virtual FNS/2021

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Committee

Resonant time-symmetry breaking in coupled oscillator

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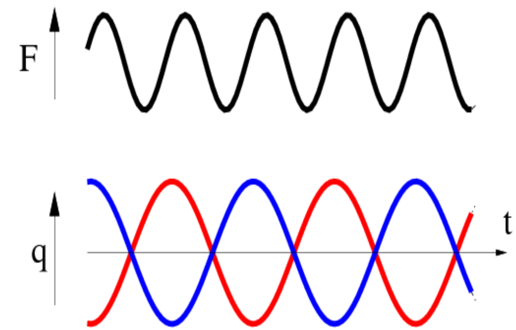
Oscillator with a periodically modulated frequency: $m\ddot{q} + 2\Gamma m\dot{q} + m(\omega_0^2 + F\cos\omega_F t)\,q + \gamma q^3 = 0$

Period doubling: the period of each state is $4\pi\,/\,\omega_F$. The states differ in phase by π -

broken discrete time-translation symmetry



squatting, $\omega_0 = \sqrt{g/l}$; two squats per period = period doubling

Breaking time-translation symmetry.

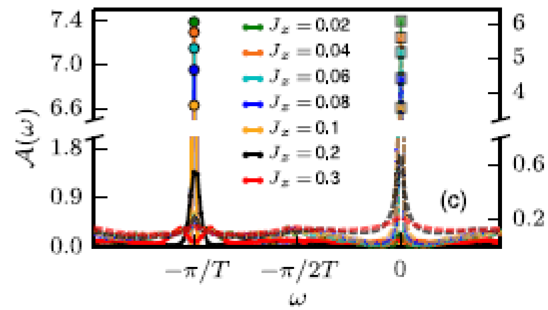
Quantum Time Crystals F. Wilczek, PRL **109,** 160401 (2012)

Absence of Quantum Time Crystals H. Watanabe and M. Oshikawa, PRL **114,** 251603 (2015)

Many-body localized states:

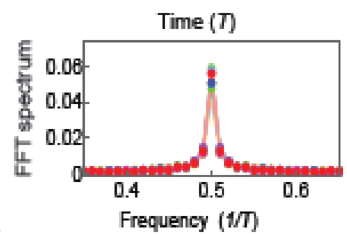
Phase Structure of Driven Quantum Systems

V. Khemani, … S. L. Sondhi, PRL **116,** 250401 (2016)



Floquet Time Crystals

D. Else, B. Bauer, & C. Nayak, PRL **117,** 090402 (2016)

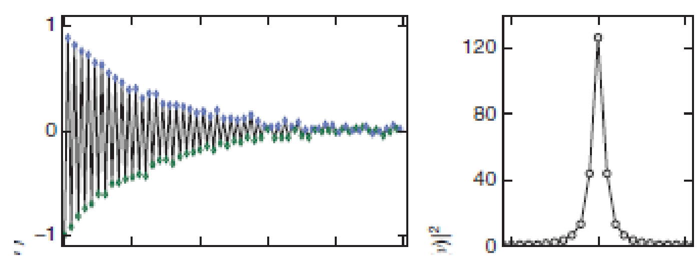


Observation of a Discrete Time Crystal

J. Zhang… C. Monroe, Nature (2017)

Observation of discrete time-crystalline order…

S. Choi… M. Lukin, Nature (2017)



Open questions:

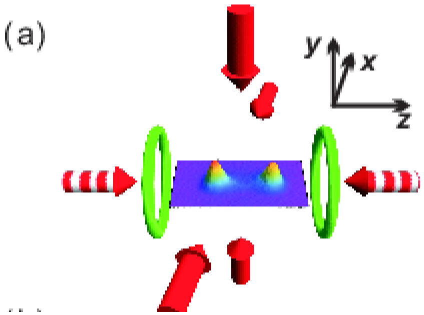
- **What is the role of many-body effects?**
- **Coherent vs dissipative**
- **Is many-body localization necessary?**
- **Heating and “prethermalization”**
- **…**
- **Well-characterized systems can help**

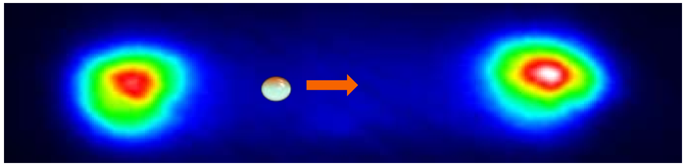
Weakly damped vibrational modes are advantageous:

the driving can be resonant and thus weak

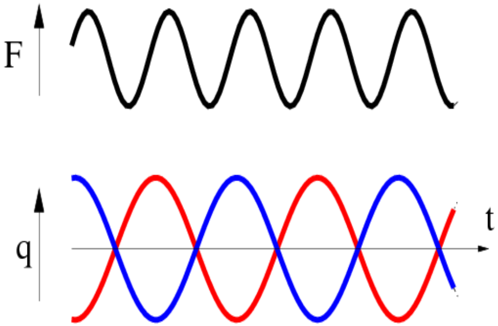
Unbroken symmetry in a magnetooptical trap

Oscillating cold-atom (^{85}Rb, $400\ \mu$K) clouds in a periodically modulated trap (Kim et al, 2006; Heo et al. 2010)





$N_{tot} < N_c \Rightarrow$ **equally populated clouds**



Many-body symmetry breaking in a magnetooptical trap

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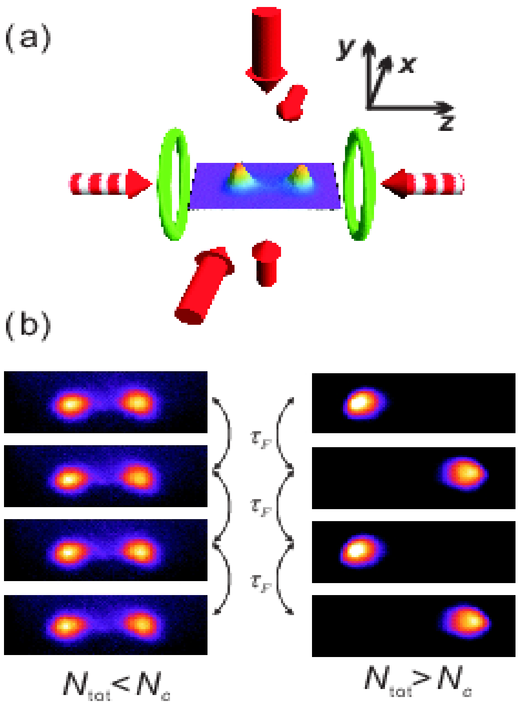
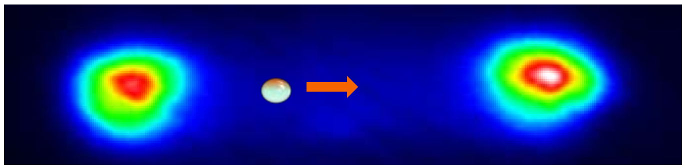


equally populated clouds, no symmetry breaking

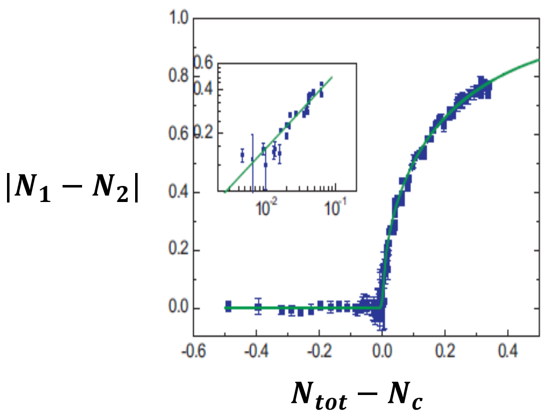
broken symmetry, period-two state



$N_{tot} < N_c \Rightarrow$ **equally populated clouds**

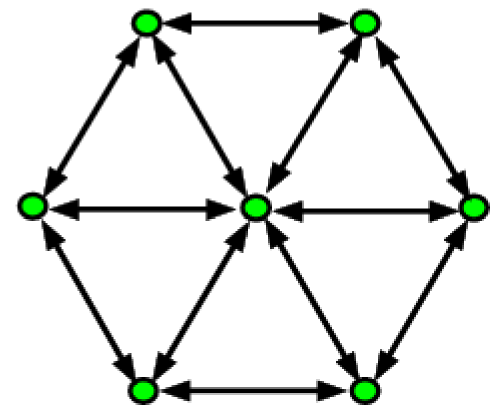
Ising transition for all-to-all coupling

All critical exponents in full agreement with the mean-field theory



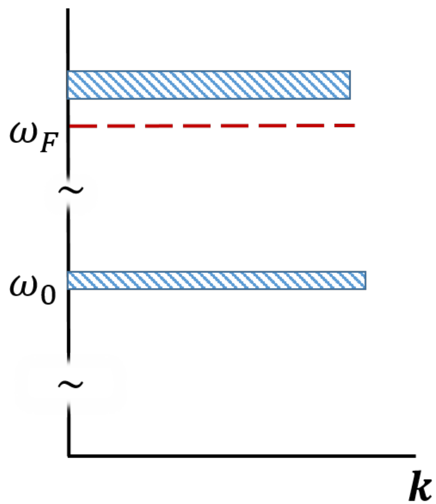
Full Hamiltonian: $H(t) = \sum_n H_n(t) - \frac{1}{2}\sum'_{mn} \epsilon_{nm} q_n q_m$

1D, 2D, not necessarily nearest-neighbor coupling



$$H_n(t) = \frac{1}{2}p_n^2 + \frac{1}{2}(\omega_0^2 + F\cos\omega_F t)\, q_n^2 + \frac{1}{4}\gamma q_n^4$$

Solid-state physics picture: modulated narrow-band optical phonons



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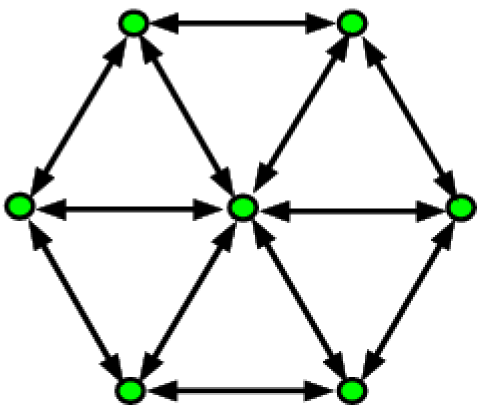


$$H_n(t) = \frac{1}{2}p_n^2 + \frac{1}{2}(\omega_0^2 + F\cos\omega_F t)\, q_n^2 + \frac{1}{4}\gamma q_n^4$$

Rotating frame, frequency $\omega_F/2$ $(\omega_F \approx 2\omega_0)$: switch to **quadratures**

$$q_n = C[-Q_n \sin(\omega_F t/2) + P_n \cos(\omega_F t/2)]$$

$$p_n = -\frac{m\omega_F}{2} C[Q_n \cos(\omega_F t/2) + P_n \sin(\omega_F t/2)]$$

$$[Q_n, P_n] = i\hbar' \propto \hbar$$

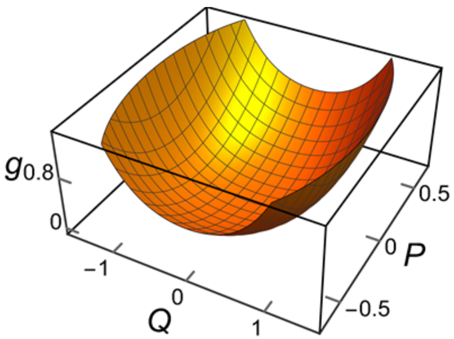
Rotating wave approximation ⇒ time-independent Hamiltonian in terms of the quadratures

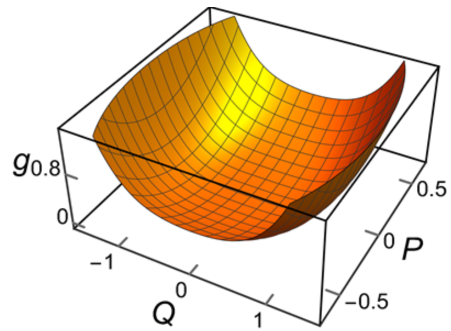
$$H(t) \Rightarrow G, \qquad G = \sum_n g_n(Q_n, P_n) - \frac{1}{2}\sum_{m\neq n} V_{nm}(Q_n Q_m + P_n P_m), \qquad V_{nm} \propto \epsilon_{nm}$$

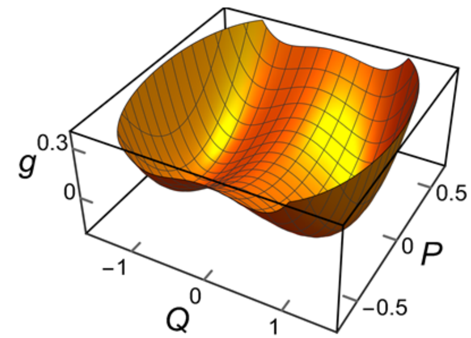
$$g(Q,P) = \frac{1}{4}(Q^2+P^2)^2 + \frac{1}{2}(1-\mu)\,P^2 - \frac{1}{2}(1+\mu)Q^2, \qquad \mu = \frac{\omega_F(\omega_F - 2\omega_0)}{|F|} \propto \frac{\text{small detuning}}{\text{small drive}}$$

$$g(Q,P) = \frac{1}{4}(Q^2+P^2)^2 + \frac{1}{2}(1-\mu)\,P^2 - \frac{1}{2}(1+\mu)Q^2, \qquad \mu = \omega_F(\omega_F - 2\omega_0)/|F|$$

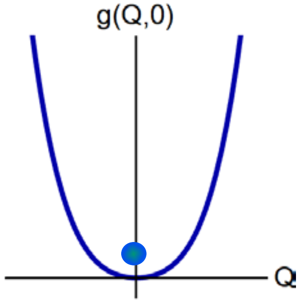
Evolution of the "Floquet" Hamiltonian with the varying scaled frequency detuning:



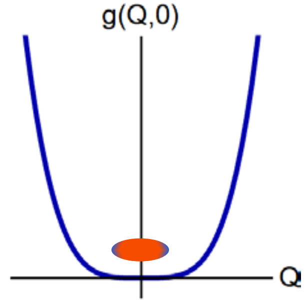




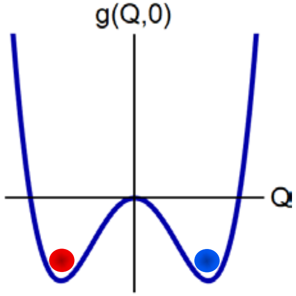
Effective "potential"



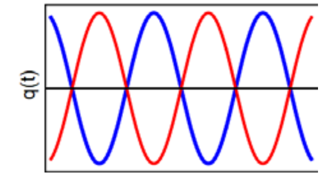
no vibrations, $\mu < -1$

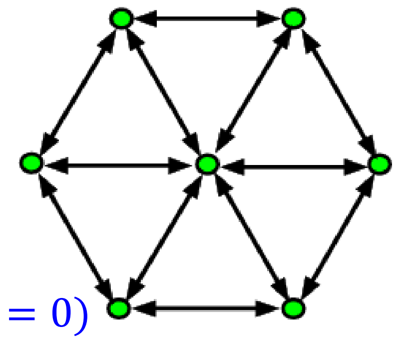


criticality, $\mu_{cr} = -1$



bistability, $\mu > -1$



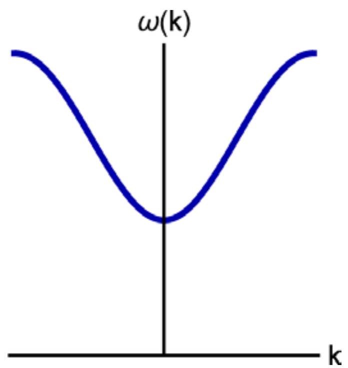


The many-body RWA Hamiltonian $G = \sum_n g(Q_n, P_n) - \frac{1}{2}\sum'_{mn} V_{mn}(Q_m Q_n + P_m P_n)$

Resonators form a periodic system, "ferromagnetic" coupling

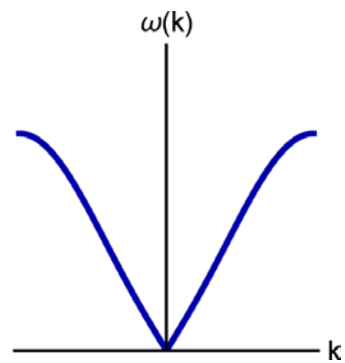
Extremum of G: $Q_n = 0, \mu < \mu_{QPT}$, and $Q_n = \pm\bar{Q},\ \bar{Q} = \left(\mu - \mu_{QPT}\right)^{1/2}, \mu > \mu_{QPT}\quad (P_n = 0)$

Excitation spectra

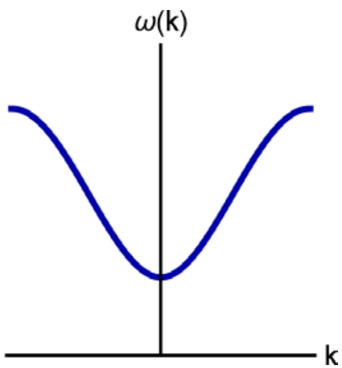


The gap above QPT $(\mu < \mu_{QPT})$

$\left[\left(\mu_{QPT} - \mu + 2\right)\left(\mu_{QPT} - \mu\right)\right]^{1/2}$



At QPT $\omega(k) \propto k, k \to 0$



The gap below QPT $(\mu > \mu_{QPT})$ is

$2|\bar{Q}| \equiv 2\left(\mu - \mu_{QPT}\right)^{1/2}$

$$\mu_{QPT} = -1 + \sum_{\mathbf{n}} V_{\mathbf{nm}}, \qquad \mu = \omega_F(\omega_F - 2\omega_0)/|F|$$

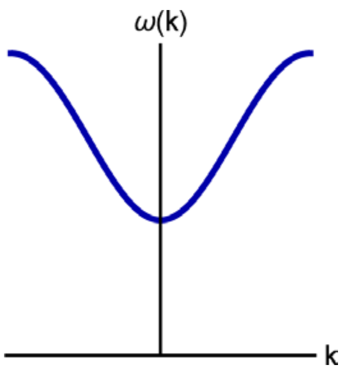
A similar, but topologically nontrivial transition, for a resonantly modulated spin chain (MD, PRA 2019)

The **full** many-body Hamiltonian $G = \sum_n g(Q_n, P_n) - \frac{1}{2}\sum'_{mn} V_{mn}(Q_m Q_n + P_m P_n) + (\hat{g}_{\mathrm{ctr-rot}} \mathrm{e}^{2\mathrm{i}\omega_F t} + \mathrm{c.c.})$

Absorption: from counter-rotating terms. Requires resonance: $n\omega(k) = 2\omega_F \Rightarrow n \gg 1 \Rightarrow$ exponentially small overlap integral away from the critical point.

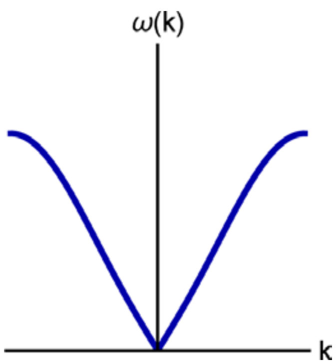
No MBL is required for the time crystal, no high-frequency limit is required to prevent heating.

Excitation spectra



The gap above QPT ($\mu < \mu_{QPT}$)

$[(\mu_{QPT} - \mu + 2)(\mu_{QPT} - \mu)]^{1/2}$



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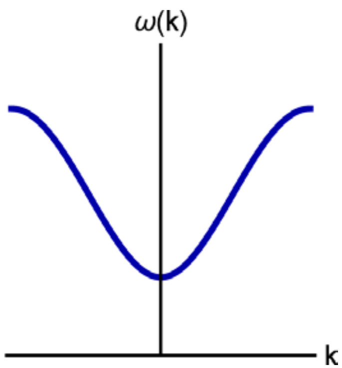

The gap below QPT ($\mu > \mu_{QPT}$) is

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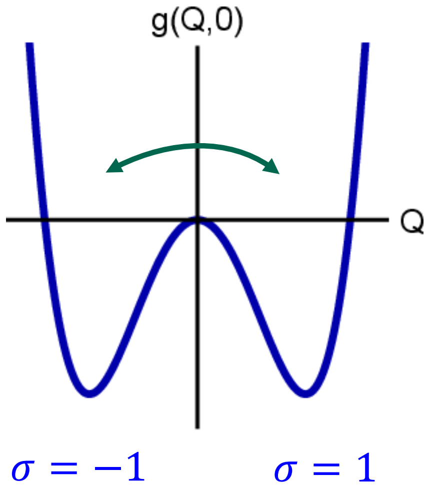
$$\mu_{QPT} = -1 + \sum_{\mathbf{n}} V_{\mathbf{nm}}, \qquad \mu = \omega_F(\omega_F - 2\omega_0)/|F|$$

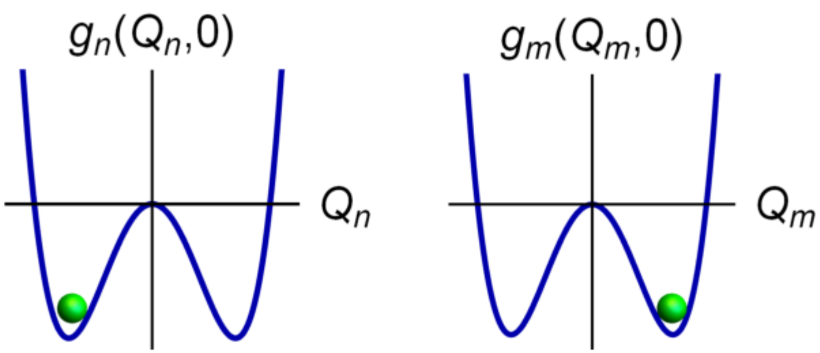
Dissipation⇒ classical- and quantum-noise-induced "over-barrier" switching between the period-two states,

$$W_\sigma^{(n)} = C\exp(-R_\sigma^{(n)}/\tilde{\hbar})$$

$\tilde{\hbar}$ –"quantum temperature", for $k_BT > \hbar\omega_0$, $\tilde{\hbar} \to k_BT$;

cf. $W \propto \exp(-\Delta U/k_BT)$

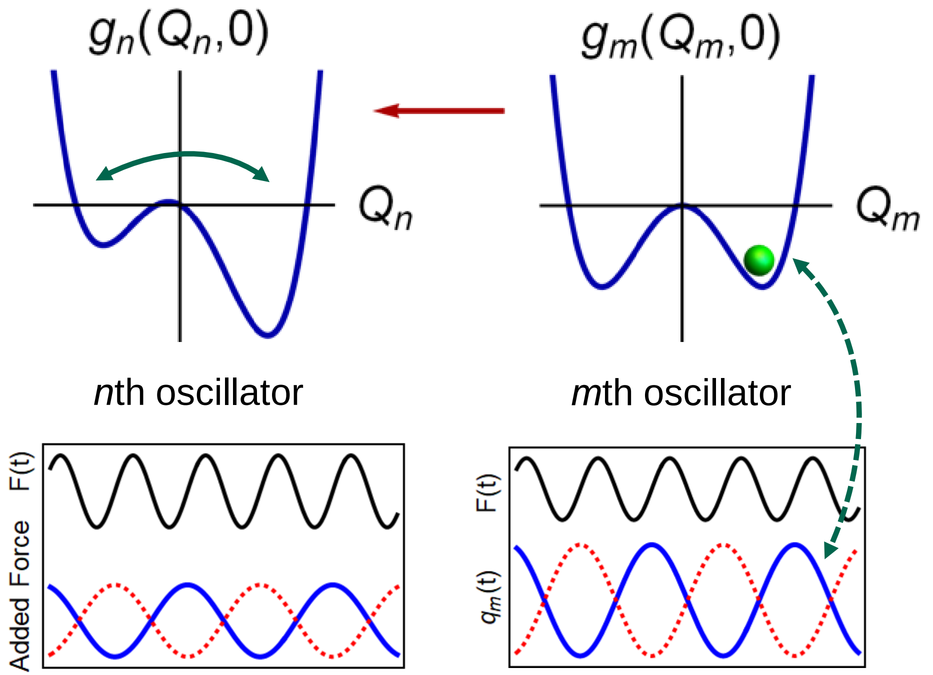




Uncoupled oscillators: $W_1^{(n)} = W_{-1}^{(n)}$. Each oscillator vibrates at the same frequency $\omega_F/2$, The phases, i.e., the occupied minima of g_n, are uncorrelated. No symmetry breaking in a large systerm

Experiment on classical switching: L. Lapidus, D. Enzer, and G. Gabrielse, PRL (1999)

Coupled oscillators *m and n:* If oscillator *m* is in state σ, the time-translation symmetry for oscillator *n* **is broken**



"weak tilt". The change of the switching "activation barrier" is linear in the coupling

$$W_\sigma^{(n)} = C\exp\left(-R_\sigma^{(n)}/\tilde{\hbar}\right), \qquad R_\sigma^{(n)} = \overline{R^{(n)}} + \delta R_\sigma^{(n)}$$

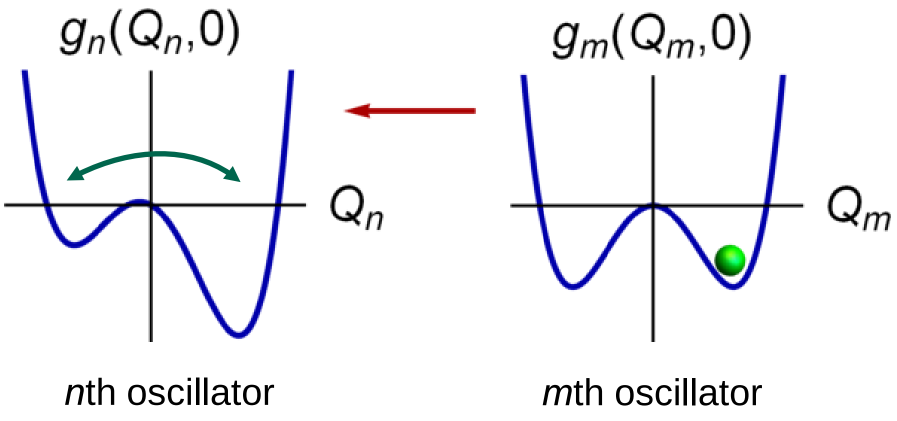
$$\delta R_\sigma^{(n)} = \sigma_n \sum\nolimits_{m\neq n} \sigma_m J_{nm}$$

The sign of $\delta R_\sigma^{(n)}$ is opposite in opposite wells; the equilibrium position σ_m has opposite signs in opposite wells

Symmetry lifting by an extra drive at $\omega_F/2$, classical: D. Ryvkine and MD, PRE (2006); observed: I. Mahboob et al., APL (2010); numerical work on coupled quantum oscillators: H. Goto, J. Phys. Soc. Japan (2019), N. Lörch et al., PRR (2019) and refs. therein.

Coupling-induced change of the switching rates

Coupled oscillators *m and n:* If oscillator *m* is in state σ, the time-translation symmetry for oscillator *n* **is broken**



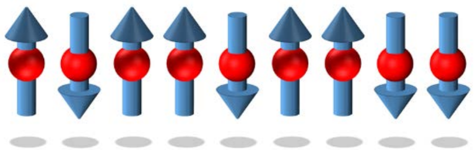
$$W_\sigma^{(n)} = C \exp\left(-R_\sigma^{(n)}/\tilde{\hbar}\right), \qquad R_\sigma^{(n)} = \overline{R^{(n)}} + \delta R_\sigma^{(n)}$$

$$\delta R_\sigma^{(n)} = \sigma_n \sum_{m \neq n} \sigma_m J_{nm}$$

Mapping on the system of coupled spins: $W_\sigma^{(n)} = \overline{W^{(n)}} \exp[-\sigma_n \sum_m J_{nm} \sigma_m / \tilde{\hbar}]$

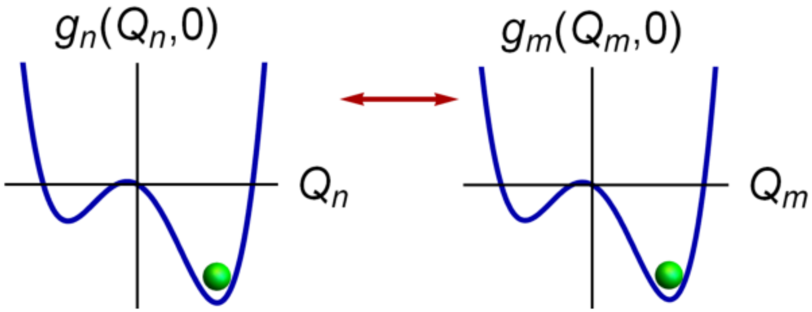
Ising system, except that $J_{nm} \neq J_{mn}$ if nanoresonators are not identical!

J_{nm} is **not** the coupling energy! Calculating it is fun

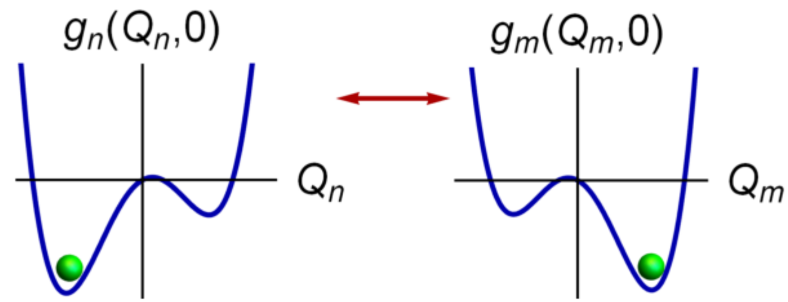


Weak coupling: order and time frustration

“ferromagnetic” coupling $J_{nm} > 0 \Rightarrow$ same phase state

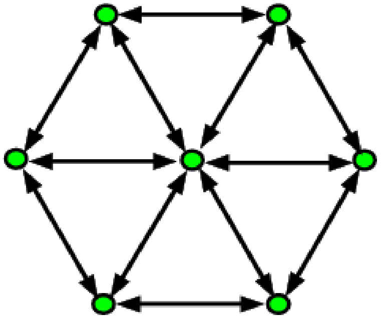


“antiferromagnetic”: $J_{nm} < 0 \Rightarrow$ possible frustration



If the coupling is attractive, oscillators vibrate with the same phase

„Effectively strong“ coupling: metastable many-body states



Quantum dynamics **near the bifurcation point** $\mu = \mu_B$ where period-two vibrations emerge

$$\frac{d\tilde{Q}_n}{dt} = -\frac{\partial U}{\partial \tilde{Q}_n} + \sum_m V_{nm}\,\tilde{Q}_m + quantum\ noise\,,$$

$$U(\tilde{Q}) = -\tfrac{1}{2}\tilde{Q}^2 + \tfrac{1}{4}\tilde{Q}^4, \quad V_{nm} \propto |\mu - \mu_B|^{-1}$$

$\Rightarrow$ simple explicit form of J_{nm}

Very close to the bifurcation point the coupling becomes strong – multiple global minima of the effective energy, no matching on a spin system

$$E = \sum_n U\left(\tilde{Q}_n\right) - \tfrac{1}{2}\sum_{m\neq n} V_{nm}\tilde{Q}_n\tilde{Q}_m$$

Quantum-noise induced diffusion of coupled particles in a potential landscape

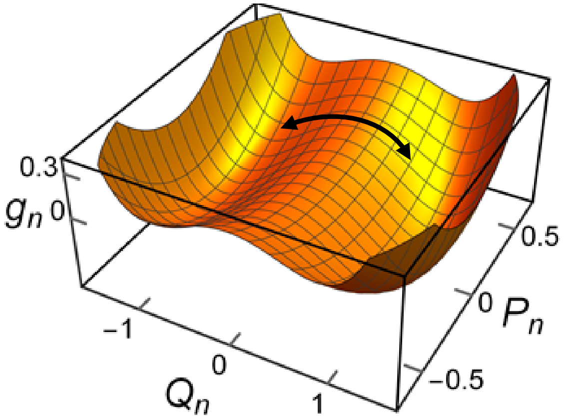
Conclusions

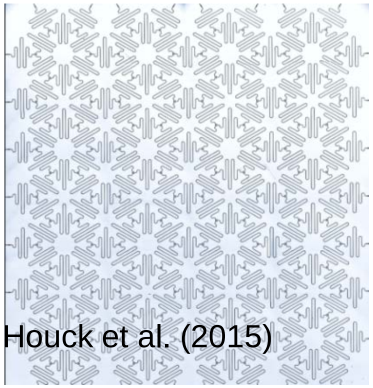
- **Interaction of quantum parametric oscillators leads to breaking of the discrete time-translation symmetry, the "time-crystal" effect.**

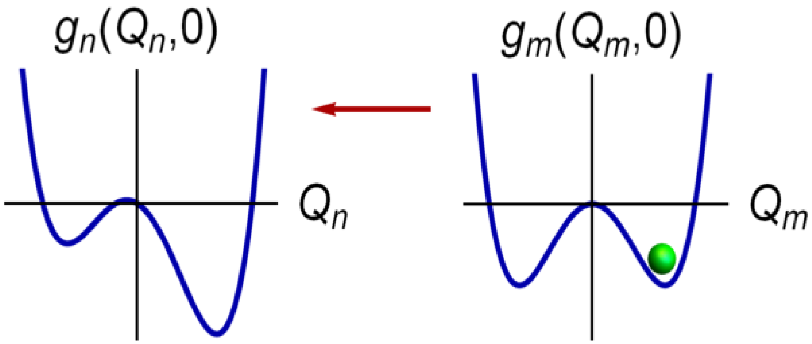
- **In the quantum coherent regime, time-symmetry breaking maps onto a far-from-equilibrium quantum phase transition. Heating is exponentially slow away from the QPT.**

- **The QPT leads to a topologically nontrivial phase for a parametrically modulated spin/qubit chain**

- **The system of coupled dissipative oscillators maps onto the Ising system with controlled connectivity. The system has a nontrivial "non-Hamiltonian" disorder and broken time symmetry**




Houck et al. (2015)



Quantum mechanics of periodically driven systems.

The Schrödinger equation: $i\hbar\dot{\psi} = H(t)\psi, \quad H(t) = H(t+\tau_F), \quad \tau_F = 2\pi/\omega_F$

No stationary eigenstates in a driven system**. But** there is time-translation symmetry

Floquet (quasienergy) states $\psi_\varepsilon(t+\tau_F) = \exp(-i\varepsilon\tau_F/\hbar)\psi_\varepsilon(t)$

Expectation value of an observable: $\langle L(t)\rangle = \langle\psi(t)|L|\psi(t)\rangle$. For a Floquet state $\langle L(t+\tau_F)\rangle = \langle L(t)\rangle$

Superposition: $\psi(t) = A_1\psi_{\varepsilon_1}(t) + A_2\psi_{\varepsilon_2}(t) \Rightarrow$

$$\langle L(t+\tau_F)\rangle = |A_1|^2\langle\psi_{\varepsilon_1}(t)|L|\psi_{\varepsilon_1}(t)\rangle + |A_2|^2\langle\psi_{\varepsilon_2}(t)|L|\psi_{\varepsilon_2}(t)\rangle$$

$$+[A_1^*A_2\langle\psi_{\varepsilon_1}(t)|L|\psi_{\varepsilon_2}(t)\rangle\exp\left[\frac{\mathrm{i}(\varepsilon_1-\varepsilon_2)\tau_F}{\hbar}\right] + \mathrm{c.c.}]$$

If $(\varepsilon_1-\varepsilon_2)\tau_F/\hbar = \pi$ then $\langle L(t+2\tau_F)\rangle = \langle L(t)\rangle$ $\Rightarrow$period doubling in the supersposition of states

Bloch theorem: $\psi_{\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{R}) = \exp(i\boldsymbol{k}\boldsymbol{R})\psi_{\boldsymbol{k}}(\boldsymbol{r})$;