

Supervise learning of time-independent Hamiltonians for gate design

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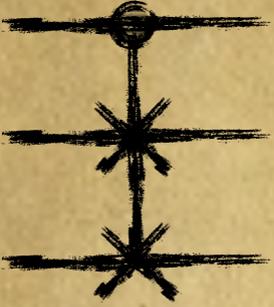


Max Planck Institute for the physics of light

Erlangen

8 May 2019

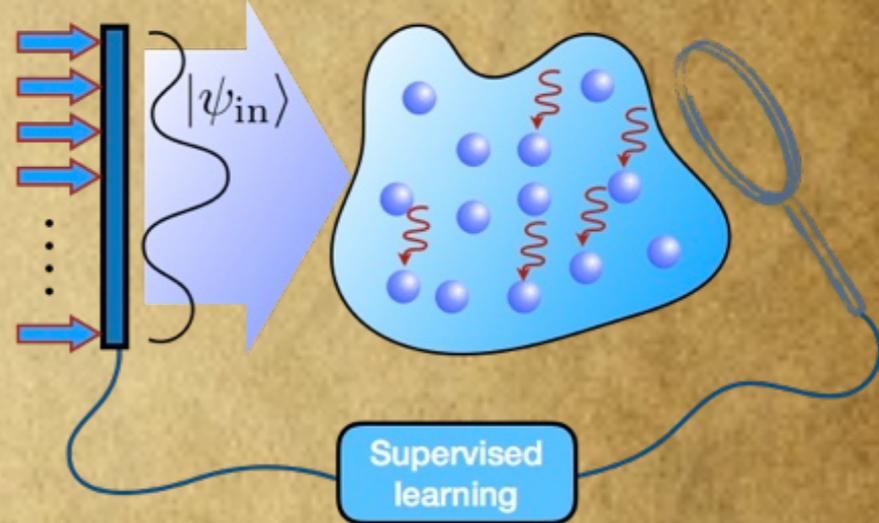
PLAN OF THE TALK



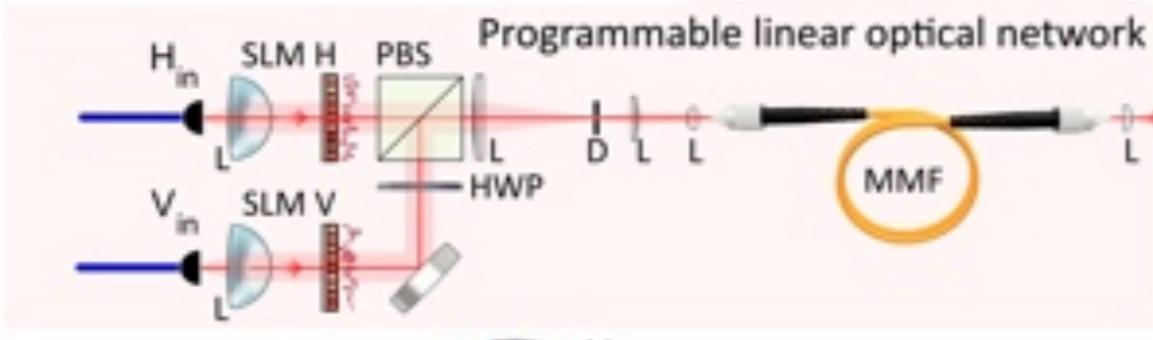
Quantum gate synthesis and design



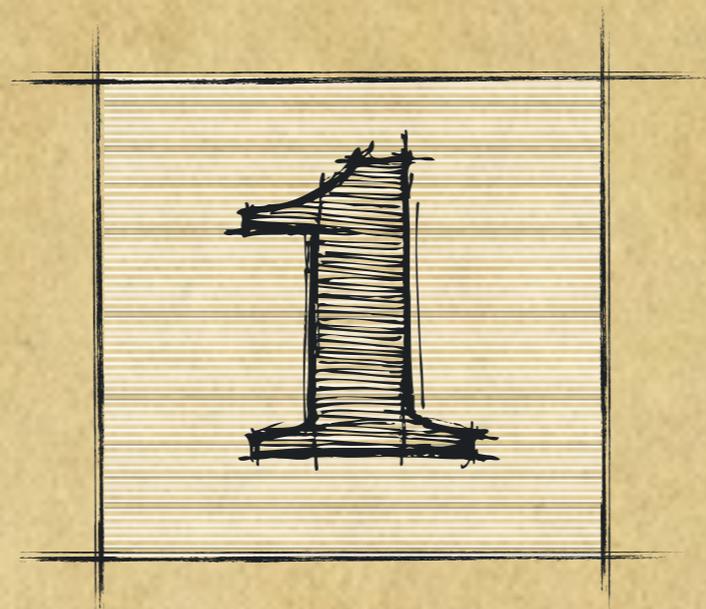
Machine learning for quantum state engineering



Programming linear networks with a fiber



PART



QUANTUM GATE

SYNTHESIS AND DESIGN



DUE CREDIT



Luca Innocenti (QUB)

Alessandro
Ferraro
(QUB)



Leonardo
Banchi
(Florence)

Sougato Bose (UCL)





MOTIVATIONS (CLASSICAL)



Sammy, 3 years old:
typical learner



What Sammy has!

What Sammy wants

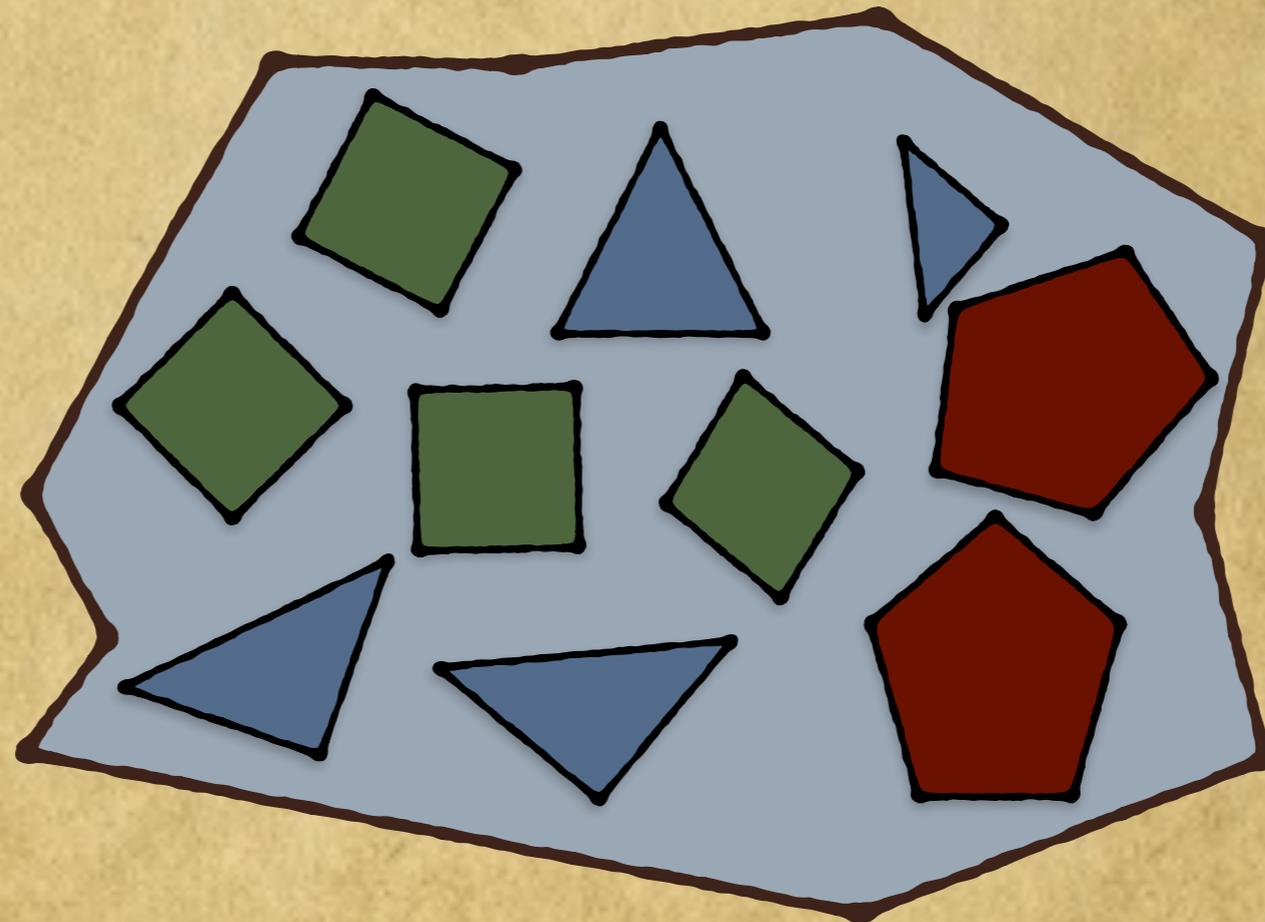
(15 of them)

A simple question

Is it possible to synthesise exactly a gate G from a generator \mathcal{H}_G comprising operations drawn only from an assigned Γ ?



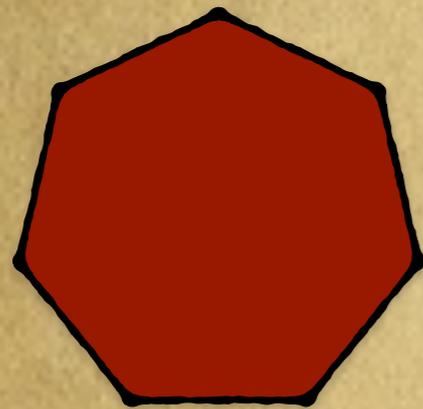
Desired
transformation



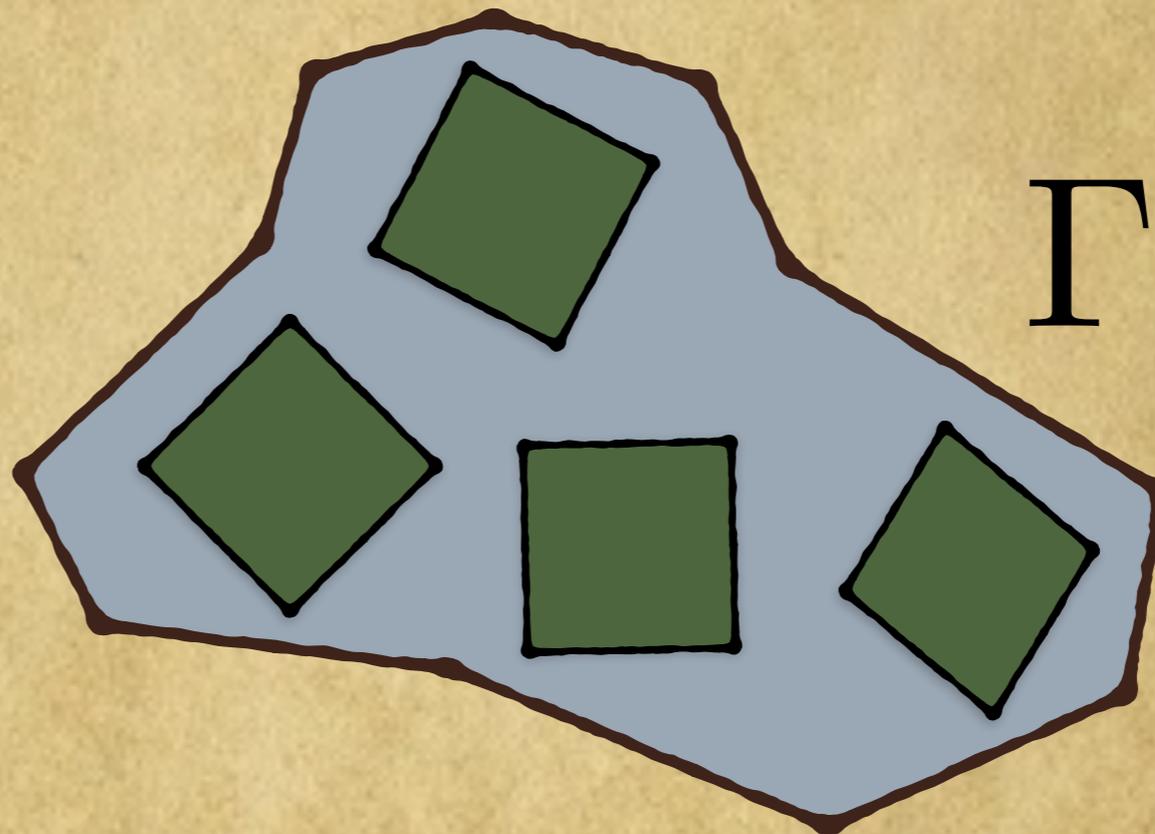
The gates that we would need

A simple question

Is it possible to synthesise exactly a gate G from a generator \mathcal{H}_G comprising operations drawn only from an assigned Γ ?



Desired
transformation



The gates that a given experimental
platform implements naturally



What to take home?

Is it possible to synthesise exactly a gate \mathcal{G} from a generator $\mathcal{H}_{\mathcal{G}}$ comprising operations drawn only from an assigned Γ ?

Take - home messages from this talk

-3 conditions to produce a $\tilde{\mathcal{H}}$ comprising only operations drawn from π such that

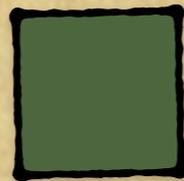
$$\mathcal{G} = \exp(i\tilde{\mathcal{H}})$$

-Present a supervised-learning optimisation technique to train qubit networks & achieve the desired synthesis

- Discuss & demonstrate algorithm-training instances of multi-qubit gates

$$\mathcal{G} = e^{i\mathcal{H}_G} = U\Lambda U^\dagger \Rightarrow \boxed{\mathcal{H}_G} = -iU \text{Log}(\Lambda)U^\dagger$$

Will contain both feasible and unfeasible interactions

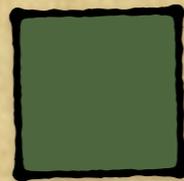


The goal: Construct $\tilde{\mathcal{H}}_G$ built using Γ only, such that $\mathcal{G} = \exp(i\tilde{\mathcal{H}}_G)$

- What we do NOT want:
- 1) Time-dependent Hamiltonians
 - 2) Ancillary systems
 - 3) Complicated quantum control

$$\mathcal{G} = e^{i\mathcal{H}_G} = U\Lambda U^\dagger \Rightarrow \boxed{\mathcal{H}_G} = -iU \text{Log}(\Lambda)U^\dagger$$

Will contain both feasible and unfeasible interactions



The goal: Construct $\tilde{\mathcal{H}}_G$ built using Γ only, such that $\mathcal{G} = \exp(i\tilde{\mathcal{H}}_G)$

What we DO want:

- 1) $\tilde{\mathcal{H}}_G$ contains only feasible interactions
- 2) $[\tilde{\mathcal{H}}_G, \mathcal{H}_G] = 0$
- 3) $\text{Eig}(\tilde{\mathcal{H}}_G - \mathcal{H}_G) = \{2\pi n_i \text{ with } n_i \in \mathbb{Z}\}$

1) $\tilde{\mathcal{H}}_G$ contains only feasible interactions ✓

$$2) [\tilde{\mathcal{H}}_G, \mathcal{H}_G] = 0 \quad 3) \text{Eig}(\tilde{\mathcal{H}}_G - \mathcal{H}_G) = \{2\pi n_i \text{ with } n_i \in \mathbb{Z}\}$$

$$\mathcal{G} = \exp(i\tilde{\mathcal{H}}_G - i\mathcal{H}_G) \exp(i\mathcal{H}_G)$$

$$\exp(i\tilde{\mathcal{H}}_G - i\mathcal{H}_G) = \mathbb{1}$$

The gist of it: given \mathcal{G} look for $\tilde{\mathcal{H}}_G = \mathcal{H}_G + \text{something else}$
satisfying 1), 2), and 3)

$\tilde{\mathcal{H}}_G$ contains only physical interactions ✓

$$2) [\tilde{\mathcal{H}}_G, \mathcal{H}_G] = 0 \quad 3) \text{Eig}(\tilde{\mathcal{H}}_G - \mathcal{H}_G) = \{2\pi n_i \text{ with } n_i \in \mathbb{Z}\}$$



A) Analytically solved in at least some situations of physical interest

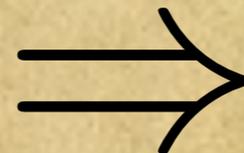
B) Help to identify efficient starting points for numeric optimizations

B) Help to identify efficient starting points for numeric optimizations

Choose $\tilde{H}_G(\lambda)$ drawn from Γ

Use condition 2) to reduce set of possible interactions
& to constrain parameters

Non-trivial step: enforce 3)



Inverse
eigenvalue
problem

Supervised
learning approach





General methodology

Strategy

Decompose $\mathcal{H}(\lambda) = \sum_i \lambda_i O_i$

Consider $\mathcal{F}(\lambda, \psi) \equiv \langle \psi | \mathcal{G}^\dagger \exp(i\mathcal{H}(\lambda)) | \psi \rangle$

Look for λ_0 such that $\mathcal{F}(\lambda_0, \psi) = 1$ for all $|\psi\rangle$

Supervised
learning approach

Supervised learning approach

Algorithm

- Make an initial choice for λ
- Generate a random set of input states $|\psi_k\rangle, k = 1, \dots, N_b$
- For each k compute $\nabla_{\lambda} \mathcal{F}(\lambda, \psi_k)$ Automatic Derivative with error backpropagation: Theano, PyTorch, TensorFlow
- Update the choice of λ using momentum gradient

$$\begin{aligned}
 \mathbf{v} &\rightarrow \gamma \mathbf{v} + \eta \nabla_{\lambda} \mathcal{F}(\lambda, \psi_k) \\
 \lambda &\rightarrow \lambda + \mathbf{v}
 \end{aligned}$$

momentum term

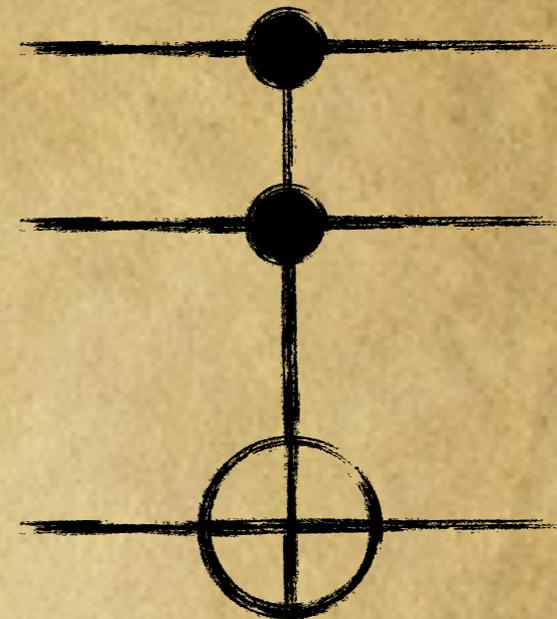
learning rate

- Repeat till a satisfactory value of fidelity is found

$$\mathcal{H}_{\text{Toff}} = \frac{\pi}{8} (1 - \sigma_1^z)(1 - \sigma_2^z)(1 - \sigma_3^x)$$

Γ : only 2-qubit terms!

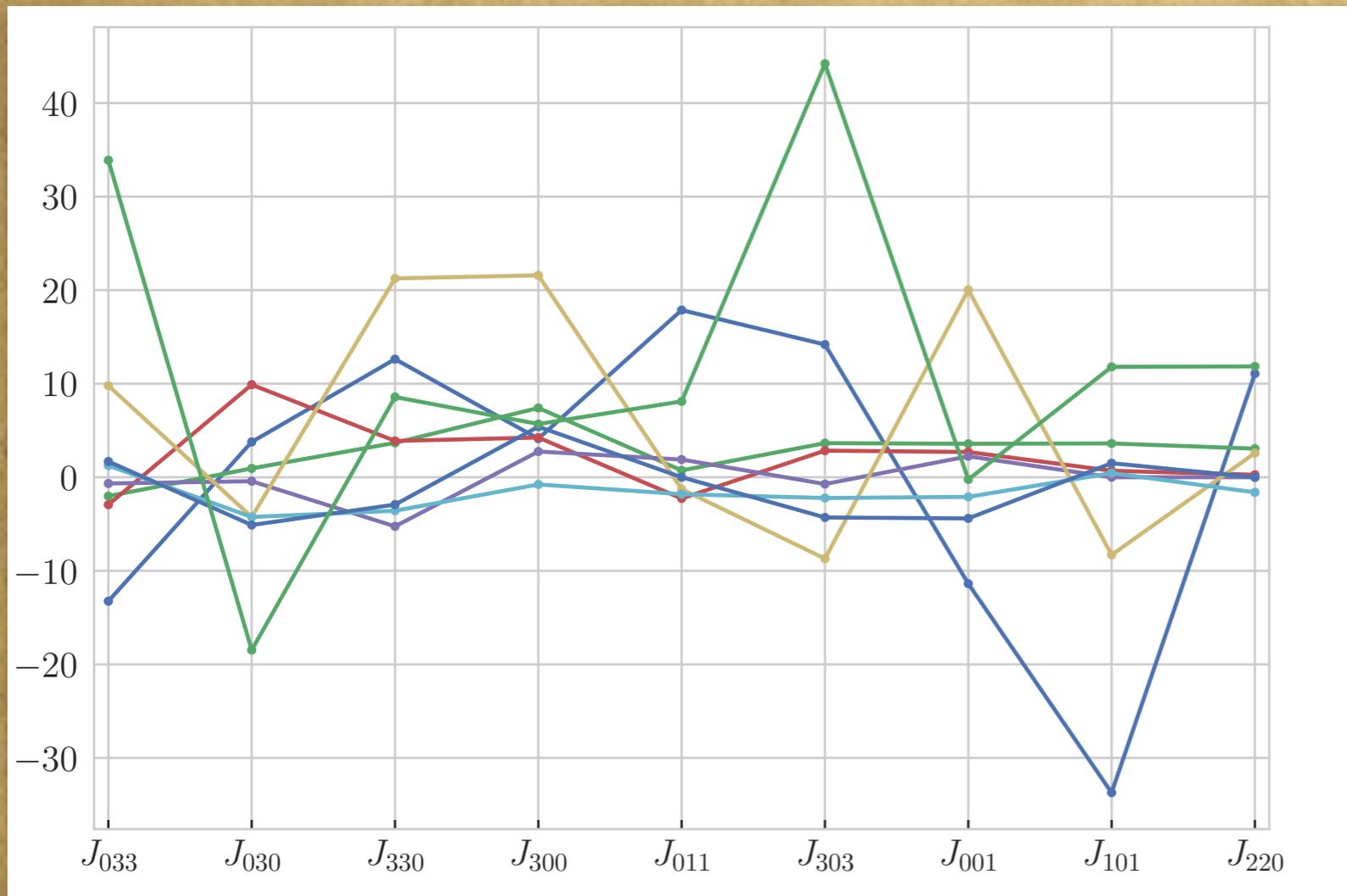
$$\tilde{\mathcal{H}}_{\text{Toff}} = h_0 \mathbb{1} + \sum h_{i,\alpha} \sigma_i^\alpha + \sum J_{i,j}^{\alpha,\beta} \sigma_i^\alpha \sigma_j^\beta$$



37 parameters \Rightarrow Enforce condition 2: reduces parameters

$$\begin{aligned} \tilde{\mathcal{H}}_{\text{Toff}} = & J_{000} + J_{300}Z_1 + J_{030}Z_2 + J_{001}X_3 + Z_3[J_{303}(1 + Z_1) + J_{033}(1 + Z_2)] \\ & + (J_{101}X_1 + J_{011}X_2)(1 + X_3) + J_{220}(X_1X_2 + Y_1Y_2) + J_{330}Z_1Z_2. \end{aligned}$$

\Rightarrow Too large for inverse eigenvalue approach



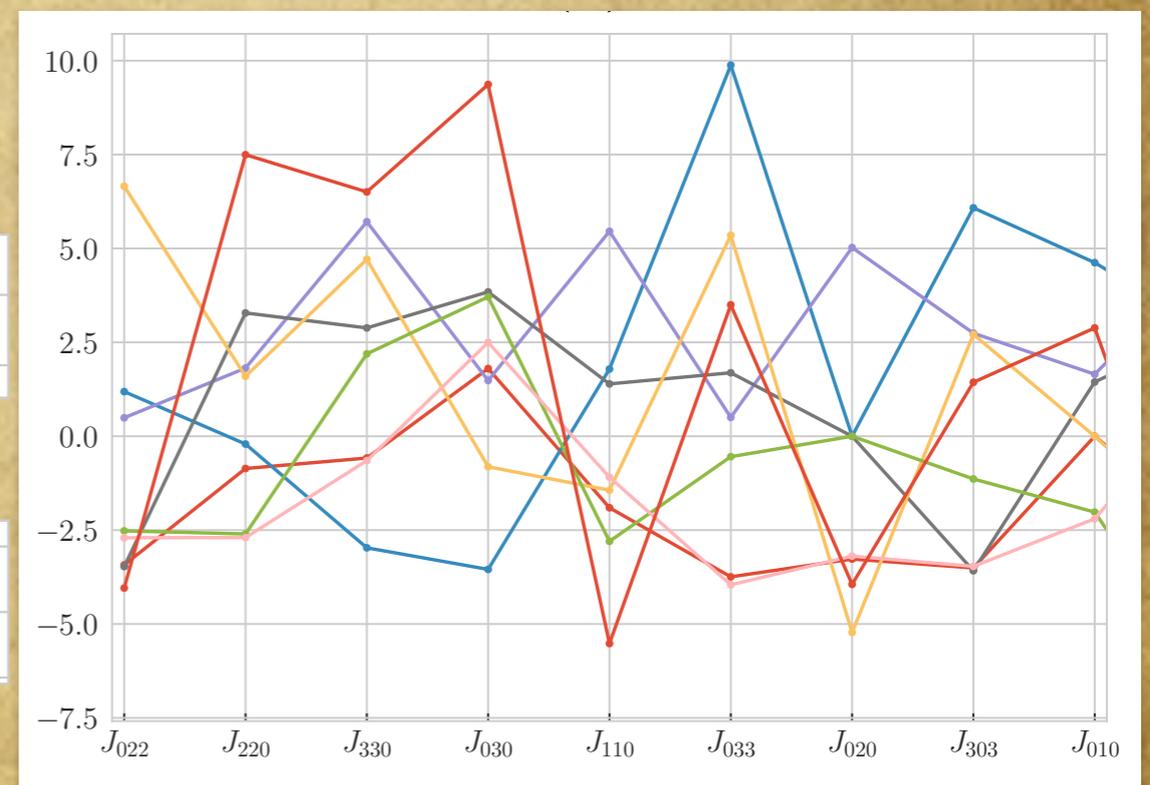
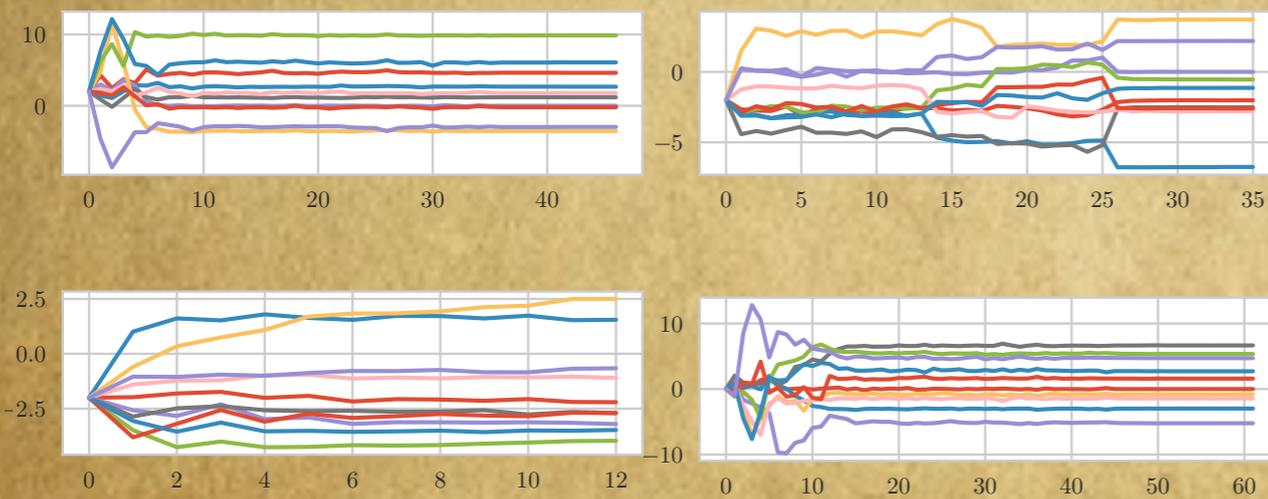
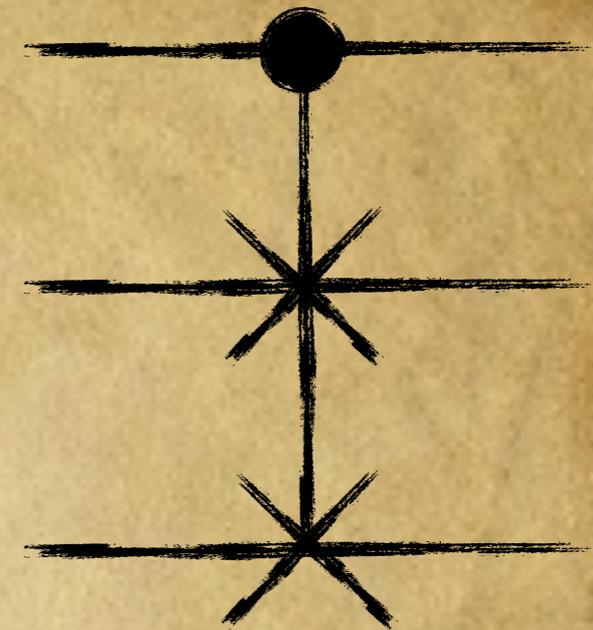
$$\begin{aligned} \tilde{\mathcal{H}}_{\text{Toff}} = & J_{000} + J_{300}Z_1 + J_{030}Z_2 + J_{001}X_3 + Z_3[J_{303}(1 + Z_1) + J_{033}(1 + Z_2)] \\ & + (J_{101}X_1 + J_{011}X_2)(1 + X_3) + J_{220}(X_1X_2 + Y_1Y_2) + J_{330}Z_1Z_2. \end{aligned}$$

Results: Fredkin

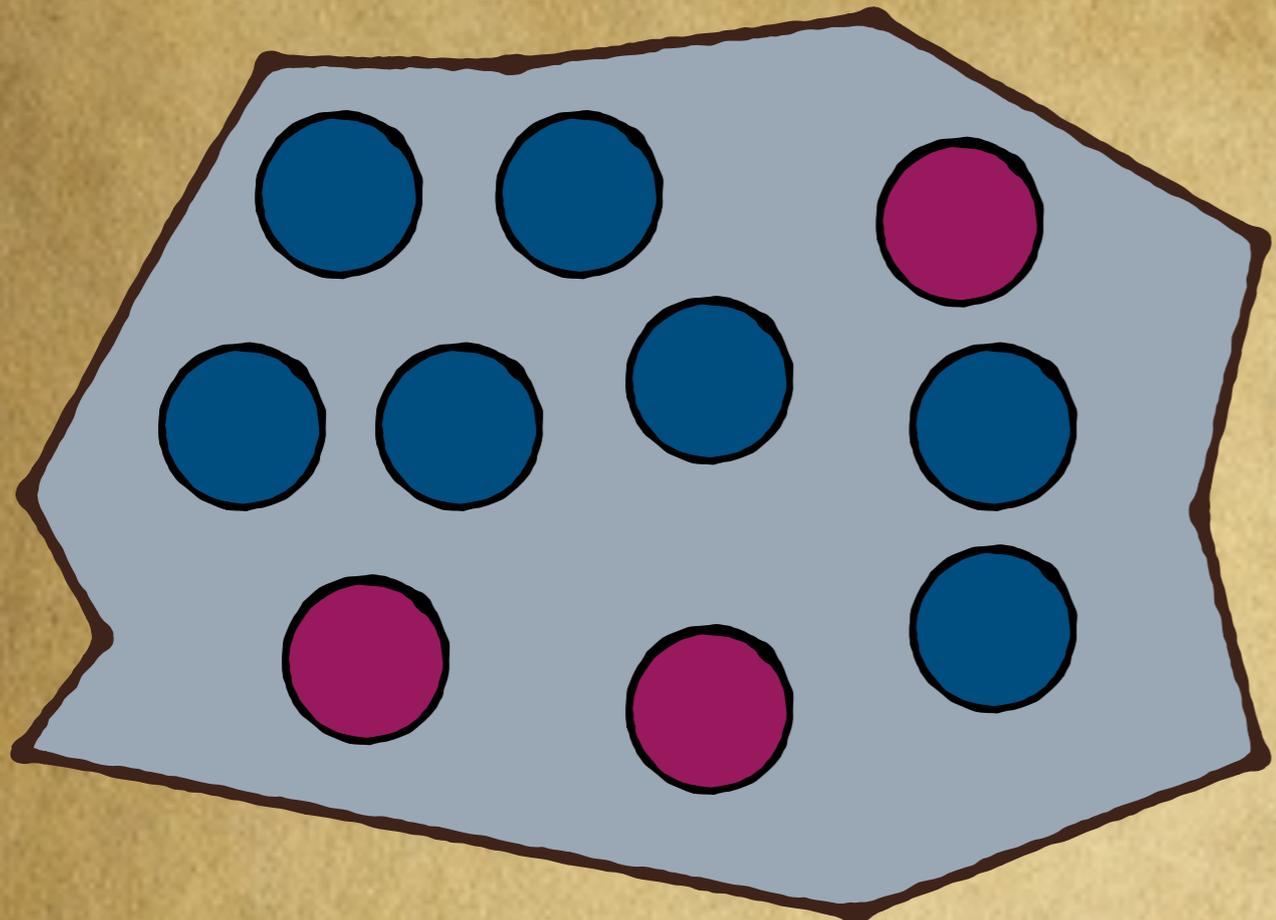
$$\mathcal{H}_{\text{Fred}} = \frac{\pi}{8} (\mathbb{1} - \sigma_1^z) \left[\mathbb{1} - \sum_{\alpha=x,y,z} \sigma_2^\alpha \sigma_3^\alpha \right]$$

Γ : only 2-qubit terms!

$$\tilde{\mathcal{H}}_{\text{Fred}} = \frac{\pi}{8} \left(\sqrt{\frac{143}{5}} \mathbb{1} + 5\sqrt{3} \sigma_1^x \right) (\sigma_2^x + \sigma_3^x) - \frac{3\pi}{8} \left(\sum_{\alpha=x,y,z} \sigma_2^\alpha \sigma_3^\alpha - \mathbb{1} \right) + \frac{\pi}{2} \sigma_1^z \left[\frac{3}{2} \sqrt{\frac{7}{5}} (\sigma_2^z + \sigma_3^z) + \mathbb{1} \right]$$

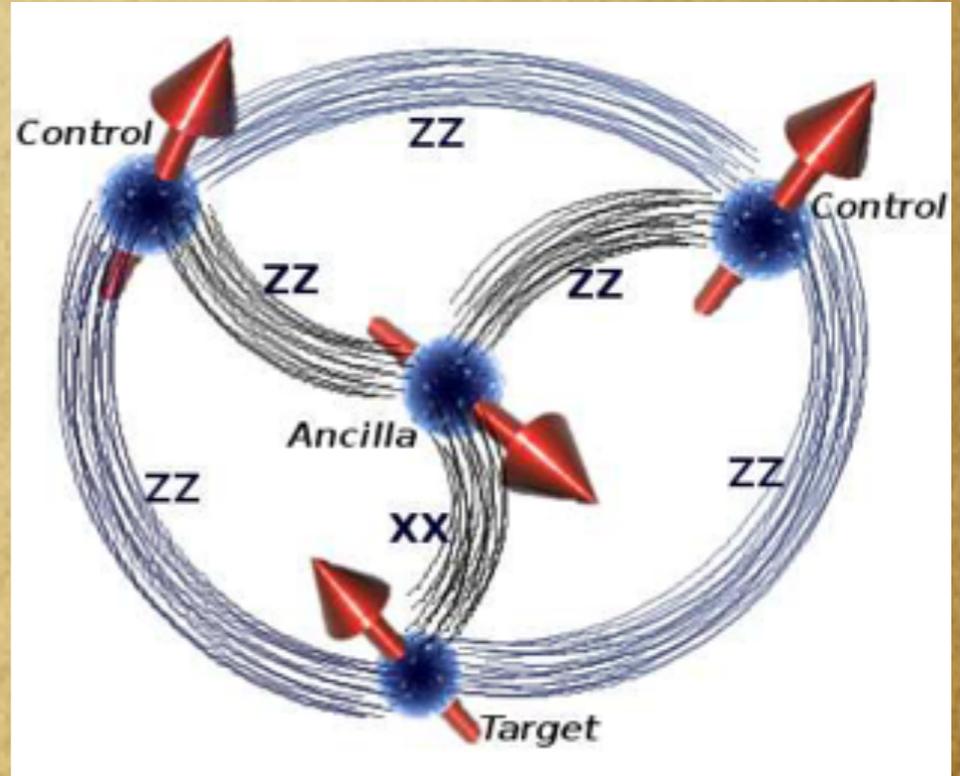


Additional results: using ancillae



$|\psi\rangle_S$ ● System
 $|\phi\rangle_A$ ● Ancilla

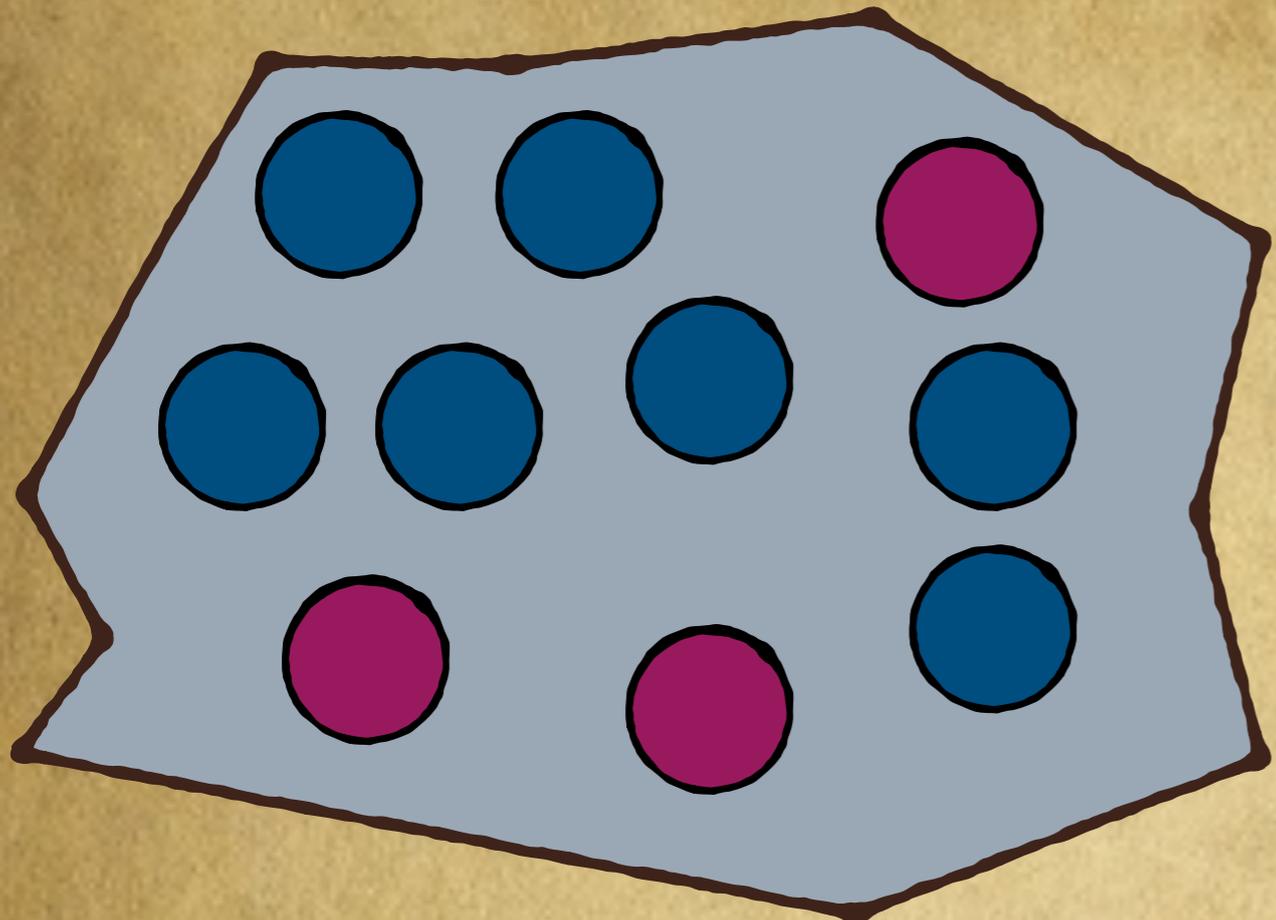
Set of information carriers



L. Banchi, N. Pancotti, S. Bose, npj QI 2, 16019 (216)

L. Innocenti, L. Banchi, S. Bose, A. Ferraro, and M. Paternostro, IJQI 16, 1840004 (218)

Additional results: using ancillae



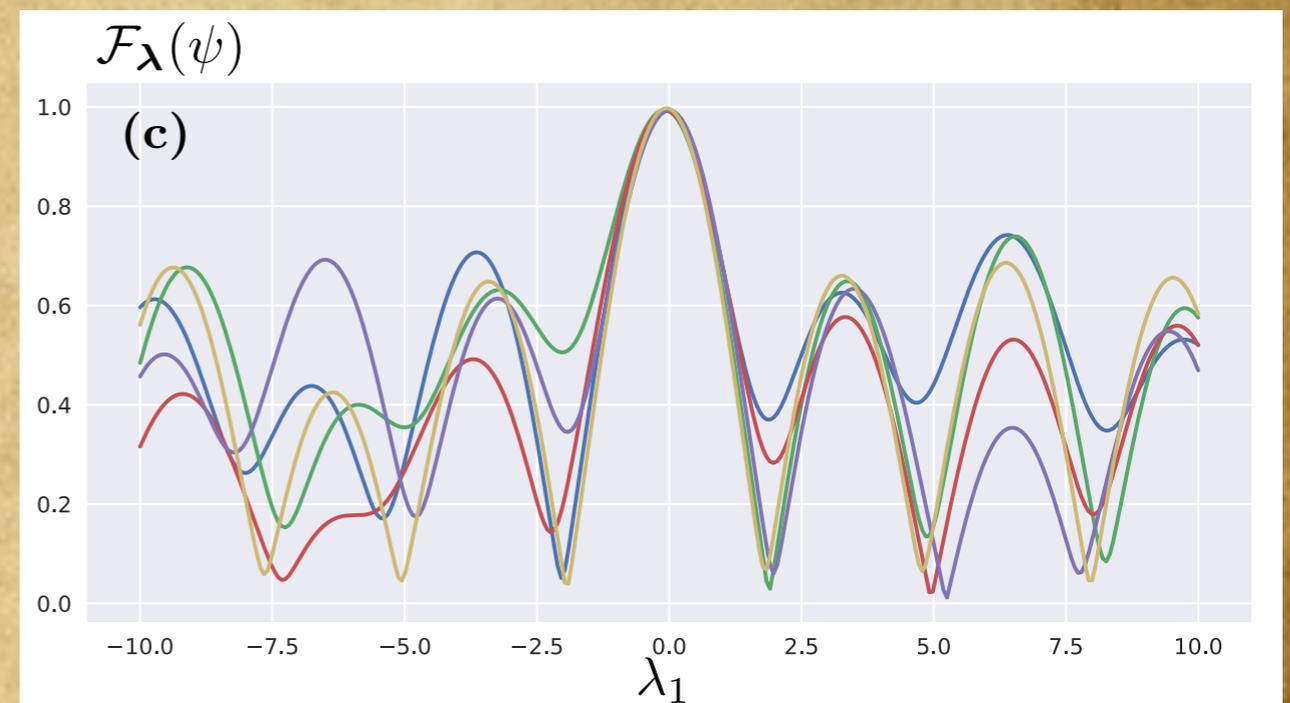
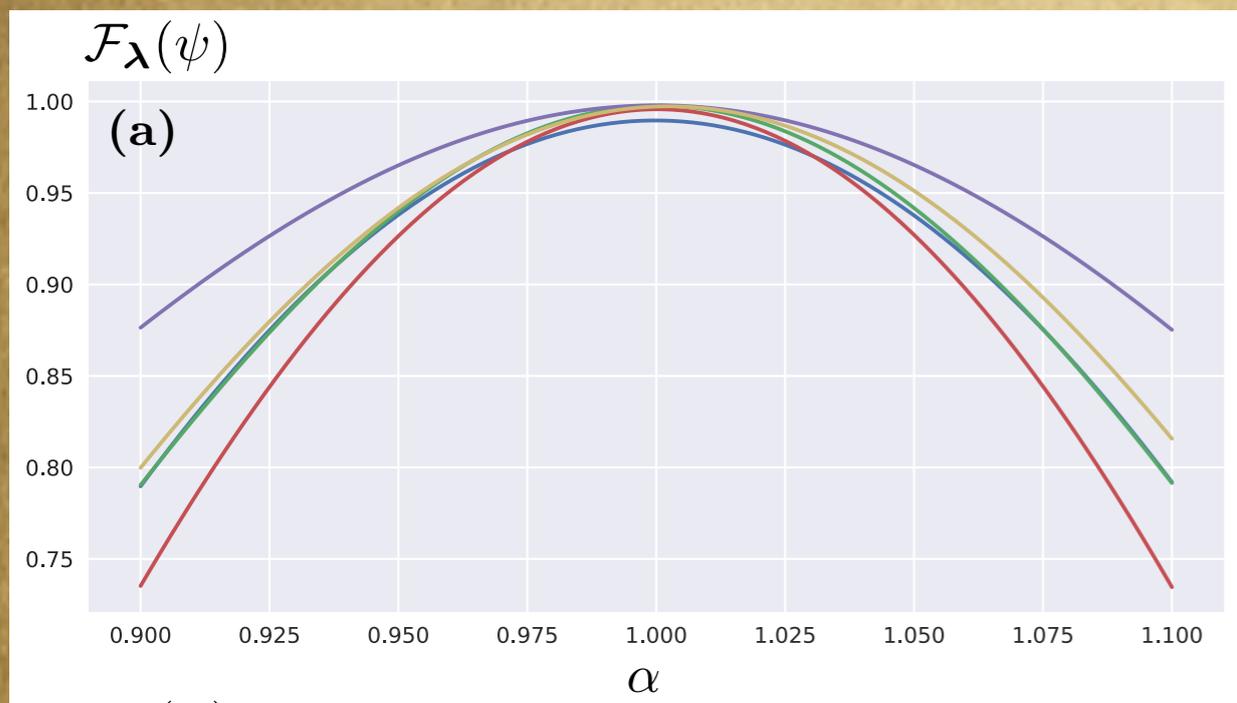
$|\psi\rangle_{\mathcal{S}}$ ● System
 $|\phi\rangle_{\mathcal{A}}$ ● Ancilla

$$\rho_{\text{out}}^{\mathcal{S}}(t, \lambda) = \text{Tr}_{\mathcal{A}} \left[\exp(it\mathcal{H}_{\lambda})(\psi \otimes \phi) \right]$$

$$\rho_{\text{out}}^{\mathcal{S}}(\lambda_0) = \mathcal{U}_{\text{target}}(|\psi\rangle\langle\psi|_{\mathcal{S}}) \mathcal{U}_{\text{target}}^{\dagger}, \quad \forall |\psi\rangle_{\mathcal{S}}$$

$$\mathcal{F}_{\lambda}(\psi) = {}_{\mathcal{S}}\langle\psi| \mathcal{U}_{\text{target}}^{\dagger} \rho_{\text{out}}(\lambda) \mathcal{U}_{\text{target}} |\psi\rangle_{\mathcal{S}}$$

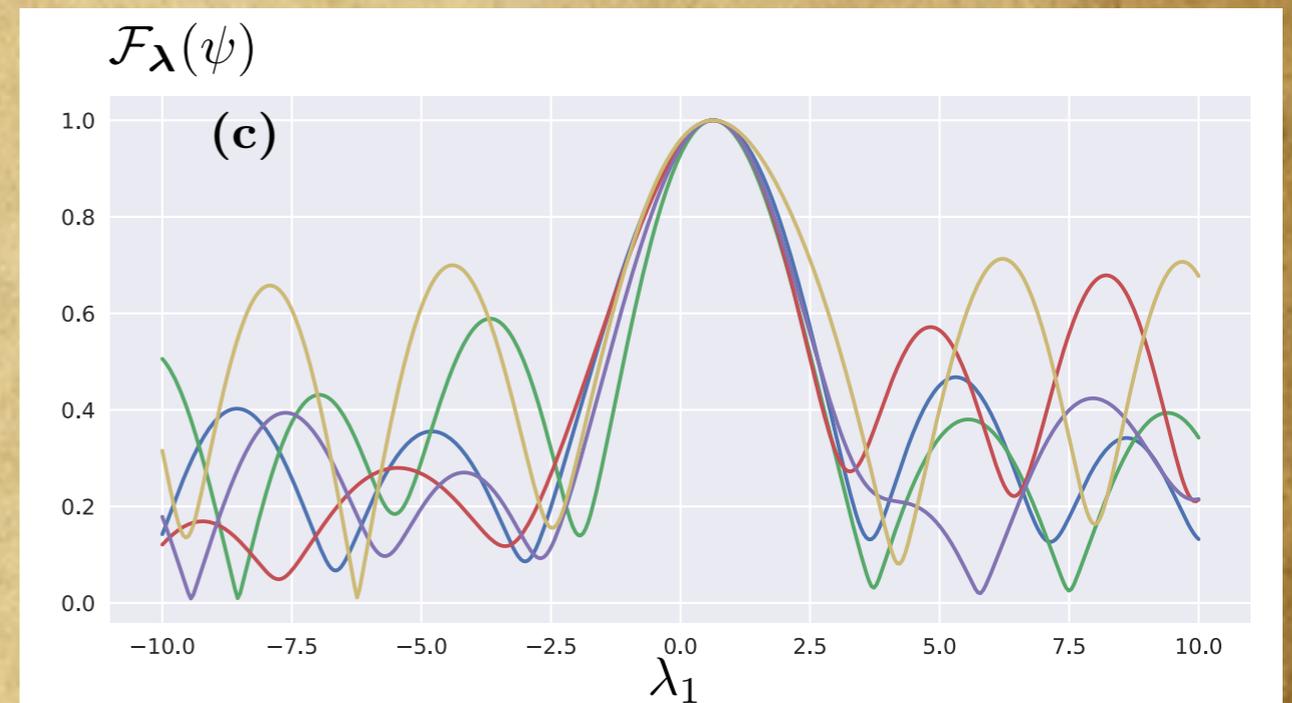
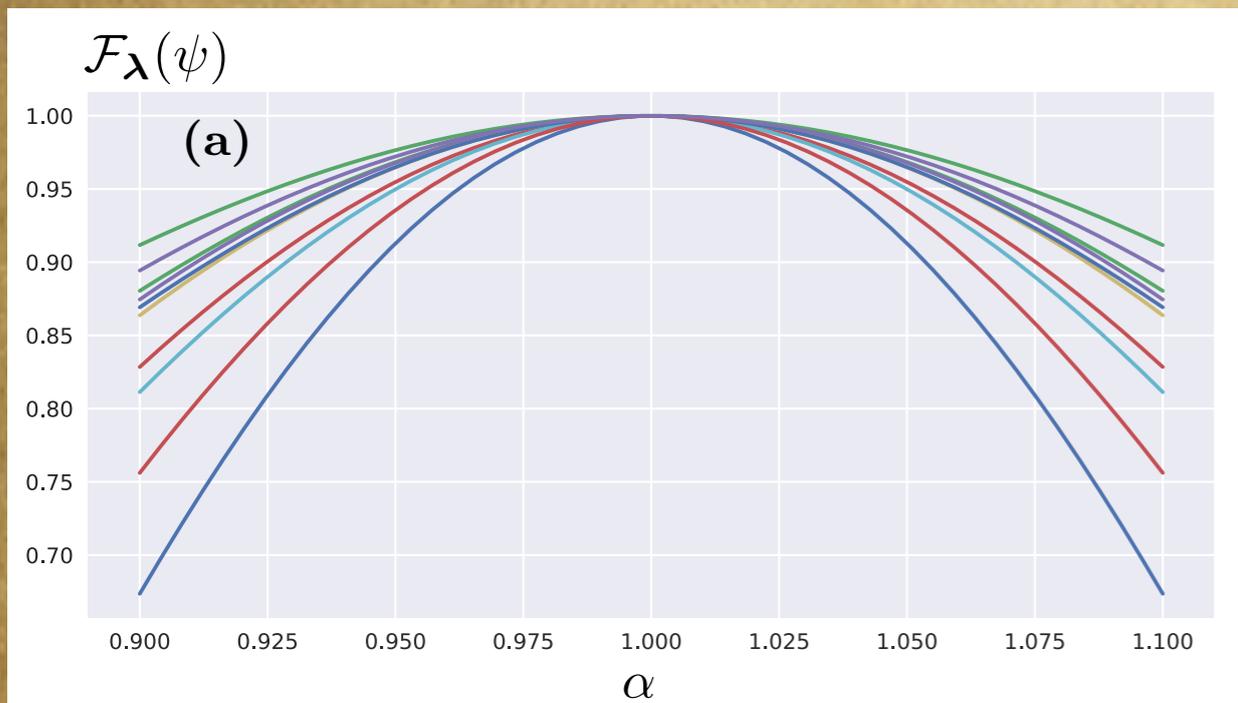
$$\text{QFT} |a\rangle_1 |b\rangle_2 |c\rangle_3 = \frac{1}{\sqrt{8}} (|0\rangle + e^{i\pi c} |1\rangle)_1 \otimes (|0\rangle + e^{i\pi(b + \frac{c}{2})} |1\rangle)_2 \otimes (|0\rangle + e^{i\pi(a + \frac{b}{2} + \frac{c}{4})} |1\rangle)_3$$



$$\lambda \rightarrow \alpha\lambda$$

Additional results: TofFredkin

$$\mathcal{U}_{\text{TF}} = |0\rangle\langle 0|_1 \otimes \text{CNOT}_{23} + |1\rangle\langle 1|_1 \otimes \text{SWAP}_{23}$$



$$\lambda \rightarrow \alpha\lambda$$



Additional results: TofFredkin

$$\mathcal{U}_{TF} = |0\rangle\langle 0|_1 \otimes \text{CNOT}_{23} + |1\rangle\langle 1|_1 \otimes \text{SWAP}_{23}$$

0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.09	0.15	0.25	0.36	0.12	0.38	0.10	
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.77	0.07	0.08	0.36	0.44	0.15	0.19	
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.05	0.54	0.74	0.07	0.08	0.34	0.13	
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.03	0.78	0.54	0.10	0.12	0.06	0.20	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.43	0.07	0.10	0.72	0.36	0.01	0.02	
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.34	0.11	0.03	0.44	0.75	0.28	0.12	
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.05	0.07	0.24	0.07	0.21	0.15	0.91	
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.31	0.21	0.14	0.01	0.21	0.78	0.22	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15



Additional results: Half-adder gate

$$\mathcal{U}_{HA} = \text{CNOT}_{12} \text{CCNOT}_{123}$$

0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
1	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
3	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
4	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
6	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
7	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.63	0.32	0.32	
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.63	0.32	0.32	
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.32	0.63	0.63	
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.32	0.63	0.63	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

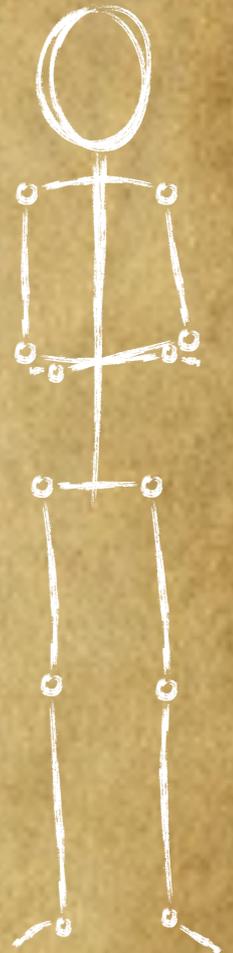
PART

2

Complexity

Open systems

State
engineering



LOOKING AHEAD:
ML FOR QUANTUM
STATE ENGINEERING



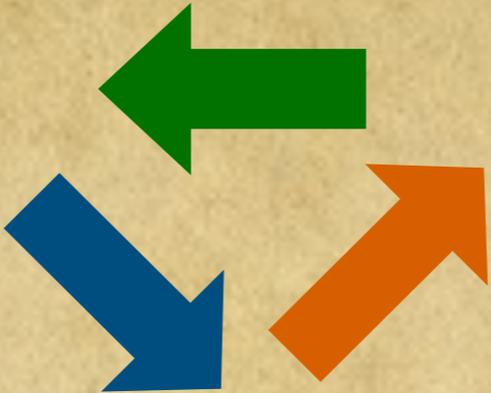
ML, quantum walk & state engineering



Taira Giordani
Emanuele Polino
Alessia Suprano
Sabrina Emiliani
Nicolo' Spagnolo
Fabio Sciarrino



Luca Innocenti
Helena Majury
Alessandro Ferraro



Lorenzo Marrucci

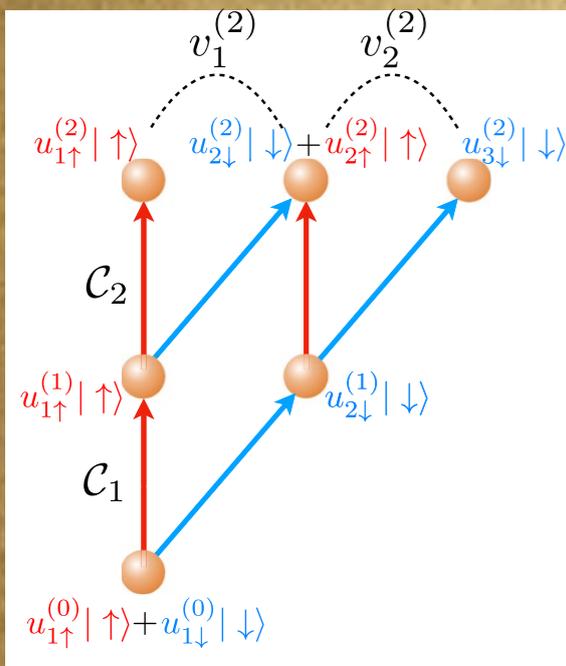
L. Innocenti, H. Majury, T. Giordani, N. Spagnolo, F. Sciarrino, M. Paternostro, A. Ferraro, PRA 96, 062326 (2017)

T. Giordani, E. Polino, S. Emiliani, A. Suprano, L. Innocenti, H. Majury, L. Marrucci, M. Paternostro, A. Ferraro, N. Spagnolo, & F. Sciarrino PRL 122, 020503 (2019)

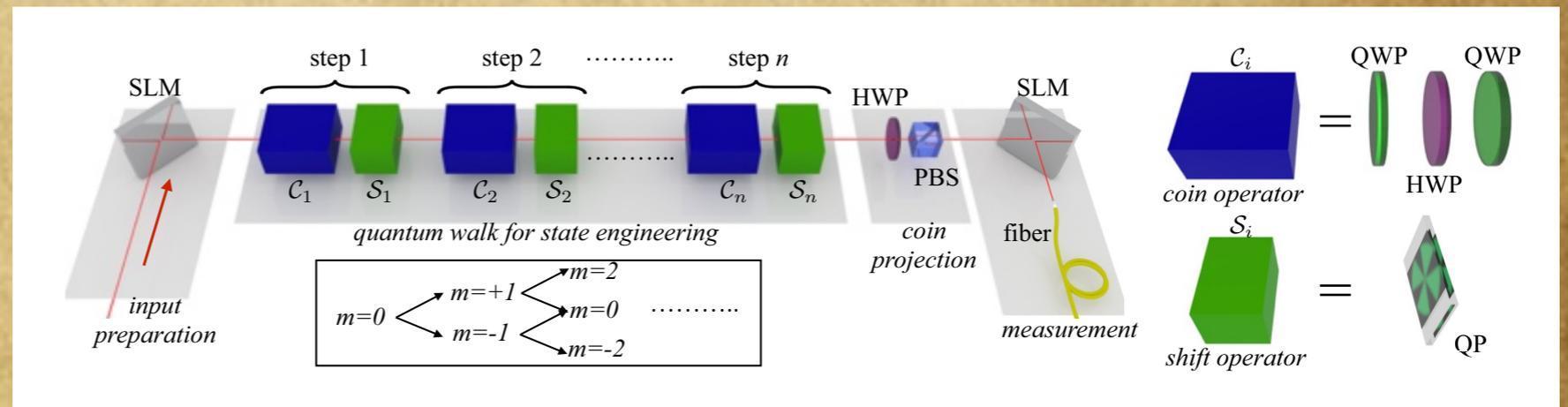
ML, quantum walk & state engineering

 quantum walker

—N Spin-N \Rightarrow Can we prepare arbitrary states? N



Use back-tracking optimisation ML approach to determine the optimal coin parameters and projection
 Synthesis of arbitrary qudit states



L. Innocenti, et al., PRA 96, 062326 (2017)

T. Giordani, E. Polino, S. Emiliani, A. Suprano, L. Innocenti, H. Majury, L. Marrucci, M. Paternostro, A. Ferraro, N. Spagnolo, & F. Sciarrino PRL 122, 020503 (2019)

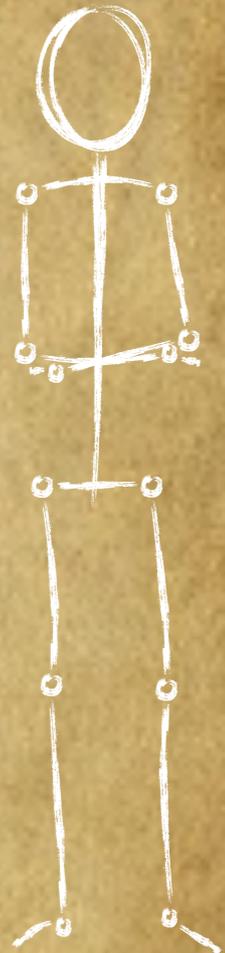
PART

3

Complexity

Open systems

State
engineering



LOOKING AHEAD:
ML FOR TAMING
COMPLEXITY



Forward look: complexity



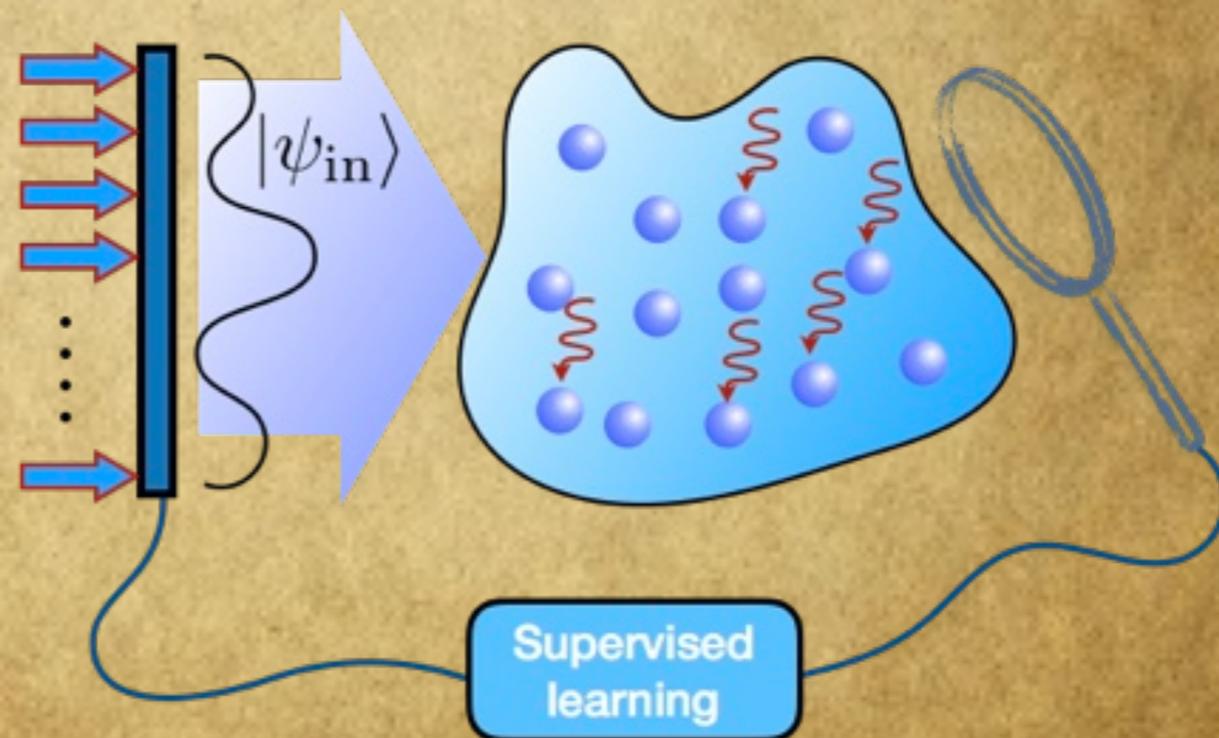
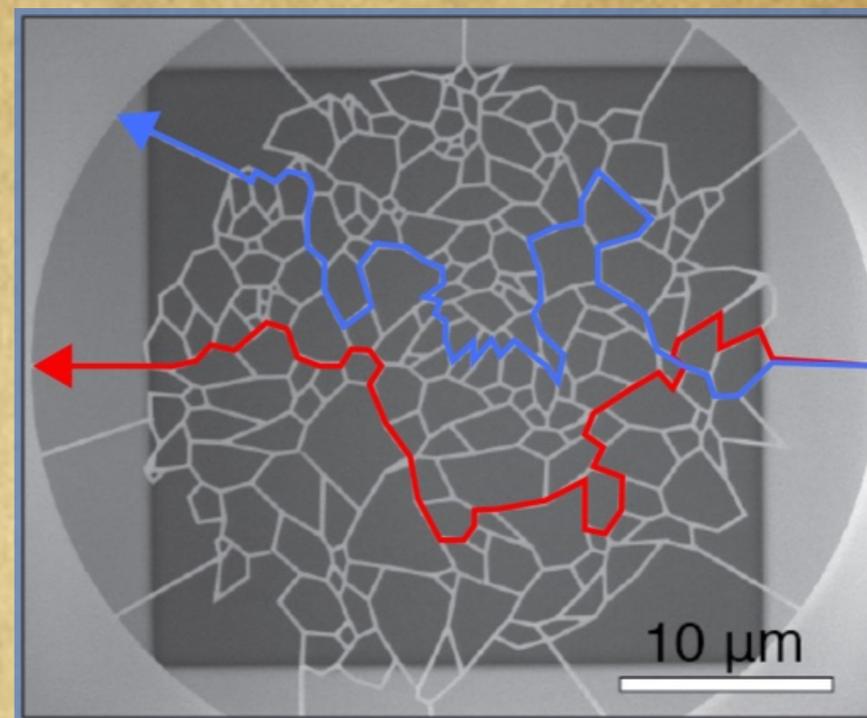
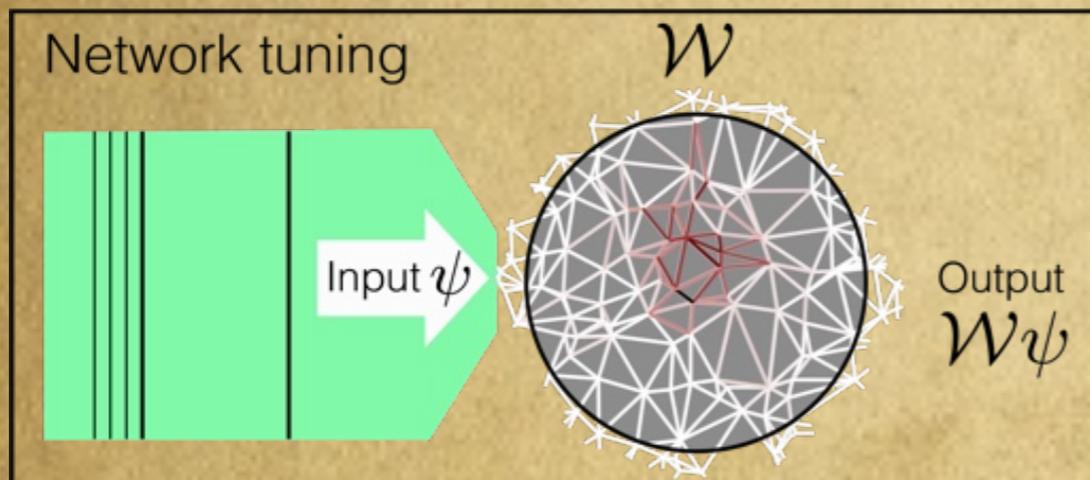
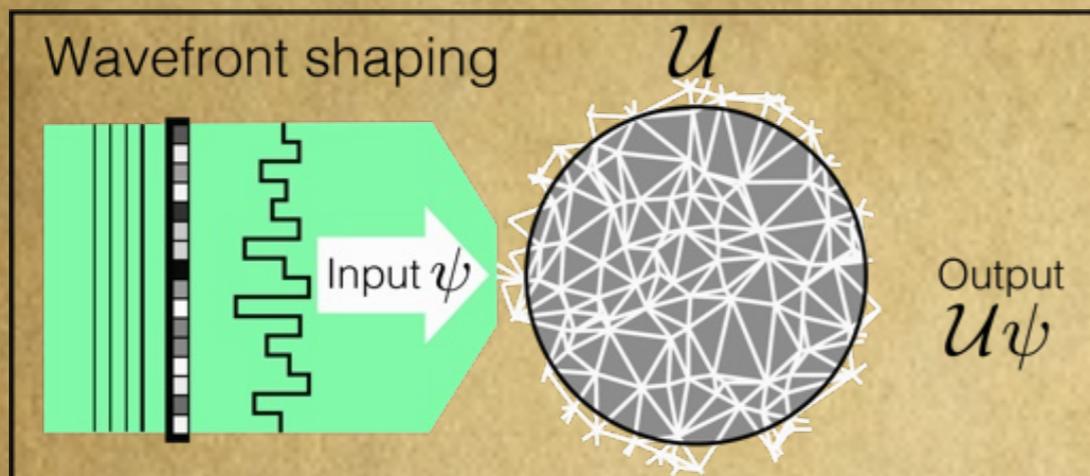
Sylvain Gigan
(LKB Paris, Sorbonne
University)

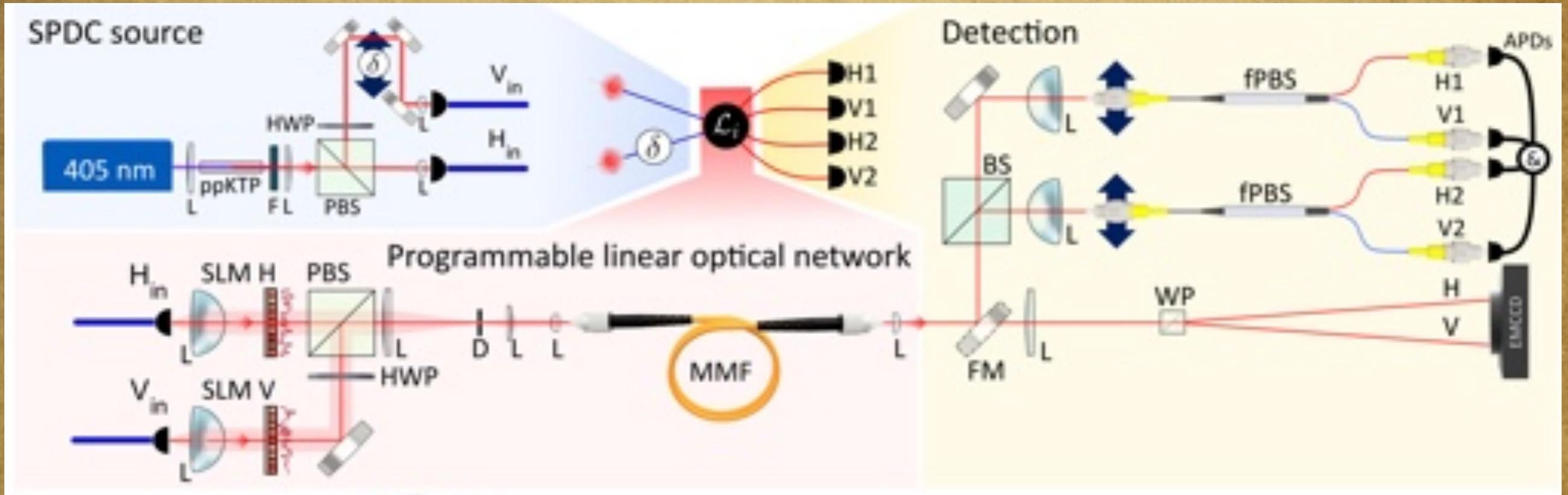
Riccardo Sapienza
(Imperial College
London)





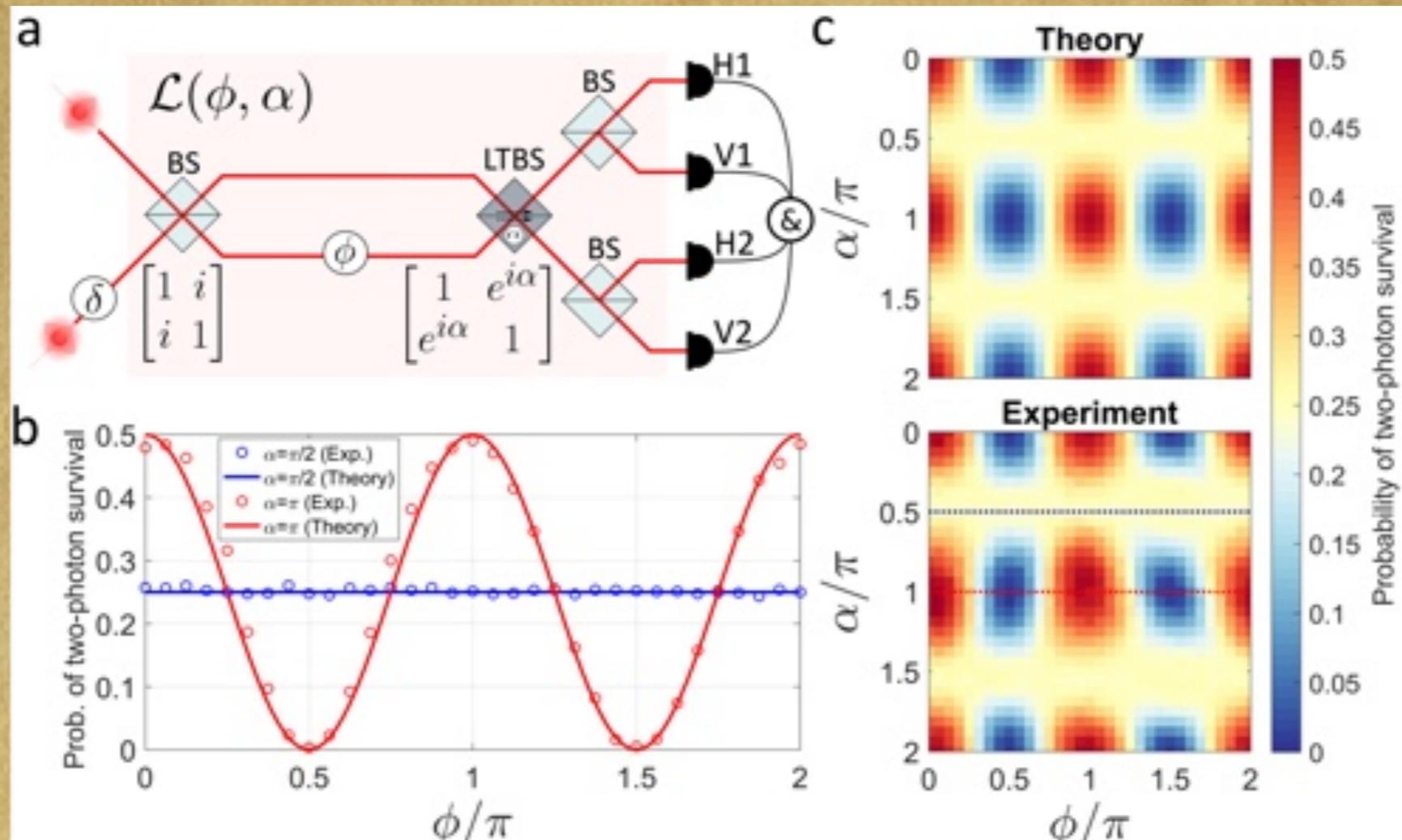
Forward look: complexity





Circuit programmed to perform unambiguous coherent absorption through wave-front shaping & high-mixing in complex media

S Leedumrongwatthanakun, L Innocenti, H Defienne, Th Juffmann, A Ferraro, M Paternostro, and S Gigan., arXiv: 1902.10678 (2019)

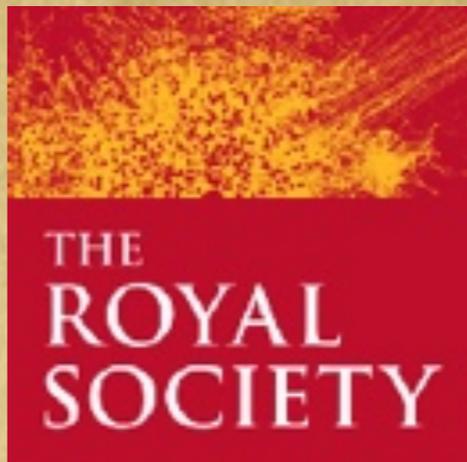


Circuit programmed to perform unable coherent absorption through wave-front shaping & high-mixing in complex media

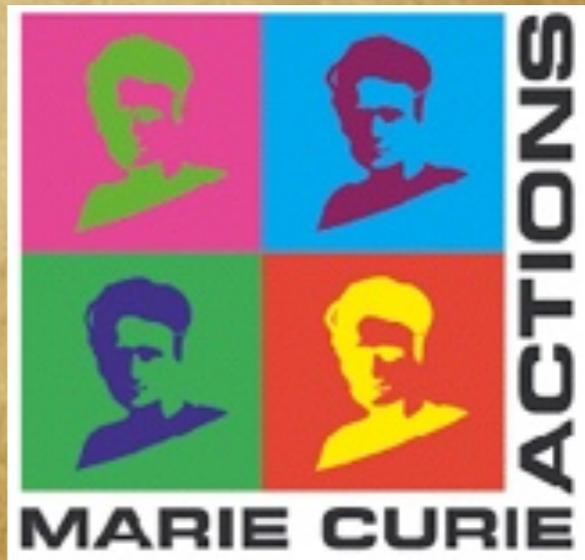
S Leedumrongwatthanakun, L Innocenti, H Defienne, Th Juffmann, A Ferraro, M Paternostro, and S Gigan., arXiv: 1902.10678 (2019)



Bread on tables..



Fondazione
Angelo Della Riccia





QTEQ
QUANTUM TECHNOLOGY at QUEEN'S

The Belfast crew



THANK YOU