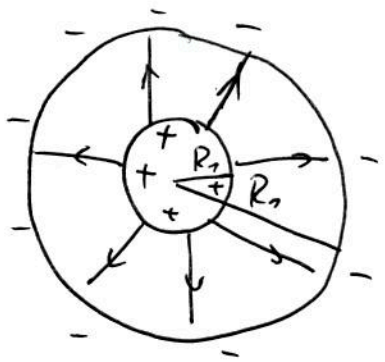
## Aufgabe 1

a)




Kugelsymmetrie: $\vec{E}(\vec{r}) = E(r)\cdot\vec{e}_r$

b) Maxwell: $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0} \iff \int_V \vec{\nabla}\vec{E}\, d^3r \overset{\text{S.v.G.}}{=} \oint_{\partial V} \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0}\int_V \rho\, d^3r$

- $\oint_{\partial V} \vec{E}\, d\vec{A} = \int_0^{2\pi}\int_0^{\pi} E(r)\,\vec{e}_r \cdot r^2 \sin\vartheta\, d\vartheta\, d\varphi\, \vec{e}_r = 4\pi r^2 E(r)$

- $\int_V \rho\, d^3r = Q(r) = \begin{cases} 0 & \text{für } r < R_1 \\ Q & \text{für } R_1 \le r \le R_2 \\ 0 & \text{für } R_2 < r \end{cases}$

$\Rightarrow E(r) = \frac{Q(r)}{4\pi\varepsilon_0}\cdot\frac{1}{r^2} = \begin{cases} 0 & \text{für } r < R_1 \\ \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} & \text{für } R_1 \le r \le R_2 \\ 0 & \text{für } R_2 < r \end{cases}$

c) $U = \frac{\varepsilon_0}{2}\int_{\mathbb{R}^3} \vec{E}^2\, d^3r = \frac{\varepsilon_0}{2}\int_{R_1}^{R_2}\int_0^{\pi}\int_0^{2\pi} \frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\frac{1}{r^4}\cdot r^2 \sin\vartheta\, dr\, d\varphi\, d\vartheta$

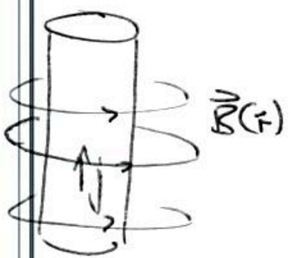
$= \frac{\varepsilon_0}{2}\cdot\frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\int_{R_1}^{R_2}\frac{1}{r^2}\,dr \cdot \int_0^{2\pi} d\varphi \cdot \int_0^{\pi}\sin\vartheta\, d\vartheta$

$= \frac{Q^2}{32\pi^2\varepsilon_0}\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\cdot 2\pi\cdot 2$

$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow C = 4\pi\varepsilon_0\left[\frac{1}{R_1}-\frac{1}{R_2}\right]^{-1}$

## Aufgabe 2




Symmetrie: $\vec{B}(\vec{r}) = |B(r)| \cdot \vec{e}_\varphi$

Maxwell: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$

S: Scheibe mit Radius r senkrecht zum Draht

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{B})\, d\vec{A} = \underbrace{\int_{\partial S} \vec{B}\, d\vec{l}}_{2\pi r \cdot |B(r)|} = \mu_0 \int_S \vec{j}\, d\vec{A}$$

$\underline{r < R}$: $$\int_S \vec{j}\, d\vec{A} = j_0 \int_0^{2\pi} d\varphi \int_0^r dr'\, r' \left(1 - \frac{r'^2}{R^2}\right) = j_0 2\pi \left[ \int_0^r dr'\, r' - \frac{1}{R^2}\int_0^r dr'\, r'^3 \right]$$

$$= j_0 \pi r^2 \left(1 - \frac{r^2}{2R^2}\right)$$

$\underline{r \geq R}$: $$\int_S \vec{j}\, d\vec{A} = j_0 \int_0^{2\pi} d\varphi \int_0^R dr'\, r' \left(1 - \frac{r'^2}{R^2}\right) = j_0 \pi R^2 \left(1 - \frac{R^2}{2R^2}\right) = \frac{j_0 \pi R^2}{2}$$

$$\Rightarrow |B(r)| = \begin{cases} \frac{\mu_0 j_0 r}{2}\left[1 - \frac{r^2}{2R^2}\right] & r < R \\ \frac{\mu_0 j_0 R^2}{4r} & r \geq R \end{cases}$$

## Aufgabe 3:

a) $\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$

$\vec{\nabla} \times \vec{B} = \underbrace{\mu_0 \vec{j}}_{=0} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E}$

~~es~~ gilt: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}\vec{E}) - \nabla^2 \vec{E}$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = \underbrace{\mu_0 \varepsilon_0}_{\frac{1}{c^2}} \frac{\partial}{\partial t} \left(-\frac{\partial}{\partial t} \vec{B}\right) = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{B}$$

$$= \vec{\nabla}(\underbrace{\vec{\nabla}\vec{B}}_{=0}) - \nabla^2 \vec{B} = -\vec{\nabla}^2 \vec{B}$$

$$\Rightarrow \boxed{\vec{\nabla}^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{B} = 0}$$

b) $(c^2 \partial_x^2 - \partial_t^2)\, \phi(x,t) = 0$

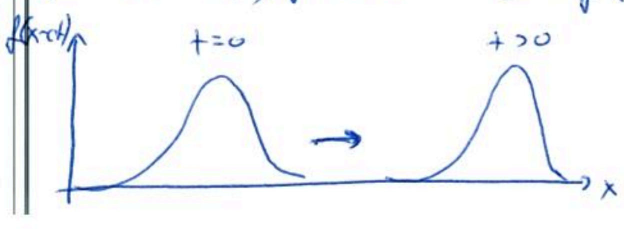
$\phi(x,t) = f(x-ct)$

$\phi' := \frac{\partial \phi}{\partial (x-ct)}$

$\frac{\partial^2}{\partial x^2} f(x-ct) = \frac{\partial}{\partial x}\left(f'(x-ct) \cdot 1\right) = f''(x-ct) \cdot 1$

$\frac{\partial^2}{\partial t^2} f(x-ct) = \frac{\partial}{\partial t}\left(f'(x-ct) \cdot (-c)\right) = f''(x-ct) \cdot (-c)^2$

$\Rightarrow (c^2 \cdot \partial_x^2 - \partial_t^2) f(x-ct) = c^2 \cdot f''(x-ct) - c^2 f''(x-ct) = 0$ ✓




nach links laufend: $f(x+ct) = \phi_L(x+ct)$

c) $E_x(x,y,z,t) = f(z-ct)$

$B_y(x,y,z,t) = K f(z-ct)$

- $\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$

$\Rightarrow$ • $\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} f(z-ct) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \partial_z f(z-ct) \\ -\partial_y f(z-ct) \end{pmatrix} = \begin{pmatrix} 0 \\ \partial_z f(z-ct) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ f'(z-ct) \end{pmatrix}$

• $-\frac{\partial}{\partial t} \begin{pmatrix} 0 \\ K f(z-ct) \\ 0 \end{pmatrix} = -K \begin{pmatrix} 0 \\ f'(z-ct)\cdot(-c) \\ 0 \end{pmatrix} = cK \begin{pmatrix} 0 \\ f'(z-ct) \\ 0 \end{pmatrix}$

$\Rightarrow cK \overset{!}{=} 1 \rightarrow \boxed{K = \frac{1}{c}}$

- $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E}$

• $\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} 0 \\ \frac{1}{c} f(z-ct) \\ 0 \end{pmatrix} = \begin{pmatrix} -\partial_z \frac{1}{c} f(z-ct) \\ 0 \\ \partial_x \frac{1}{c} f(z-ct) \end{pmatrix} = -\frac{1}{c} \begin{pmatrix} f'(z-ct) \\ 0 \\ 0 \end{pmatrix}$

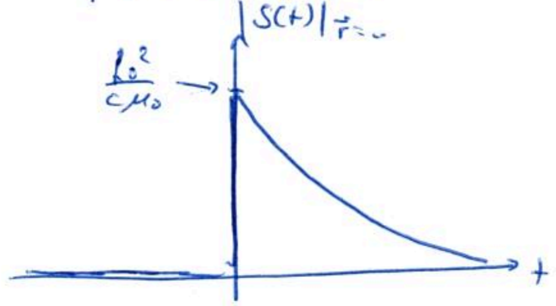
• $\frac{1}{c^2} \frac{\partial}{\partial t} \begin{pmatrix} f(z-ct) \\ 0 \\ 0 \end{pmatrix} = \frac{1}{c^2} \begin{pmatrix} f'(z-ct)\cdot(-c) \\ 0 \\ 0 \end{pmatrix} = -\frac{1}{c} \begin{pmatrix} f'(z-ct) \\ 0 \\ 0 \end{pmatrix}$ ✓

- $\vec{\nabla} \vec{E} = 0$ (trivial) ($E_x$ hängt nicht von x ab)

- $\vec{\nabla} \vec{B} = 0$ (trivial) ($B_y$ hängt nicht von y ab)

d) $|\vec{S}(t)|_{\vec{r}=0} = \left| \frac{1}{c\mu_0} \left[ f(-ct) \right]^2 \right|$ $\qquad f(\omega) = \begin{cases} f_0 e^{\omega/z_0} & \omega \leq 0 \\ 0 & \omega > 0 \end{cases}$

$\Rightarrow$




$t < 0 \Rightarrow \omega > 0 \Rightarrow f = 0$

$t > 0 \Rightarrow \omega < 0 \Rightarrow f \sim e^{-\frac{ct}{z_0}}$

$\Rightarrow |S| \sim e^{-\frac{2c}{z_0} t}$

## Aufgabe 4

a) $$E = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

$$C_1 = \varepsilon_0 \varepsilon_r \frac{A_1}{d} \qquad C_2 = \varepsilon_0 \frac{A_2}{d}$$

$$A_1 = h_1 \cdot l \qquad A_2 = h_2 \cdot l = (h - h_1) l$$

$$Q_1 = \sigma_1 A_1 \qquad Q_2 = \sigma_2 A_2$$

$$\Rightarrow E = \frac{\sigma_1^2 A_1^2 d}{2 \varepsilon_0 \varepsilon_r A_1} + \frac{\sigma_2^2 A_2^2 d}{2 \varepsilon_0 A_2} = \frac{d}{2\varepsilon_0} \left( \frac{\sigma_1^2 A_1}{\varepsilon_r} + \sigma_2^2 A_2 \right)$$

b) $$\sigma_1 A_1 + \sigma_2 A_2 = Q$$

$$\rightarrow \sigma_2 = \frac{Q - \sigma_1 A_1}{A_2}$$

in Energie

$$E(\sigma_1) = \frac{d}{2\varepsilon_0} \left( \frac{\sigma_1^2 A_1}{\varepsilon_r} + \frac{(Q - \sigma_1 A_1)^2}{A_2} \right)$$

$$\frac{dE}{d\sigma_1} = \frac{d}{2\varepsilon_0} \left( \frac{2\sigma_1 A_1}{\varepsilon_r} - \frac{2(Q - \sigma_1 A_1) A_1}{A_2} \right) = \frac{d}{\varepsilon_0} \left( \frac{\sigma_1 A_1}{\varepsilon_r} - \frac{A_1}{A_2} Q + \frac{A_1^2}{A_2} \sigma_1 \right)$$

$$\frac{d^2E}{d\sigma_1^2} > 0 \Rightarrow \text{Minimum}$$

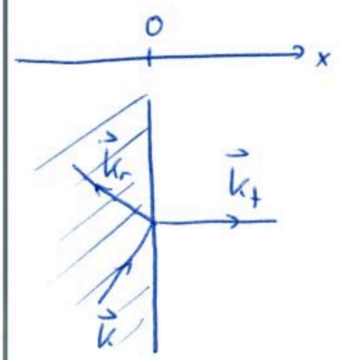
$$\frac{dE}{\delta\sigma_1} \overset{!}{=} 0$$

$$\Rightarrow \frac{\sigma_1 \cancel{A_1}}{\varepsilon_r} - \frac{\cancel{A_1}}{A_2} Q + \frac{A_1^{\cancel{2}}}{A_2} \sigma_1 = 0$$

$$\sigma_1 \left( \frac{1}{\varepsilon_r} + \frac{A_1}{A_2} \right) = \frac{Q}{A_2}$$

$$\Rightarrow \sigma_1 = \frac{Q / A_2}{\frac{1}{\varepsilon_r} + \frac{A_1}{A_2}} = \frac{Q}{\frac{A_2}{\varepsilon_r} + A_1} = \frac{Q/L}{\frac{h-h_1}{\varepsilon_r} + h_1}$$

# Aufgabe 5




a) Randbedingung auf der gesamten Oberfläche ($x=0$)

"$E_{in\parallel} = E_{out\parallel}$"

$$\vec{E}_{0\parallel}\, e^{i(\vec{k}_\parallel \vec{r} - \omega t)} + \vec{E}_{r_\parallel}\, e^{i(\vec{k}_r \vec{r}_\parallel - \omega t)} - \vec{E}_{t\parallel}\, e^{i(\vec{k}_{t\parallel} \vec{r} - \omega t)} =$$

$$\Rightarrow \vec{k}_\parallel = \vec{k}_{r\parallel} = \vec{k}_{t\parallel}$$

b) In Medium: $|k| = |\vec{k}_r|$ & $\vec{k}_\parallel = \vec{k}_{\parallel r}$

$\Rightarrow$ Einfallswinkel = Ausfallswinkel

$$\frac{c}{n} |\vec{k}| = c\,|\vec{k}_t|$$

$$k_\perp = k \cos\theta \quad \Rightarrow |k_\parallel| = k \cdot \sin\theta$$

$$\frac{|\vec{k}|^2}{n^2} = |\vec{k}_t|^2 \quad \Rightarrow \frac{1}{n^2}\left( \vec{k}_\perp^{\,2} + \vec{k}_\parallel^{\,2} \right) = k_{t_\perp}^2 + \underbrace{|\vec{k}_{t\parallel}|^2}_{= \vec{k}_\parallel^{\,2}}$$

$$\rightarrow k_{t_\perp} = \sqrt{\frac{1}{n^2}\left( \underbrace{k_\perp^2}_{k^2 \cdot \cos^2\theta} + \underbrace{|k_\parallel|^2}_{k^2 \cdot \sin^2\theta} \right) - \underbrace{|k_\parallel|^2}_{k^2 \cdot \sin^2\theta}}$$

$$= |k| \cdot \sqrt{\frac{1}{n^2} - \sin^2\theta}$$

Totalreflexion wenn $\sqrt{\phantom{x}} \notin \mathbb{R}$

$$\rightarrow \frac{1}{n^2} \leq \sin^2\theta \quad \Leftrightarrow \quad \frac{1}{n} \leq \sin^2\theta$$

c) Abklinglänge:

$$|\vec{E}|^2 \sim \left| e^{i(\vec{k}_t \vec{r} - \omega t)} \right|^2$$

$$= \left| e^{i|\vec{k}_t \vec{r}|}\, e^{-i\omega t} \right|^2$$

Vorsicht:

$$k_t = |k| \sqrt{\underbrace{\frac{1}{n^2} - \sin^2\theta}_{\leq 0}} \in i\mathbb{R}$$

$$= e^{i(\vec{k}_t \vec{r})} e^{-i\omega t} e^{-i(\vec{k}_t^* \vec{r})} e^{i\omega t}$$

$$= e^{ik_{t\perp}x} e^{i\vec{k}_{\parallel}\vec{r}} e^{-ik_{t\perp}^*x} e^{-i\vec{k}_{t\parallel}\vec{r}}$$

$\in \mathbb{R}!$

$$= e^{ik_{t\perp}x} e^{-ik_{t\perp}^*x}$$

$$\left[ k_{t\perp} = |k| \cdot \sqrt{\frac{1}{n^2} - \sin^2\theta} = i|k| \cdot \alpha \right. \quad \alpha \in \mathbb{R}$$

$$= e^{-|k|\alpha x} \cdot e^{-|k|\alpha x} = e^{-2|k|\alpha x}$$

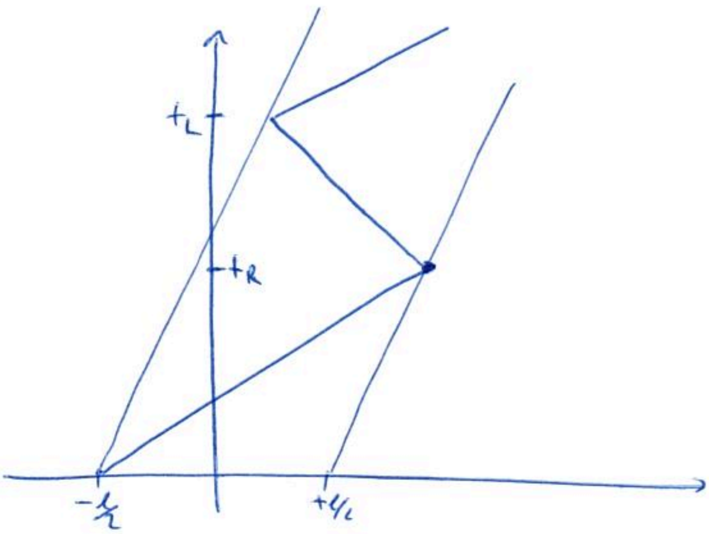
$$\Rightarrow \frac{1}{\ell} = 2|k|\alpha = 2|k| \cdot \left| \sqrt{\frac{1}{n^2} - \sin^2\theta} \right|$$

$$= \frac{2\omega}{c} \sqrt{\sin^2\theta - \frac{1}{n^2}}$$

$$\Rightarrow \boxed{\ell = \frac{c}{2\omega} \cdot \left( \sin^2\theta - \frac{1}{n^2} \right)^{-1/2}}$$

## Aufgabe 6

a)




b) $t_R = ?$

Im Laborsystem müssen gleiche Strecken zurückgelegt werden:

$\rightarrow \quad c \cdot t_R = v \cdot t_R + l \qquad \Rightarrow \quad \boxed{t_R = \frac{l}{c-v}}$

„Rückflug" des Lichts

$$c\,(t_L - t_R) = -v\,(t_L - t_R) + l \qquad \Rightarrow \quad \boxed{t_L = \frac{l}{c+v} + t_R}$$

c) $x' = \gamma(x - vt)$

$t' = \gamma\left(t - \frac{vx}{c}\right)$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

3 Ereignisse (in 10)

$t = 0 \;\&\; x(0) = -\frac{l}{2}$

$t = t_R \;\&\; x(t_R) = \frac{l}{2} + v t_R$

$t = t_L \;\&\; x(t_L) = -\frac{l}{2} + v t_L$

Transformation in das bewegt Sys.

$t' = \gamma\left(0 + \frac{vl}{2c^2}\right) = \gamma \frac{vl}{2c^2}$

$$t_R' = \gamma\left(t_R - \frac{v}{c^2}\left(\frac{l}{2} + vt_R\right)\right)$$
$$= \gamma\left(t_R\left(1-\frac{v^2}{c^2}\right) - \frac{vl}{2c^2}\right)$$
$$t_L' = \gamma\left(t_L - \frac{v}{c^2}\left(-\frac{l}{2} + vt_L\right)\right)$$
$$= \gamma\left(t_L\left(1-\frac{v^2}{c^2}\right) + \frac{vl}{2c^2}\right)$$

- $\Delta T_1 = t_R' - t' = \gamma t_R\left(1-\frac{v^2}{c^2}\right) - \gamma\left(\frac{vl}{c^2}\right) = \gamma\left(1-\frac{v^2}{c^2}\right)\frac{l}{c-v} - \gamma\frac{vl}{c^2}$
- $\Delta T_2 = t_L' - t_R' = \gamma t_L\left(1-\frac{v^2}{c^2}\right) + \gamma\frac{vl}{2c^2} - \gamma t_R\left(1-\frac{v^2}{c^2}\right) + \gamma\frac{vl}{2c^2}$

$$= \gamma\left(1-\frac{v^2}{c^2}\right)\underbrace{(t_L - t_R)}_{=\frac{l}{c+v}} + \gamma\frac{vl}{c^2} = \gamma\left(1-\frac{v^2}{c^2}\right)\frac{l}{c+v} + \gamma\frac{vl}{c^2}$$

Gleiche Abstände:

$$\Delta T_1 - \Delta T_2 \stackrel{?}{=} 0$$

$$\left(1-\frac{v^2}{c^2}\right)\frac{l}{c-v} - \frac{vl}{c^2} - \left(1-\frac{v^2}{c^2}\right)\frac{l}{c+v} - \frac{vl}{c^2} \stackrel{?}{=} 0$$

$$\underbrace{\left(1-\frac{v^2}{c^2}\right)}_{\frac{c^2-v^2}{c^2}}\underbrace{\left(\frac{1}{c-v} - \frac{1}{c+v}\right)}_{\frac{c+v-c+v}{c^2-v^2}} - \frac{2vl}{c^2} \stackrel{?}{=} 0$$

$$\frac{2v}{c^2} - \frac{2v}{c^2} = 0 \quad \checkmark$$