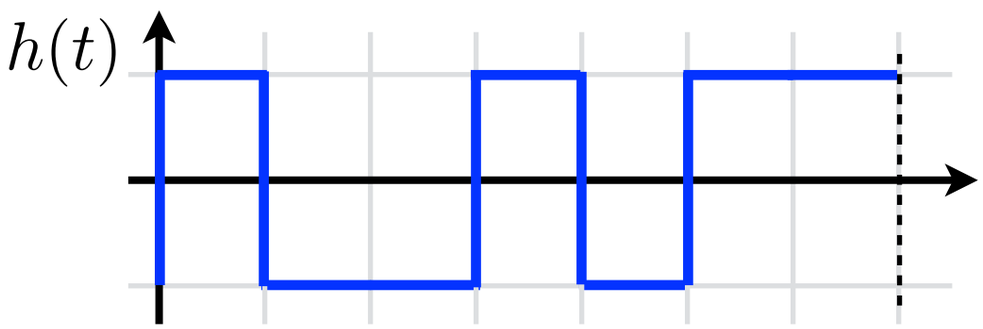

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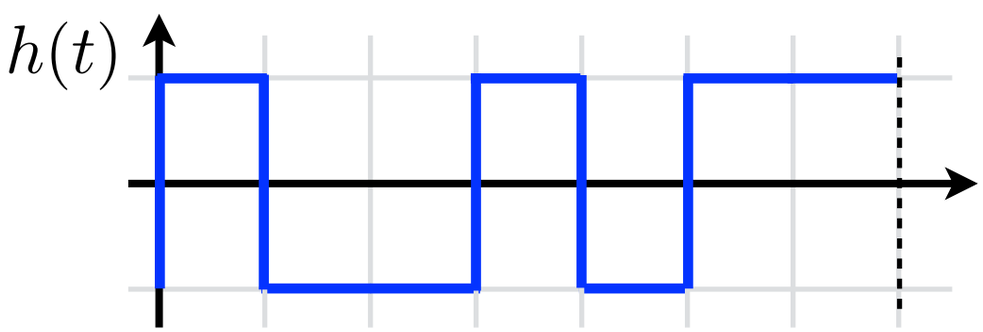
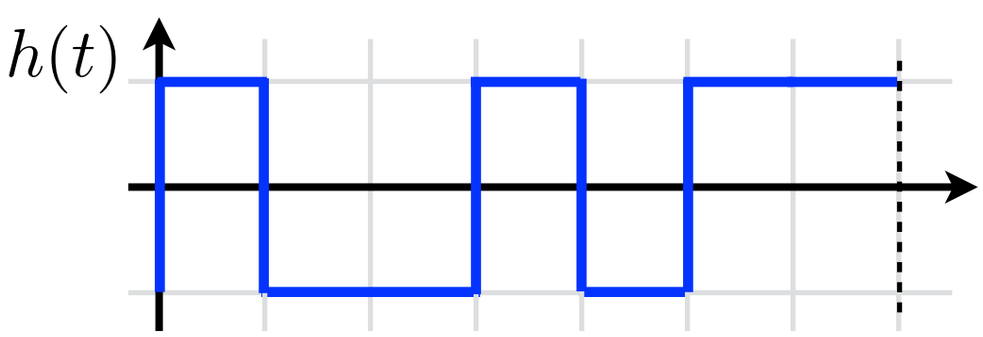


measurement: $F_h = 0.592$

initial state could not be prepared perfectly: more headache!



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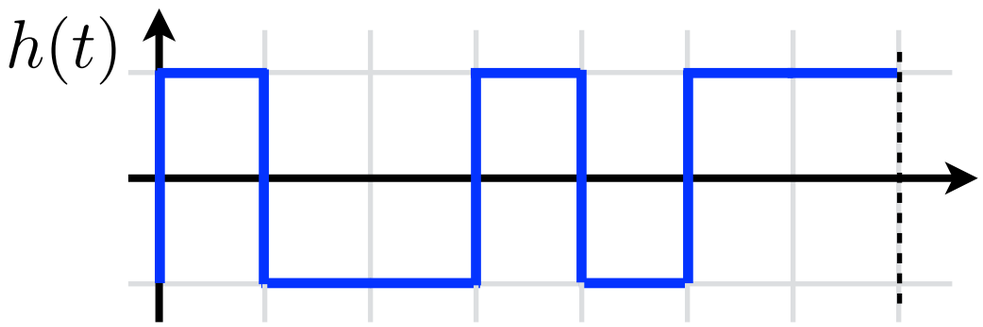
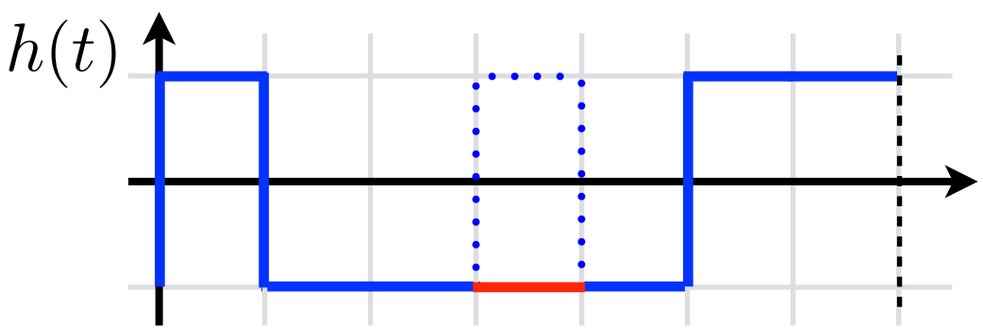
what if we fix the initial state:

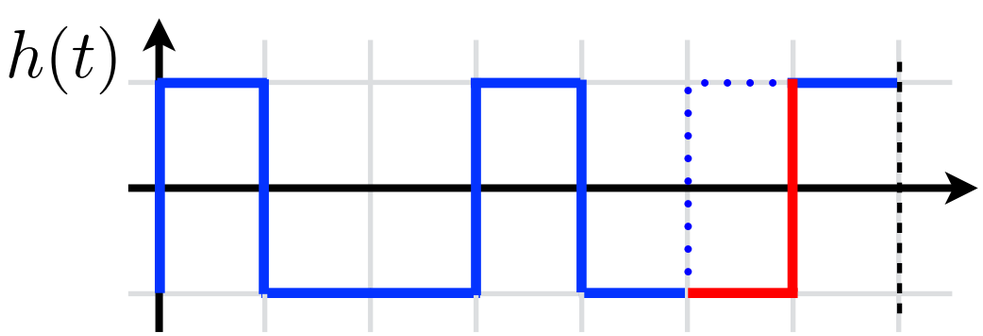
→ repeat protocol again!



measurement: $F_h = 0.627$



measurement: $F_h = 0.572$



measurement: $F_h = 0.657$

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what if we fix the initial state:

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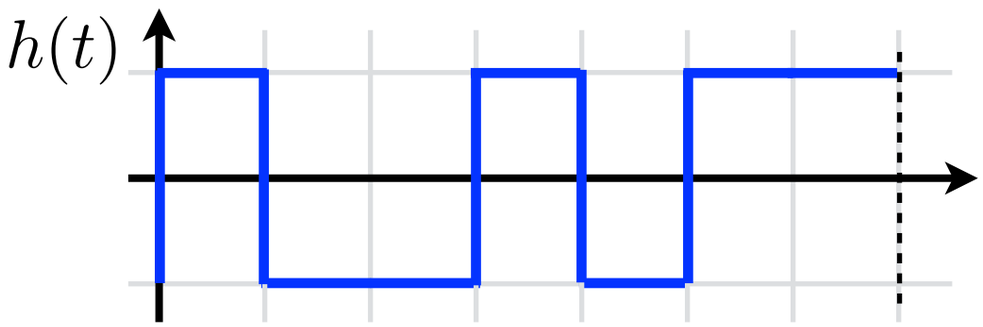

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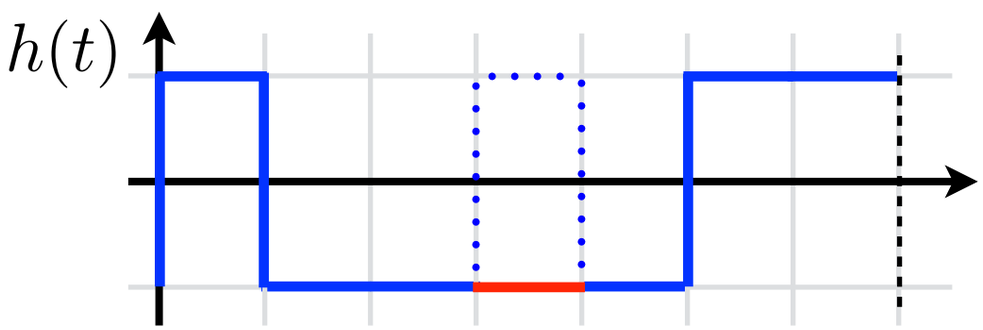
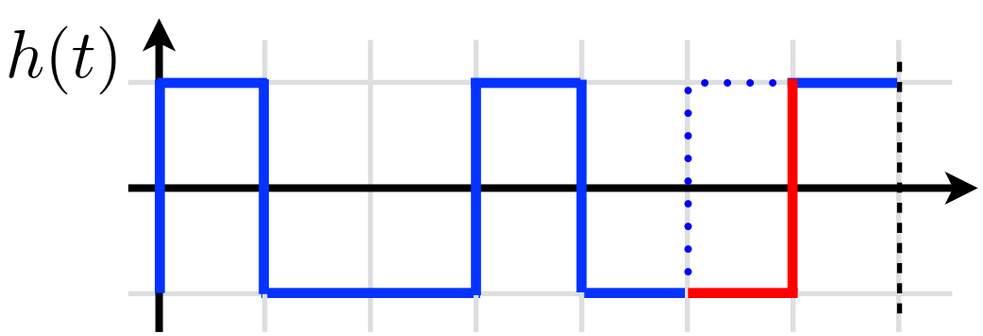


measurement: $F_h = 0.572$

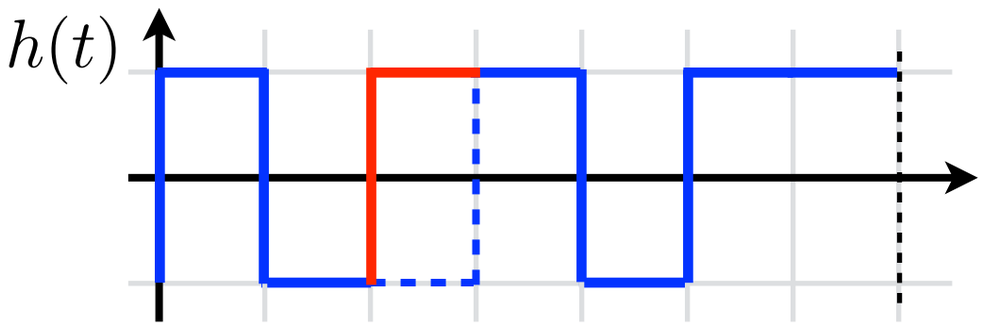
control apparatus failed: it can't be!



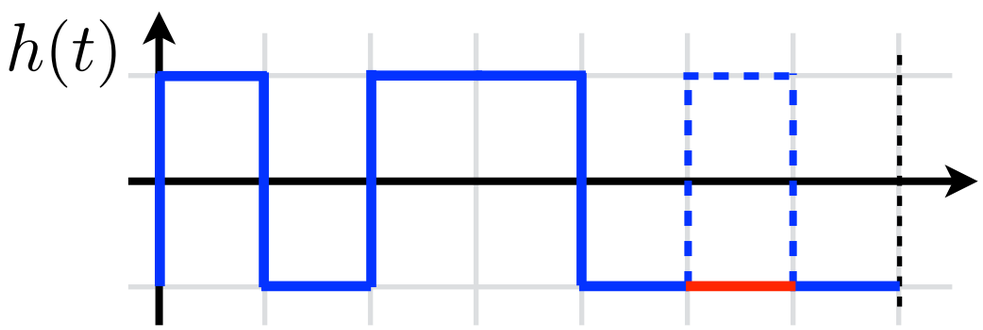
measurement: $F_h = 0.657$

PRB 98, 224305 (2018)

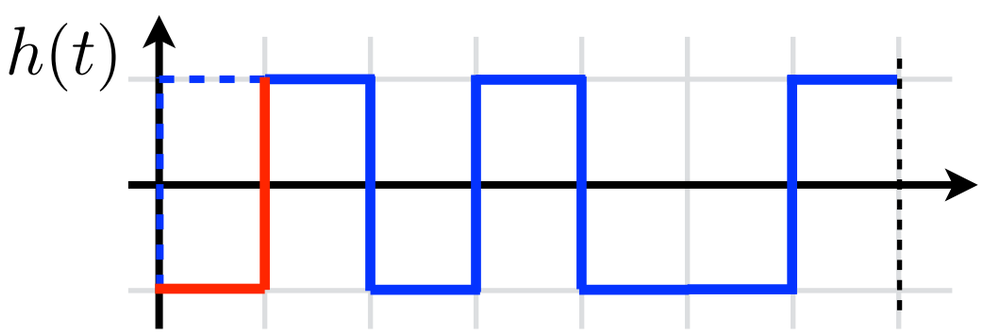
The Cruel Reality: all together (and probably much more!)



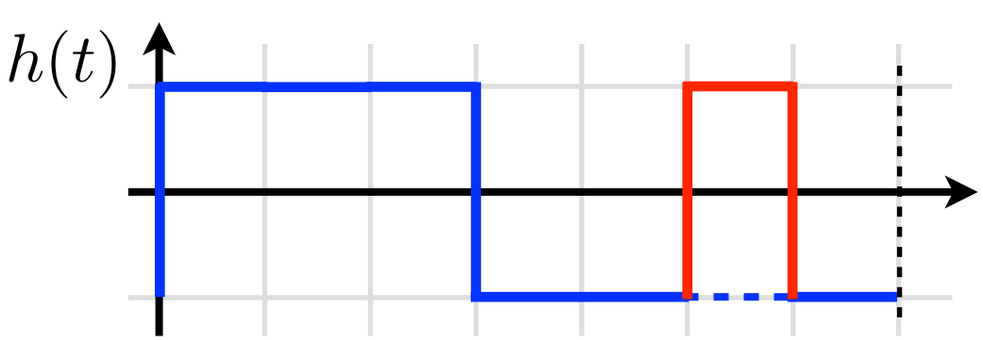
measurement: -1



measurement: $+1$



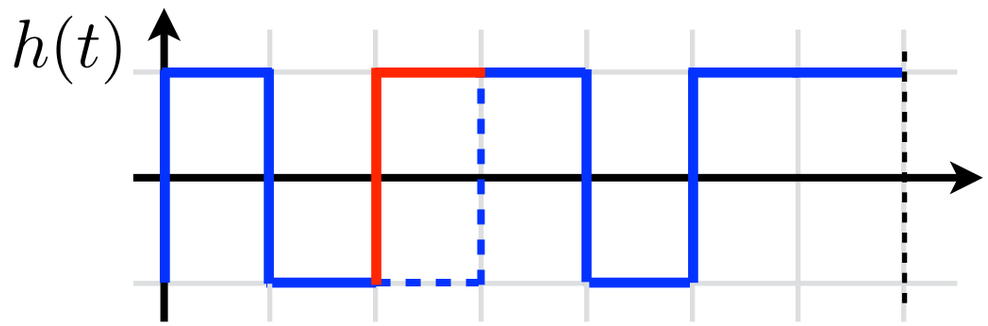
measurement: -1



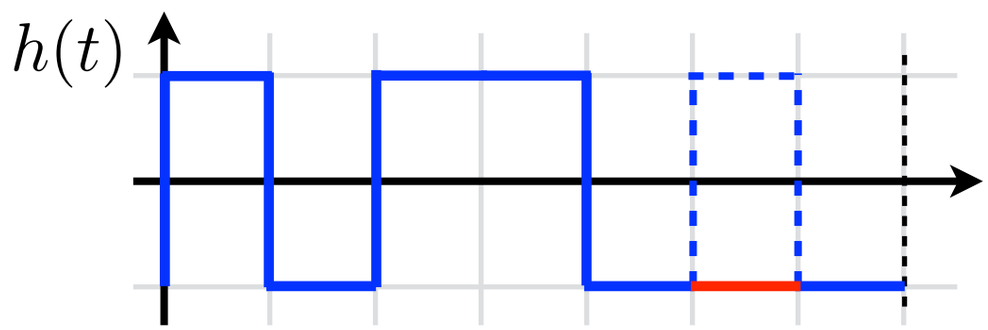
measurement: -1

PRB 98, 224305 (2018)

The Cruel Reality:
all together (and probably much more!)

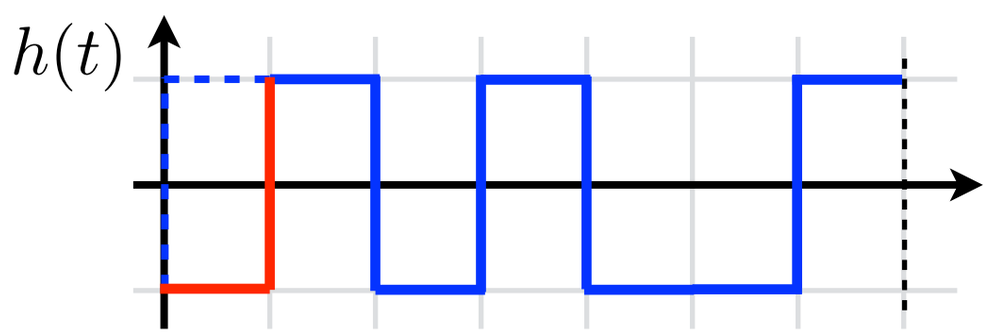


measurement: -1

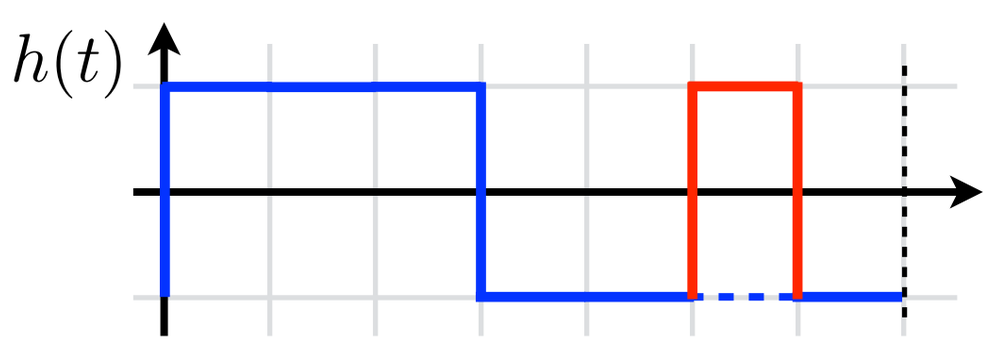


measurement: $+1$

extremely tedious task!



measurement: -1



measurement: -1

PRB 98, 224305 (2018)

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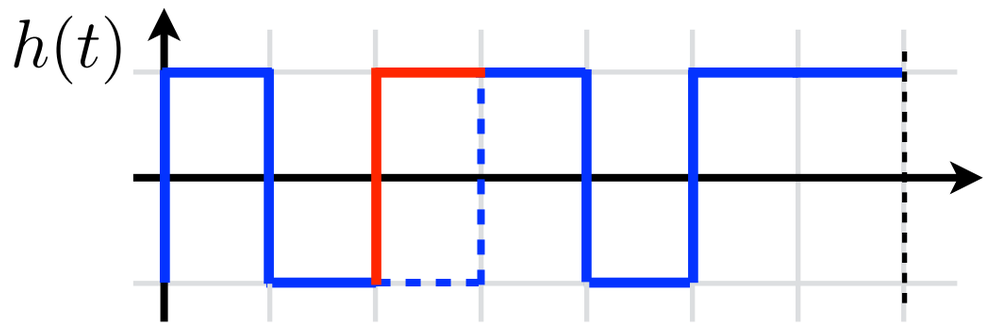
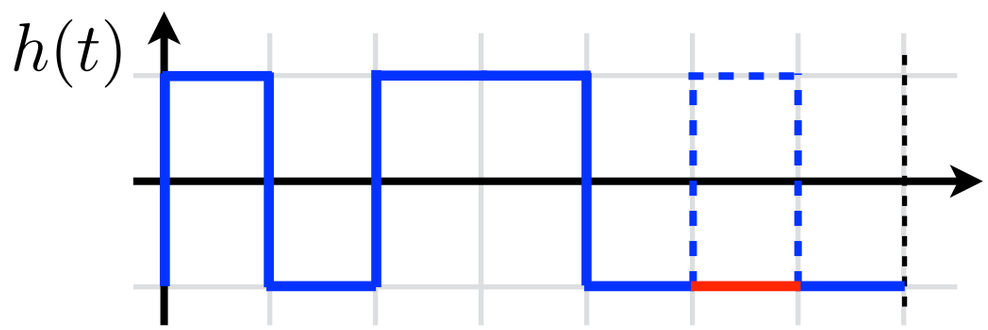


measurement: -1



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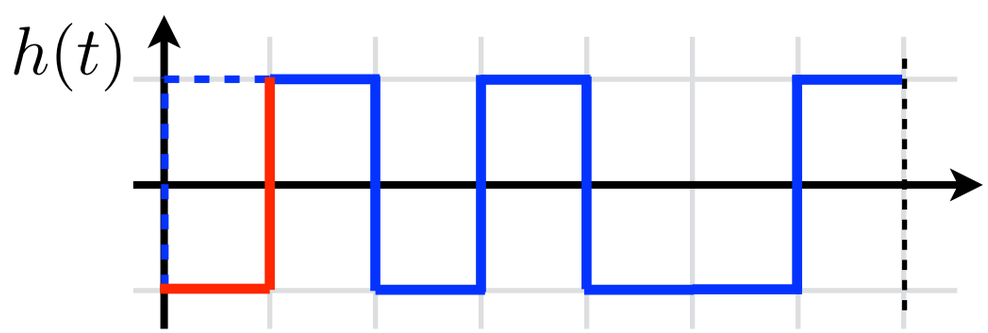
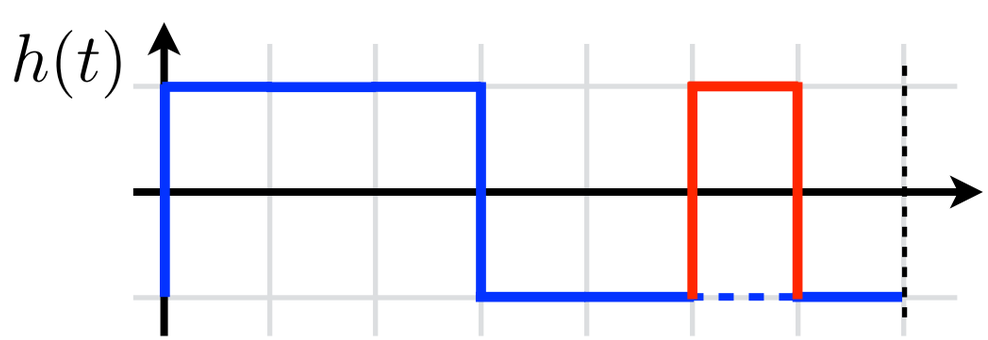


measurement: -1

how do we solve it efficiently?



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PRB 98, 224305 (2018)

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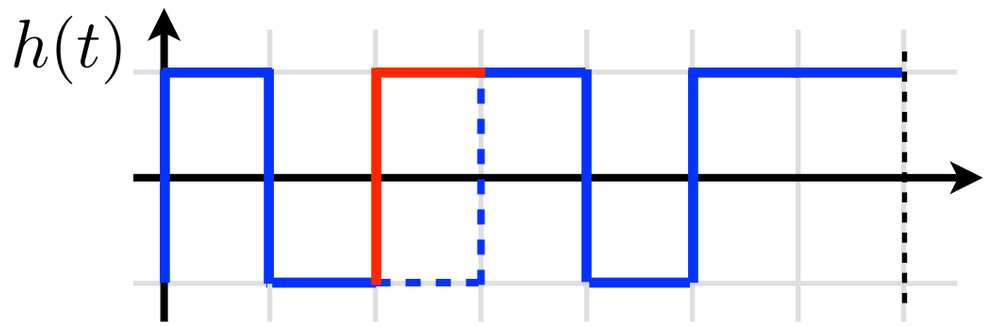

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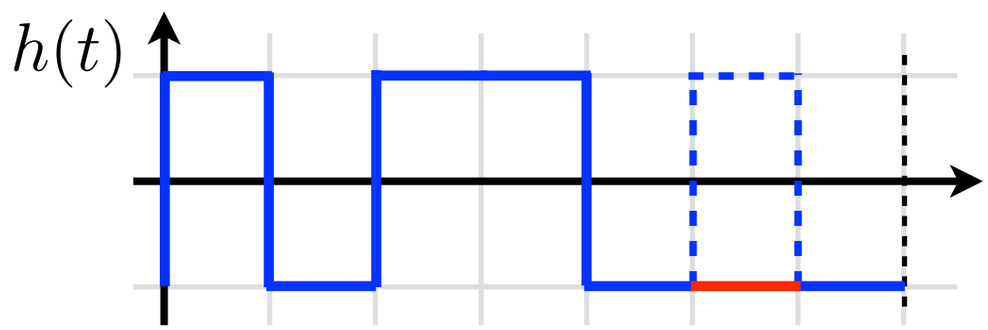

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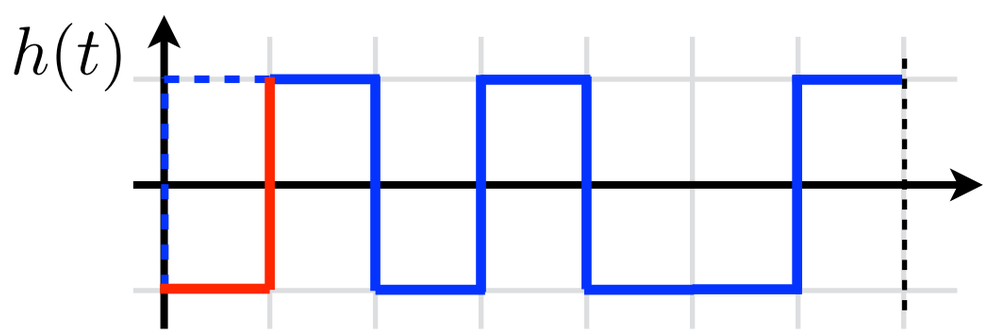

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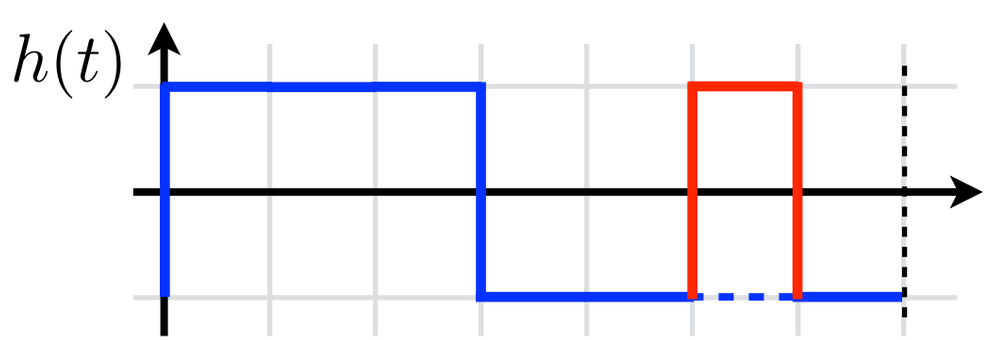
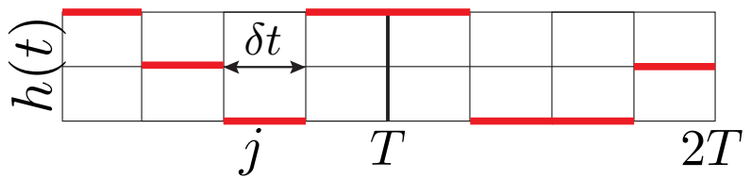


measurement: -1

can we automate it?

PRB 98, 224305 (2018)

Reinforcement Learning to Prepare the Inverted Position Floquet State

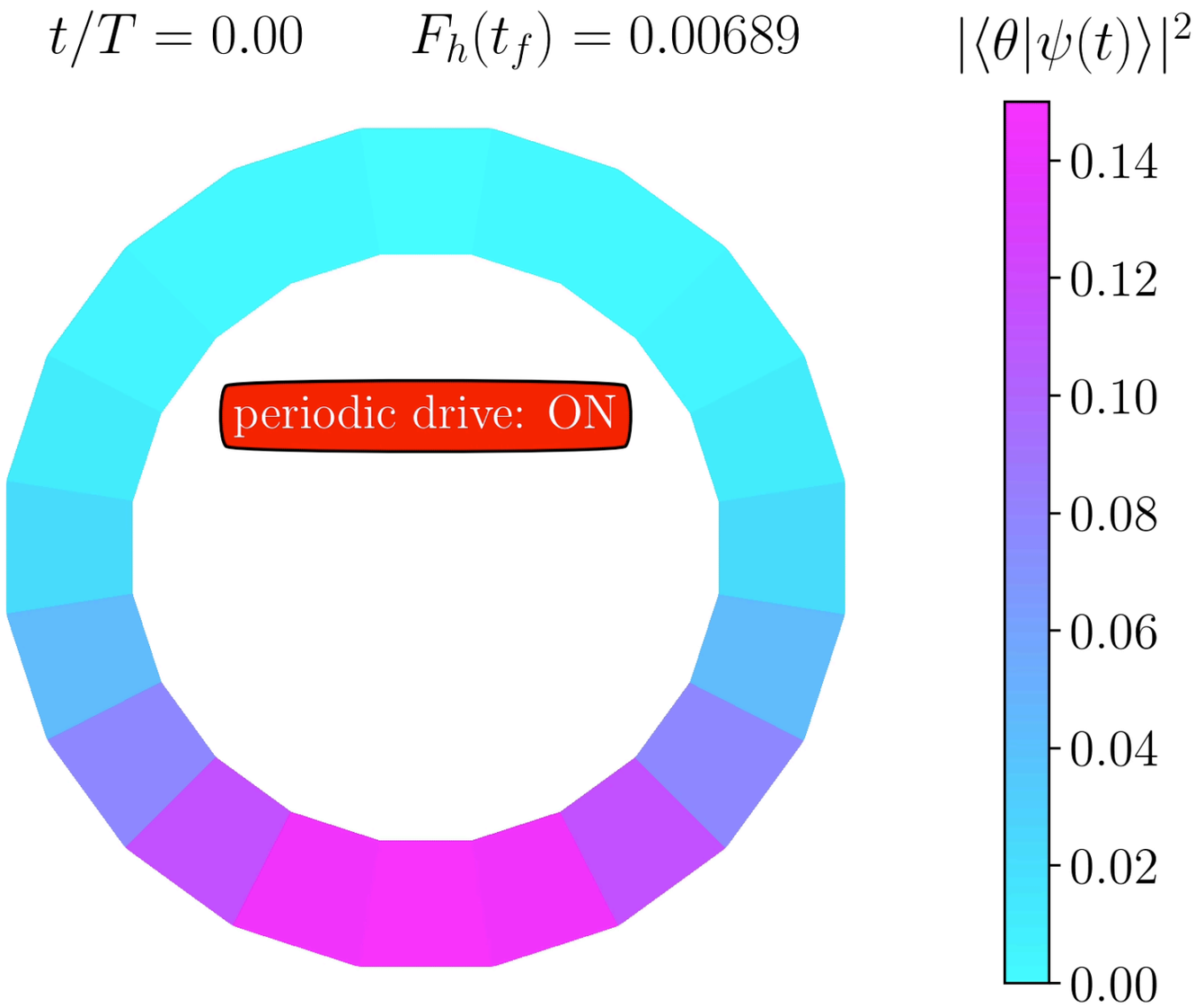


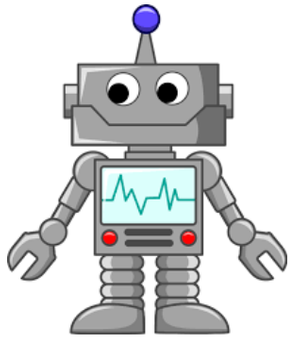
15 driving cycles (periods), 120 steps (8 per period)

$$|\mathcal{A}|^{N_T} = 3^{120} \approx 10^{57}$$

quantum Kapitza oscillator

$t/T = 0.00$ $\quad F_h(t_f) = 0.00689$





$h_{\max}/(m\omega_0) = 4.0$ $\quad \Omega/(m\omega_0) = 10.0$ $\quad A/(m\omega_0) = 2.0$

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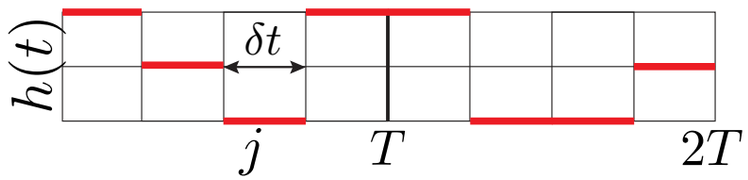
PRB 98, 224305 (2018)

Reinforcement Learning to Prepare the Inverted Position Floquet State



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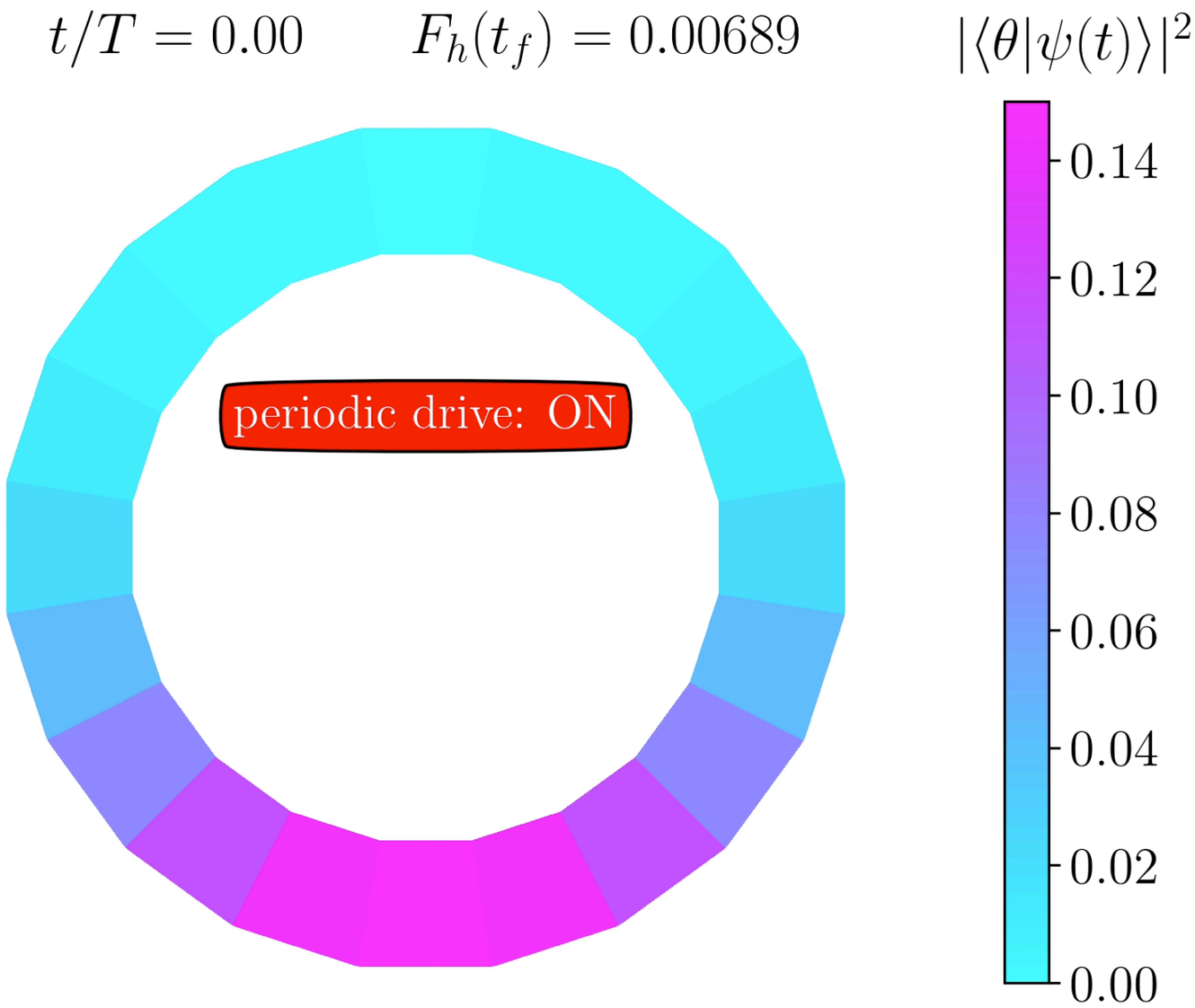

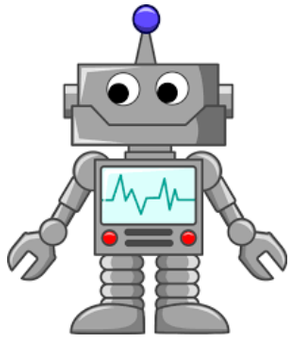

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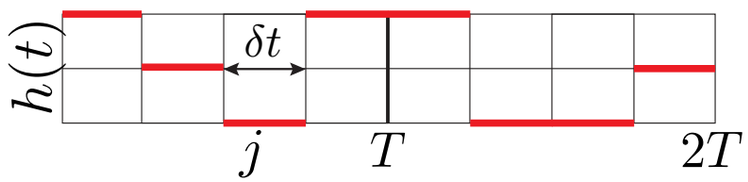
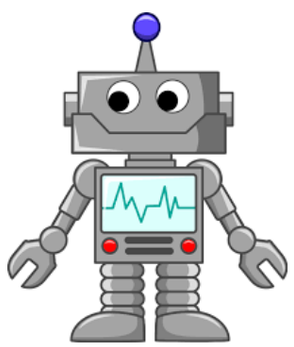
Reinforcement Learning
Classical Kapitza Pendulum



4 driving cycles (periods), 32 steps (8 per period)

Kapitza pendulum

$$t/T = 0.00,\ \theta(t) = 0.00\pi,\ p_\theta(t) = 0.00,\ r(t) = 0.00$$



periodic drive: ON

$h_{\max}/(m\omega_0) = 4.0$ $\Omega/(m\omega_0) = 10.0$ $A/(m\omega_0) = 2.0$

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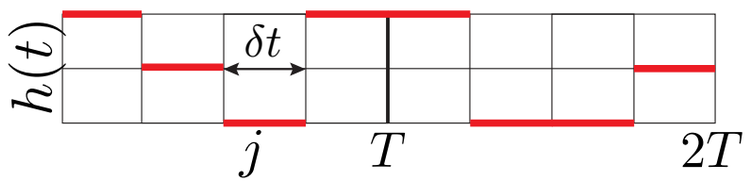
Reinforcement Learning
Classical Kapitza Pendulum



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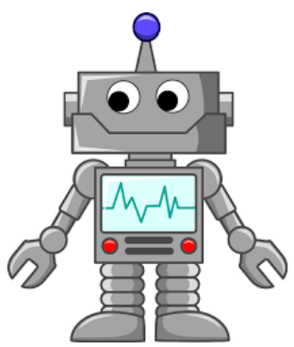

periodic drive: ON

$h_{\max}/(m\omega_0) = 4.0$ $\Omega/(m\omega_0) = 10.0$ $A/(m\omega_0) = 2.0$

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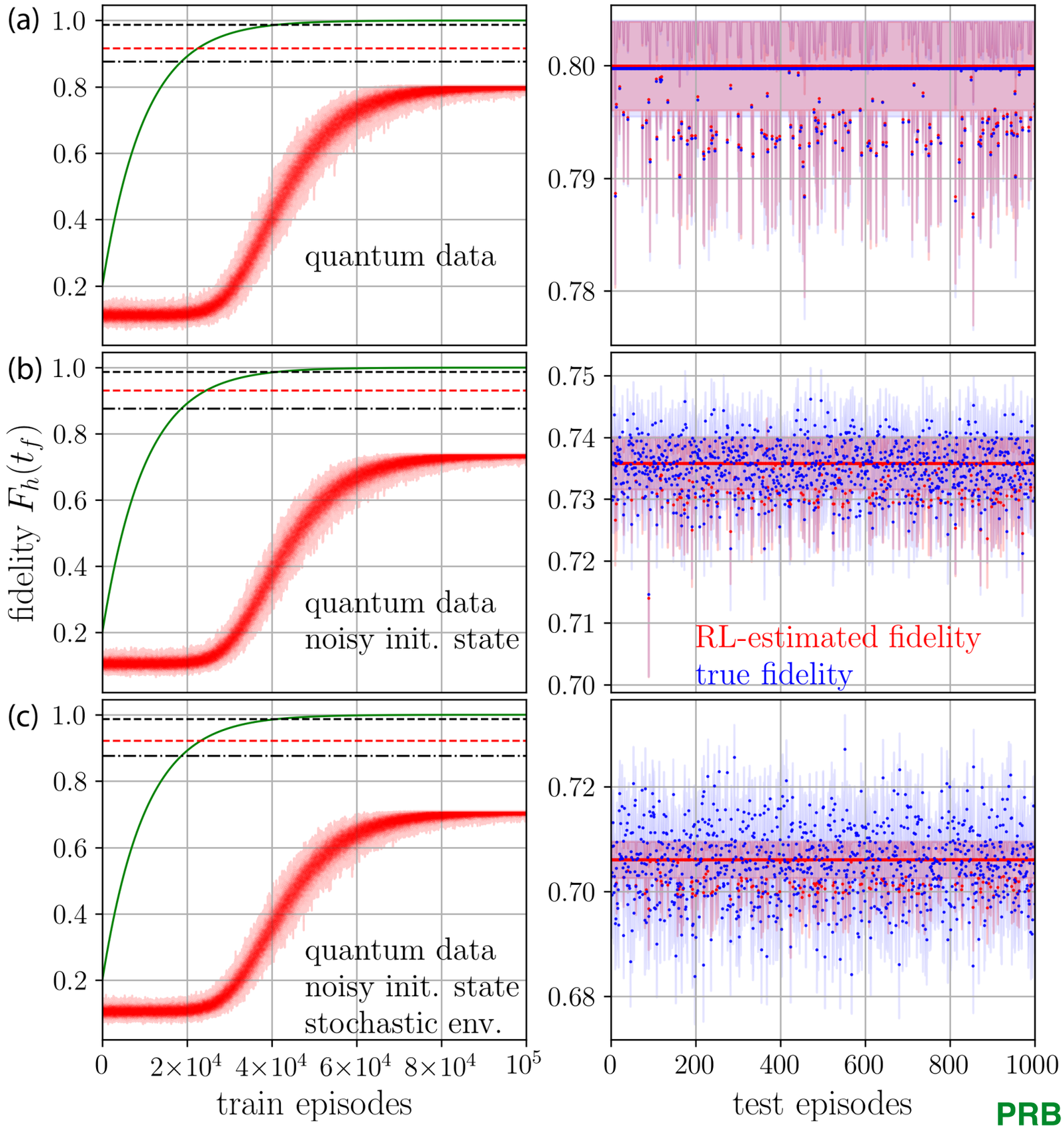
Kapitza Learning Curves



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PRB 98, 224305 (2018)