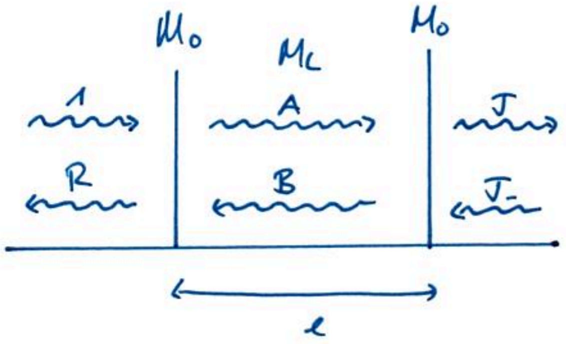
Aufgabe 2: Fabry_Perot-Kavität



$$M_0 = \frac{1}{t}\begin{pmatrix} 1 & r \\ -r & 1 \end{pmatrix} \qquad r = i\sqrt{1-t^2} \Rightarrow t^2 = 1 + r^2$$

$$M_L = \begin{pmatrix} z & 0 \\ 0 & z^* \end{pmatrix} \qquad z = e^{ikL}$$

a) es gilt:

- $$\begin{pmatrix} J e^{ikL} \\ J_- e^{-ikL} \end{pmatrix} = \underbrace{M_0 M_L M_0}_{:= M} \begin{pmatrix} 1 \\ R \end{pmatrix}$$

$$M = \frac{1}{t}\begin{pmatrix} 1 & r \\ -r & 1 \end{pmatrix}\begin{pmatrix} z & 0 \\ 0 & z^* \end{pmatrix}\frac{1}{t}\begin{pmatrix} 1 & r \\ -r & 1 \end{pmatrix} = \frac{1}{t^2}\begin{pmatrix} z - r^2 z^* & r(z+z^*) \\ -r(z+z^*) & z^* - r^2 z \end{pmatrix}$$

Bedingung: $J_- = 0 \quad \Rightarrow \quad 0 = M_{21}\cdot 1 + M_{22}\cdot R \quad \Rightarrow \quad R = -\frac{M_{21}}{M_{22}}$

- $$\begin{pmatrix} A \\ B \end{pmatrix} = M_0 \begin{pmatrix} 1 \\ R \end{pmatrix}$$

$$A = (M_0)_{11}\cdot 1 + (M_0)_{12}\cdot R = (M_0)_{11} - \frac{M_{21}}{M_{22}}(M_0)_{12}$$

$$= \frac{1}{t} - \frac{r}{t}\,\frac{\frac{1}{t^2}\, r(z+z^*)}{\frac{1}{t^2}\,(z^* - r^2 z)} = \frac{1}{t} - \frac{r^2}{t}\cdot\frac{\overbrace{z+z^*}^{= e^{ikL}+e^{-ikL} = 2\cos(kL)}}{z+z^* - t^2 z} =$$

$$= \frac{1}{t} - \frac{r^2}{t}\,\frac{2\cos(kL)}{2\cos(kL) - t^2 e^{ikL}}$$

$$\Rightarrow \quad \boxed{A = \frac{1}{t} - \frac{r^2}{t}\,\frac{2\cos(kL)}{2\cos(kL) - t^2 e^{ikL}}}$$

(b) es gilt $t \ll 1$

- $\underline{\cos(kl) = 0}$ $\Rightarrow$ $kl = \frac{\pi}{2}$ $\Rightarrow$ $e^{ikl} = i$ (Resonanz)

$$\Rightarrow A = \frac{1}{t} + \frac{r^2}{t} \frac{0}{0 + t^2 e^{ikl}} = \frac{1}{t} \quad \text{(sehr groß)}$$

- $\underline{\cos(kl) \neq 0}$

$$\Rightarrow A = \frac{1}{t} + \frac{r^2}{t} \frac{2\cos(kl)}{2\cos(kl) - \underbrace{\frac{t^2}{2} e^{ikl}}_{\ll 1}} = \frac{1}{t} + \frac{r^2}{t} = \frac{1+r^2}{t} = \frac{t^2}{t} = t \quad \text{(sehr klein)}$$

Betrachte nun Umgebung der Resonanz: $kl = \frac{\pi}{2} + \delta k \cdot l$

- $e^{ikl} = \underbrace{e^{i\frac{\pi}{2}}}_{=i} \cdot e^{i\delta k l} = i \cdot e^{i\delta k l} \approx i(1 + i\delta k l) = \underline{\underline{i - \delta k l}}$

$$\left[e^{i\delta k l} \approx e^{i\delta k l}\Big|_{\delta k = 0} + \left(\frac{\partial}{\partial \delta k} e^{i\delta k l}\right)_{\delta k = 0} \cdot \delta k \right.$$

$$\approx 1 + \left(i l \, e^{i\delta k l}\right)_{\delta k = 0} \cdot \delta k$$

$$\left. \approx 1 + i\delta k l \right]$$

- $2 \cdot \cos(\delta k l) = e^{i\delta k l} + e^{-i\delta k l} \approx i - \delta k l - i - \delta k l = -2\delta k l$

$$\Rightarrow A = \frac{1}{t} + \frac{r^2}{t} \left(\frac{-2\delta k l}{-2\delta k l - t^2(i - \delta k l)} \right)$$

$$= \frac{1}{t} \left[1 + \frac{-2\delta k l \, r^2}{-2\delta k l - t^2(i - \delta k l)} \right]$$

$$= \frac{1}{t} \left[\frac{-2\delta k l \overbrace{(1 + r^2)}^{=t^2} - t^2(i - \delta k l)}{-2\delta k l - t^2(i - \delta k l)} \right]$$

$$= \frac{1}{t}\left[\frac{-2\,\delta k\, l\, t^2 - it^2 + \delta k\, l\, t^2}{-2\,\delta k\, l - t^2(i - \delta k\, l)}\right]$$

$$= \frac{1}{t}\left[\frac{-\delta k\, l\, t^2 - it^2}{-2\delta k\, l - t^2(i - \delta k)}\right]$$

$$= \frac{t^2}{t}\left[\frac{\delta k\, l + i}{2\delta k\, l + t^2(i - \delta k)}\right]$$

$\mathcal{O}(\delta k l) \ll \mathcal{O}(i)$

$\mathcal{O}(t^2 \delta k l) \ll \mathcal{O}(\delta k l)$

$$\approx t\left[\frac{i}{2\,\delta k l + it^2}\right]$$

$$\approx \frac{it}{2\delta k l + it^2}$$

Definieren: $A_0 := \frac{it}{2}$ $\quad \varepsilon := \frac{it^2}{2}$

$$\Rightarrow A \approx \frac{2A_0}{2\delta k l + 2\varepsilon} = \underline{\underline{\frac{A_0}{\delta k l + \varepsilon}}}$$