5. Interpretations of Quantum Mechanics

5.1 "Kopenhagen interpretation"

1. Quantum evolution $i\hbar \frac{\partial \psi}{\partial t} = H\psi$
2. Measurement: Interaction with classical, macroscopic apparatus
   $\rightarrow$ probabilities according to $|\psi(x)|^2$ (etc.)
   $\rightarrow$ collapse of ψ

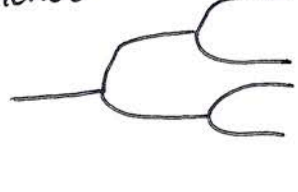
Values of observables become real only via msmt

($\rightarrow$ "No trajectories")

"Many worlds": ($\rightarrow$ see later)

Only coherent evolution
$\Rightarrow$ highly entangled $|\psi_{sys+App.}\rangle$

"branches"



5.2 Bohm's hidden variable theory

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"Can we use trajectories $\vec{x}(t)$?"

Usual answer: No, $\hat{x}$ & $\hat{p}$ cannot be precisely defined simultaneously!
$\Rightarrow$ "no $\vec{x}(t)$"

Bohm 1952 "pilot wave theory"
(also de Broglie 1927)

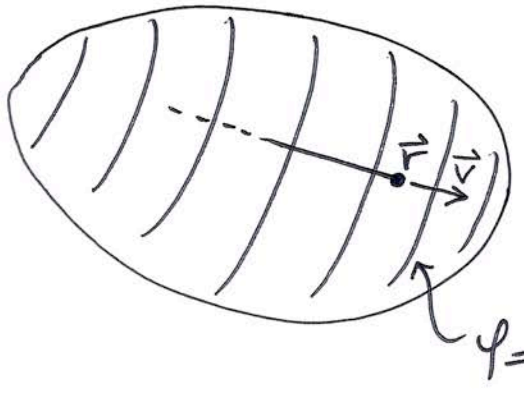
Particle at time t described by <u>both</u> $\psi(\vec{x},t)$ and $\underline{\underline{\vec{r}(t)}}$

- At $t=0$: $\vec{r}(0)$ distributed according to $|\psi(\vec{r}_0)|^2$
- At $t>0$: ψ evolves acc. to Schrödinger eq.

$\vec{r}(t)$ evolves via

$$\dot{\vec{r}}(t) = \frac{\hbar \vec{\nabla} \varphi(\vec{r}(t),t)}{m} \equiv \vec{v}(\vec{r}(t),t)$$

where $\psi(\vec{r}) = |\psi(\vec{r})| e^{i\varphi(\vec{r})}$



e.g. plane wave: $\varphi = \vec{k}\vec{r} \Rightarrow \dot{\vec{r}} = \frac{\hbar \vec{k}}{m}$ ✓

Let $\rho(\vec{r},t)$ = prob. density of Bohm's ensemble

Claim: If $\rho(\vec{r},0) = |\psi(\vec{r},0)|^2$

then $\rho(\vec{r},t) = |\psi(\vec{r},t)|^2 \;\; \forall t > 0$

Proof:

Check: $\partial_t \rho = -\operatorname{div} \vec{j}_{Bohm} \overset{?}{=} \partial_t |\psi|^2 = -\operatorname{div} \vec{j}_{QM}$

$$\vec{j}_{Bohm} = \rho \cdot \vec{v}_{Bohm} = \rho(\vec{r}) \cdot \frac{\hbar \vec{\nabla} \varphi(\vec{r})}{m}$$

$$\equiv |\psi(\vec{r},t)|^2 \frac{\hbar}{m} \vec{\nabla} \varphi(\vec{r},t)$$

if ~~correct~~ $\rho = |\psi|^2$ up to t

$$\equiv \mathrm{Re}\left[\psi^* \frac{-i\hbar\nabla}{m} \psi\right]$$

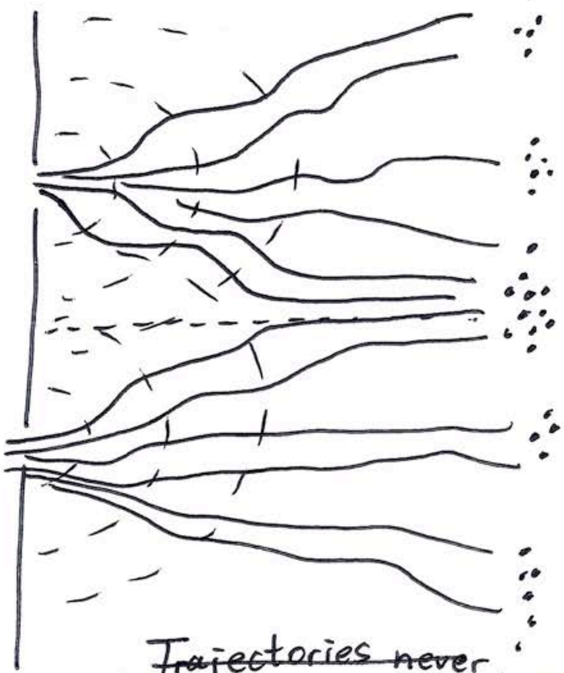
$$= \vec{j}_{QM}$$

$\Rightarrow$ ✓

$\Rightarrow$ If all msmts are ultimately reduced to position msmts, then this <u>exactly reproduces</u> all statistical predictions of standard QM!

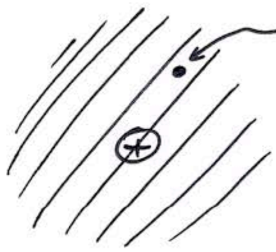
Example:

Double-slit



~~Trajectories never cross nodal ~~Line~~ surfaces $\psi = 0$~~

Hydrogen atom ground state



e^- just sits there!

$\vec{\nabla} \varphi = 0$ in any stationary state!
(for $\vec{B} = 0$)

Time evolution of velocity field

$$\vec{v} = \frac{\hbar \vec{\nabla}\varphi}{m} \cancel{= \frac{\hbar}{m} \frac{\mathrm{Im}\left[\psi^* \vec{\nabla}\psi\right]}{|\psi|^2}} \qquad (\psi = |\psi| e^{i\varphi})$$

$$\Rightarrow \quad \partial_t \vec{v} \overset{?}{=} \frac{\hbar}{m} \cancel{\frac{1}{|\psi|^2} \mathrm{Im}\left[(\partial_t \psi)^* \vec{\nabla}\psi + \psi^* \vec{\nabla}\partial_t \psi\right]}$$

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m}\Delta\psi + V\psi$$

$$\varphi = \mathrm{Im}\, \ln \psi$$

$$\Rightarrow \partial_t \varphi = \mathrm{Im}\, \frac{\partial_t \psi}{\psi}$$

$$= \mathrm{Im}\left\{\frac{1}{i\hbar}\left[-\frac{\hbar^2}{2m}\frac{\Delta\psi}{\psi} + V\right]\right\}$$

$$= \frac{\hbar}{2m} \mathrm{Re}\left(\frac{\Delta\psi}{\psi}\right) - \frac{1}{\hbar} V$$

$$\Delta\psi = \mathrm{div}\left[e^{i\varphi}\vec{\nabla}|\psi| + |\psi|\, i(\vec{\nabla}\varphi) e^{i\varphi}\right]$$

$$= e^{i\varphi}\Delta|\psi| + 2i(\vec{\nabla}\varphi)(\vec{\nabla}|\psi|)e^{i\varphi}$$

$$+ |\psi|\, i(\Delta\varphi)e^{i\varphi}$$

$$- |\psi| (\vec{\nabla}\varphi)^2 e^{i\varphi}$$

$$= \frac{\hbar}{2m}\frac{\Delta|\psi|}{|\psi|} - \frac{\hbar}{2m}(\vec{\nabla}\varphi)^2 - \frac{1}{\hbar} V$$

$$\left[\cancel{\vec{v}(\vec{\nabla}\vec{v}) =} \; (\vec{v}\vec{\nabla})\vec{v} = \left(\frac{\hbar}{m}\right)^2 \overbrace{[(\vec{\nabla}\varphi)\vec{\nabla}]\vec{\nabla}\varphi}^{(\partial_i\varphi)\partial_i\partial_j\varphi} \, \frac{1}{2}\partial_j(\partial_i\varphi)^2 \right]$$

π

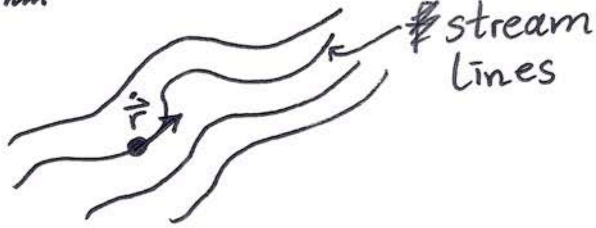
$$\Rightarrow \quad \partial_t \vec{v} = \frac{\hbar}{m}\vec{\nabla}\partial_t \varphi$$

$$= \underbrace{-\left(\frac{\hbar}{m}\right)^2 \frac{1}{2}\,\vec{\nabla}\,(\vec{\nabla}\varphi)^2}_{} - \frac{1}{m}\vec{\nabla}\Big[V \underbrace{- \frac{\hbar^2}{2m}\frac{\Delta|\psi|}{|\psi|}}_{V_{Qu.}}\Big]$$

$$= -(\vec{v}\vec{\nabla})\vec{v} + \frac{1}{m}\vec{F}_{Bohm}$$

"advective term", like in hydrodynamics!

$$\Rightarrow \quad \underbrace{\partial_t \vec{v} + (\vec{v}\vec{\nabla})\vec{v}}_{\vec{a} = \ddot{\vec{r}} = \frac{d}{dt}[\vec{v}(\vec{r}(t),t)]} = \frac{1}{m}\vec{F}_{Bohm}$$

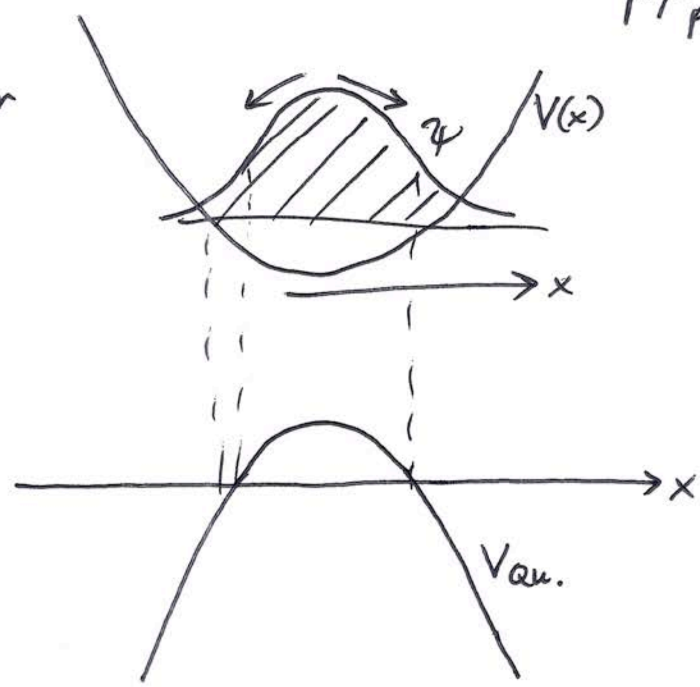


where $\vec{F}_{Bohm} = -\vec{\nabla} V_{Bohm}$

$$= -\vec{\nabla}(V + V_{Qu.})$$

V: physical pot.; $V_{Qu.}$: "quantum potential"

Example:
Harmonic oscillator



"Quantum pressure", ~~reflects~~ ~~yields~~ ~~zero point~~ finite ~~motion~~ ground state width (Heisenberg!) stabilizes ground state

$$\Rightarrow \quad V + V_{Qu.} = \text{const} \Rightarrow \vec{F}_{Bohm} = 0$$

$$\partial_t \vec{v} + (\vec{v}\vec{\nabla})\vec{v} = \frac{1}{m}\vec{F}_{Bohm} \quad \& \quad \partial_t \rho + \mathrm{div}(\rho\vec{v}) = 0$$

→ Madelung's "hydrodynamic formulation" of QM, equivalent to SEQ

$$\psi \in \mathbb{C} \leftrightarrow \rho, \vec{v}$$

Note: $\dot{\vec{r}} = \vec{v}$ is <u>not</u> connected to momentum distribution!
(e.g. $\vec{v} \equiv 0$ in stationary states, but $\langle \hat{p}^2 \rangle \neq 0$)

But at least:

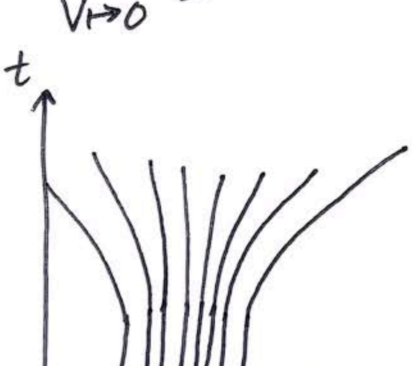
$$\underbrace{\langle \dot{\vec{r}} \rangle}_{\text{Bohm}} = \int \rho \vec{v}\, d^3\vec{r} = \int |\psi|^2 \frac{\hbar \vec{\nabla}\varphi}{m}\, d^3\vec{r}$$

$$= \int \vec{j}_{QM}\, d^3\vec{r} \underset{(!)}{=} \langle \psi | \frac{-i\hbar\vec{\nabla}}{m} | \psi \rangle$$

$$= \underbrace{\langle \frac{\hat{\vec{p}}}{m} \rangle}_{\text{QM}}$$

Measure mom. (veloc. distr.) by time-of-flight

⇒ correct predictions!



Many particles:

$$\psi = \psi(\vec{r}_1, \vec{r}_2, ..) = |\psi| e^{i\varphi}$$

$$\varphi = \varphi(\vec{r}_1, \vec{r}_2, \ldots)$$

$$\boxed{\dot{\vec{r}}_j(t) = \frac{\hbar}{m_j} \vec{\nabla}_j \varphi}$$

$\Rightarrow$ treat many-body problems like sys + msmt apparatus
configuration $(\vec{r}_1, \vec{r}_2, \ldots)$ follows one branch in Everett's picture

product state $\Rightarrow$ $\varphi = \varphi_1(\vec{r}_1) + \varphi_2(\vec{r}_2) + \ldots$

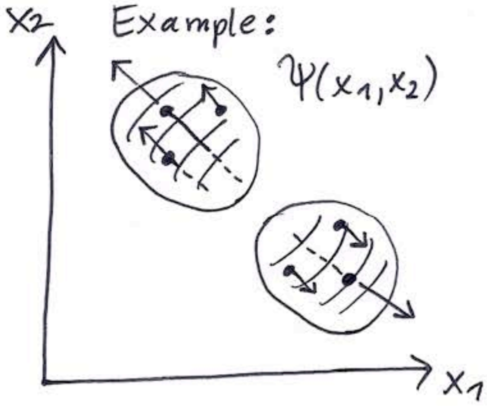
$\psi = \psi_1(\vec{r}_1) \cdot \psi_2(\vec{r}_2) \cdot \ldots$ $\Rightarrow$ $\dot{\vec{r}}_j = \frac{\hbar}{m_j} \vec{\nabla}_j \varphi_j(\vec{r}_j)$ depends only on $\vec{r}_j$ ✓

entangled state $\Rightarrow$ $\dot{\vec{r}}_j$ depends on other $\vec{r}_{l \neq j}$!

$\Rightarrow$ nonlocal theory ?

Not so quick!

Example:



Two stories

1. "$\dot{x}_2 > 0$ or $\dot{x}_2 < 0$ depending on x_1 $\Rightarrow$ nonlocal?"
2. Particle states are correlated: either ~~both~~ $\cancel{\dot{x}_1, \dot{x}_2}$
   $\dot{x}_1 > 0$, $\dot{x}_2 < 0$
   or $\dot{x}_1 < 0$, $\dot{x}_2 > 0$

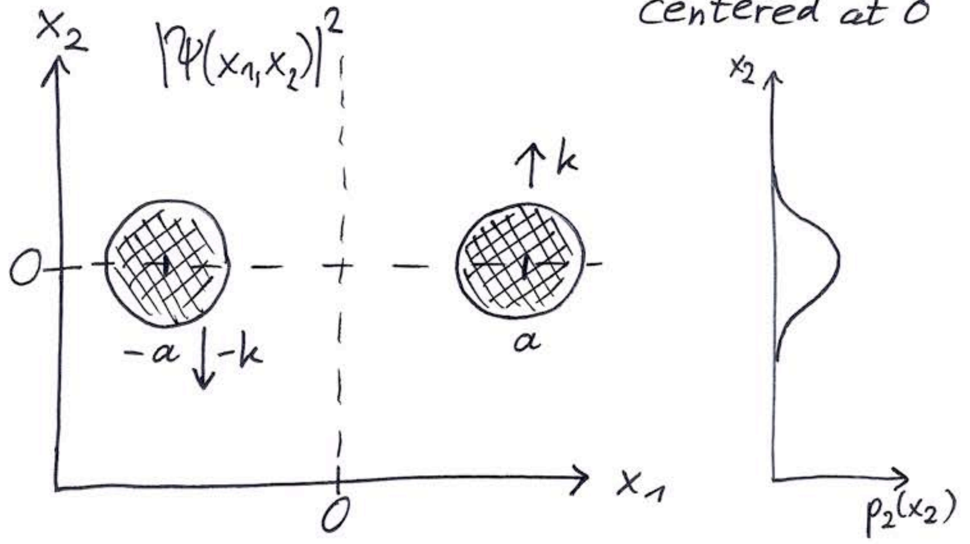
$\Rightarrow$ no mystery

Have to show: $\dot{x}_2$ can change depending on ~~what we~~ which forces we apply to x_1, in the absence of interactions (i.e. when 1 & 2 are far apart)

Two particles in state

$$\Psi(x_1, x_2) = \phi_1(x_1 - a)\phi_2(x_2)e^{ikx_2} + \phi_1(x_1+a)\phi_2(x_2)e^{-ikx_2}$$

wave packet, centered at 0



entangled!

Now: Apply potential $\bigcup$ to x_1

$\Rightarrow$ accelerate wave packets towards each other

$\Rightarrow$ interference

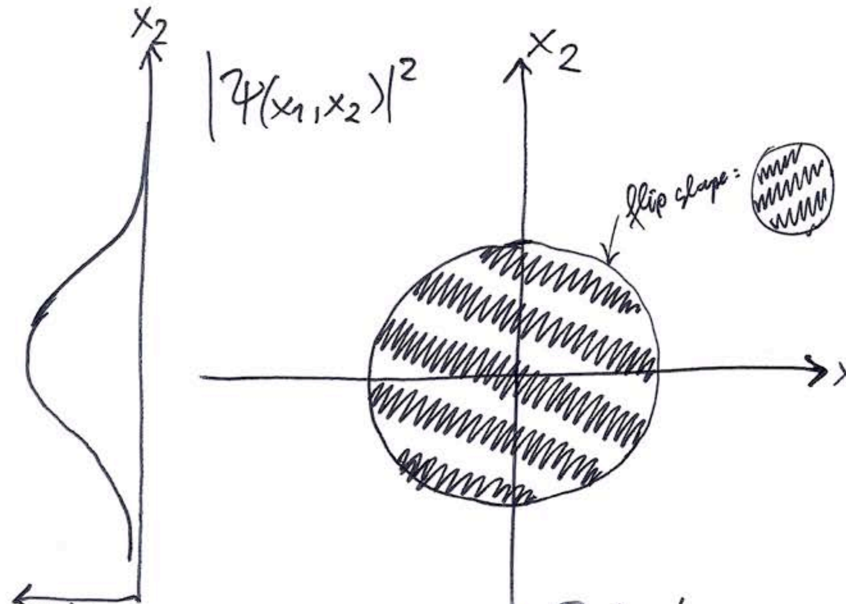
$$\Psi(x_1,x_2) \sim \phi_1(x_1)\phi_2(x_2)\underbrace{\left[e^{+i(\overset{-q}{\cancel{q}}x_1+kx_2)} + e^{-i(-qx_1+kx_2)}\right]}_{2\cos(-qx_1+kx_2)}$$

$\downarrow$

$\vec{\nabla}_1 \varphi = \vec{\nabla}_2 \varphi = 0$ at this point in time! $\Rightarrow \dot{x}_2 = 0$

in <u>contrast</u> to situation without the potential applied to x_1

$\Rightarrow$ trajectory $x_2(t)$ changes in response to action on x_1!



Bohm's theory is nonlocal! $\Leftarrow$

Note: $p_2(x_2) = \int |\Psi(x_1,x_2)|^2 \, dx_1$

does <u>not</u> ~~change~~ depend on force applied to 1 !

Proof: For $\hat{H} = \hat{H}_1 + \hat{H}_2$ (no interactions)

obs. of sys. 2: $\frac{d}{dt}\hat{A}_2 = \frac{1}{i\hbar}[\hat{A}_2, \hat{H}] = \frac{1}{i\hbar}[\hat{A}_2, \hat{H}_2]$

only depends on observ. of sys. 2

$\Rightarrow \hat{A}_2(t) =$ fct (sys 2 operators at $t=0$)

<u>independent</u> of $\hat{H}_1(t)$!

$\Rightarrow$ with $\hat{A}_2 \equiv \delta(\hat{x}_2 - x_2)$ have

$\langle \Psi(t) | \hat{A}_2 | \Psi(t) \rangle = p_2(x_2)$

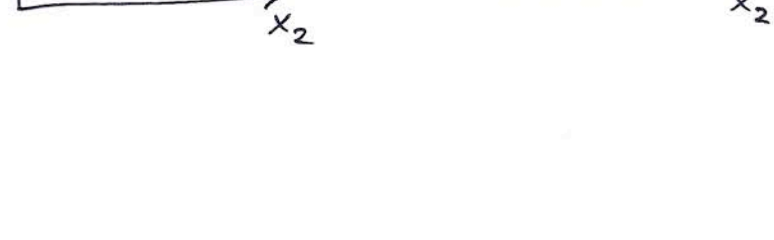
is independent of $\hat{H}_1(t)$!

$\Rightarrow$ By measuring x_2, <u>we cannot tell</u> whether force was applied to 1

$\Rightarrow$ <u>no</u> signalling faster than light!

(though each trajectory changes,)

but cannot observe traj's!



different trajectories, but the same $p_2(x_2)$!

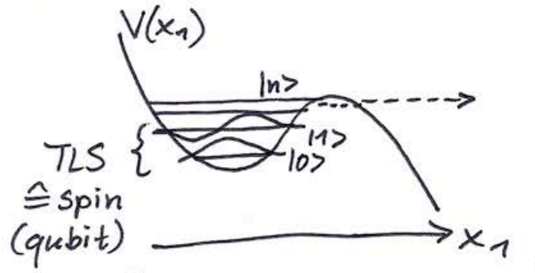
Bell: This must be the case,
~~…~~ since Bohm's theory reproduces QM predictions
$\Rightarrow$ cannot be <u>local</u> hidden var. theory, i.e.

correlations in outcomes <u>cannot</u> be explained by correlations in $\lambda = (x_1(0), x_2(0), \psi(x_1, x_2, 0))$
but "msmt setting at 1 influences 2"

Q: What about spin?

$\rightarrow$ $\exists$ extensions of Bohm's theory with spin
$\ast \rightarrow$ see 3b

$\rightarrow$ Can set up EPR/Bohm expt. with artificial two-level sys. based on continuous $X_{1,2}$:



msmt of "$\hat{\sigma}_z$":
drive $|1\rangle \rightarrow |n\rangle$ transition $\Rightarrow$ tunneling decay <u>only</u> if we were in $|1\rangle$

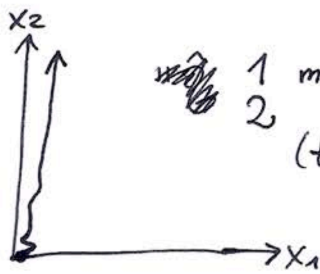
["Josephson phase qubit"
x_1 = superconducting phase difference]

Spin rotations:
- change energy levels $\Rightarrow$ rot. around $\hat{\sigma}_z$
- drive Rabi oscillations $\Rightarrow$ rot. around $\hat{\sigma}_x, \hat{\sigma}_y$

Consider $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|1\rangle_1 |0\rangle_2 - |0\rangle_1 |1\rangle_2\right]$

$\hat{=}$ singlet state

rotate spins & measure $\hat{\sigma}_{z1}$ & $\hat{\sigma}_{z2}$



1 measured in $|0\rangle$
2 " in $|1\rangle$
(tunneling decay!)

Bohm with spin:

(e.g. spin ½) $\psi_\sigma(\vec{r},t)$

$$\sigma = \pm 1$$

$$\Rightarrow \quad \vec{v}(\vec{r},t) \equiv \frac{\vec{j}(\vec{r},t)}{\rho(\vec{r},t)}$$

where

$$\vec{j}(\vec{r},t) \equiv \sum_{\sigma=\pm 1} \mathrm{Re}\left[\psi_\sigma^* \frac{-i\hbar\vec{\nabla}}{m} \psi_\sigma\right]$$

$$\rho(\vec{r},t) \equiv \sum_{\sigma=\pm 1} |\psi_\sigma(\vec{r},t)|^2$$

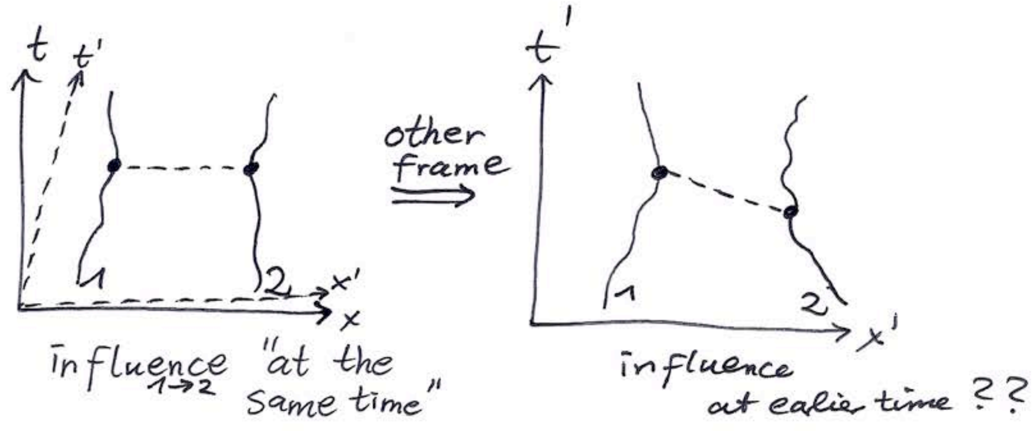
[more particles: $\sum$ over all σ in $\psi_{\sigma_1\sigma_2\ldots\sigma_N}(\vec{r}_1,\vec{r}_2,\ldots)$]

$\Rightarrow$ Spin msmt / Stern Gerlach / Bell

Note: Bohm's theory not relativistic

Extensions are being explored (easy to use Dirac eq., for example)

But:



Note: Bohm's theory not unique,

every $\vec{v}' = \dfrac{\vec{j}\,'}{\varrho}$ with $\operatorname{div}(\vec{j}\,' - \vec{j}_{QM}) = 0$

will do!

(i.e. reproduce correct $\varrho = |\psi|^2$)

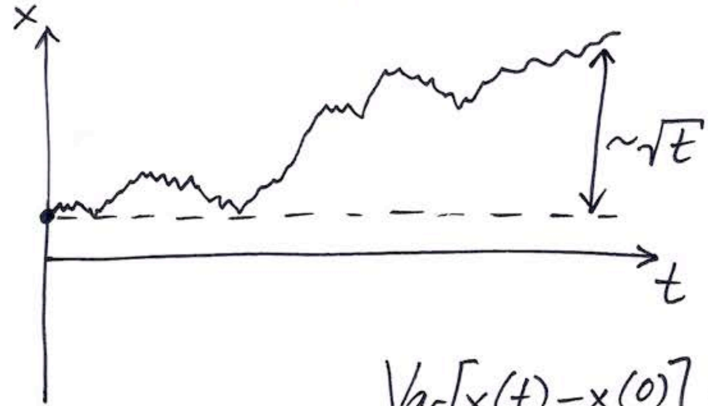
5.3 Nelson's Stochastic Quantization

Brownian motion:

$$x(t+\Delta t) = x(t) + \overset{\text{random variable, independent for each step}}{\widetilde{\Delta x(t)}}$$

$\langle \Delta x \rangle = 0$, and $\mathrm{Var}\,\Delta x = 2\underset{\text{diffusion constant}}{D}\,\Delta t$

$\Delta t \to 0$



$$\mathrm{Var}[x(t)-x(0)] = 2Dt$$

$$\overset{!}{=} \Big\langle \Big(\sum_{j=1}^{t/\Delta t} \Delta x_j\Big)^2 \Big\rangle$$

$$\overset{!}{=} \sum \langle \Delta x_j^2 \rangle = \frac{t}{\Delta t}\cdot 2D\Delta t = 2Dt$$

"Instantaneous velocity"?

$$\frac{x(t+\Delta t)-x(t)}{\Delta t} = \frac{\Delta x}{\Delta t} = \tilde{v}$$

$$\Rightarrow \mathrm{Var}\,\tilde{v} = \frac{\mathrm{Var}\,\Delta x}{\Delta t^2} = \frac{2D}{\Delta t} \to \infty \text{ for } \Delta t \to 0 \ ↯$$

white noise: $\langle \tilde{v}(t)\tilde{v}(0) \rangle = 2D\,\delta(t)$

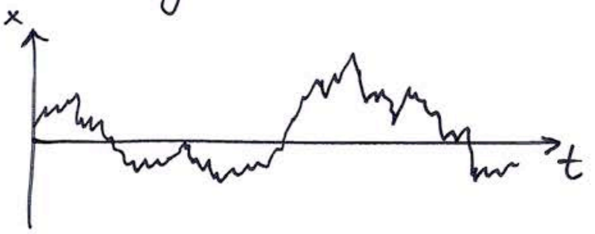
Drift-diffusion process:

$$x(t+\Delta t) = x(t) + \Delta x(t) + \Delta t \cdot \underbrace{v(t)}_{\text{drift velocity}}$$

$$\frac{dx}{dt} = \underbrace{\tilde{v}(t)}_{\text{noise}} + \underbrace{v(t)}_{\text{drift}}$$

e.g. $v(t) = -\gamma\, x(t) = v(x(t))$ "drift field"

$\Rightarrow$



$\updownarrow \; \mathrm{Var}\, x = \frac{D}{\gamma}$

"Ornstein-Uhlenbeck process"

(e.g. Brownian motion in harmonic potential; particle overdamped, e.g. in liquid)

$$\Big[\quad x(t) = e^{-\gamma t} x(0) + \int_0^t e^{-\gamma(t-t')}\, \tilde{v}(t')\, dt'$$

$$\Rightarrow \langle x(t)^2\rangle \underset{\substack{\uparrow \\ \gamma t \to \infty}}{=} \left\langle \left(\int_0^t e^{-\gamma(t-t')}\, \tilde{v}(t')\, dt' \right)^2 \right\rangle$$

(leave away $x(0)$-term etc.)

$$= \int_0^t e^{-\gamma(t-t_1') - \gamma(t-t_2')} \overbrace{\langle \tilde{v}(t_1')\, \tilde{v}(t_2') \rangle}^{2D\,\delta(t_1'-t_2')} dt_1'\, dt_2'$$

$$= \int_0^t e^{-2\gamma(t-t')}\, 2D\, dt' \underset{\substack{\uparrow \\ \gamma t \to \infty}}{=} \frac{D}{\gamma} \quad \Big]$$

Probability density $\rho(x,t)$:

$$\partial_t \rho = -\partial_x j_{Drift} + D\partial_x^2 \rho$$
$$= \underbrace{-\partial_x(\rho(x)v(x))}_{\text{drift}} + \underbrace{D\partial_x^2\rho}_{\text{diffusion}}$$

$\Rightarrow$ stationary case: $\partial_t \rho = 0$

$\Leftrightarrow$

$$D\partial_x\rho - \rho v(x) = \text{const} = 0$$

only normalizable solution!

$$\frac{\partial_x \rho}{\rho} = \frac{v(x)}{D}$$

$$\partial_x \ln\rho = \frac{v(x)}{D}$$

$\Rightarrow$ if $v(x) = -\mu U'(x)$ (mobility, potential)

then

$$\partial_x \ln\rho = -\frac{\mu}{D}\partial_x U(x)$$

$$\Rightarrow \quad \rho(x) = \mathcal{N}\cdot \exp\left[-\frac{\mu}{D}U(x)\right]$$

[e.g. in classical statistical physics:

$D = \mu kT$ "Einstein relation"

$\Rightarrow$ Boltzmann distribution!]

Conditional expectation values: (statistical, not related to QM!)

$$E(X \,|\, Y=y_0) \equiv \frac{\langle X\,\delta(Y-y_0)\rangle}{\langle \delta(Y-y_0)\rangle}$$

normalization

Example:

uncorrelated, by assumption

$$E(\Delta x(t) \,|\, x(t)=x_t) = \frac{\langle \Delta x(t)\,\delta(x(t)-x_t)\rangle}{\langle \delta(x(t)-x_t)\rangle}$$

$$= \langle \Delta x(t)\rangle \underbrace{\frac{\langle \delta(x(t)-x_t)\rangle}{\langle \quad '' \quad \rangle}}_{1}$$

$$= 0$$

But what is:

$$E(\Delta x(t) \,|\, x(t+\Delta t)=x_+) = \frac{\langle \Delta x(t)\,\delta(x(t+\Delta t)-x_+)\rangle}{\langle \delta(x(t+\Delta t)-x_+)\rangle}$$

$$x(t+\Delta t) = x(t) + \underbrace{\Delta x(t)}_{\mathcal{O}(\sqrt{\Delta t})} + \underbrace{v(x(t))\Delta t}_{\mathcal{O}(\Delta t)}$$

$$\Rightarrow \delta(x(t+\Delta t)-x_+) \approx \delta(x(t)+\Delta x(t)-x_+)$$
$$\approx \delta(x(t)-x_+) + \Delta x(t)\,\delta'(x(t)-x_+) + \ldots$$

$$\Rightarrow \langle \Delta x(t)\,\delta(..)\rangle = \underbrace{\langle \Delta x(t)\rangle}_{0}\langle \delta(x(t)-x_+)\rangle$$
$$+ \underbrace{\langle \Delta x(t)^2\rangle}_{2D\Delta t}\ \underbrace{\langle \delta'(x(t)-x_+)\rangle}$$
$$= \int \delta'(x(t)-x_+)\,\rho(x(t))\,dx(t)$$
$$\underset{\text{partial integr.}}{=} -\rho'(x_+)$$

$$\Rightarrow E(\Delta x(t) \,|\, x(t+\Delta t)=x_+) = -2D\Delta t\,\frac{\rho'(x_+)}{\rho(x_+)}$$

$\Rightarrow$

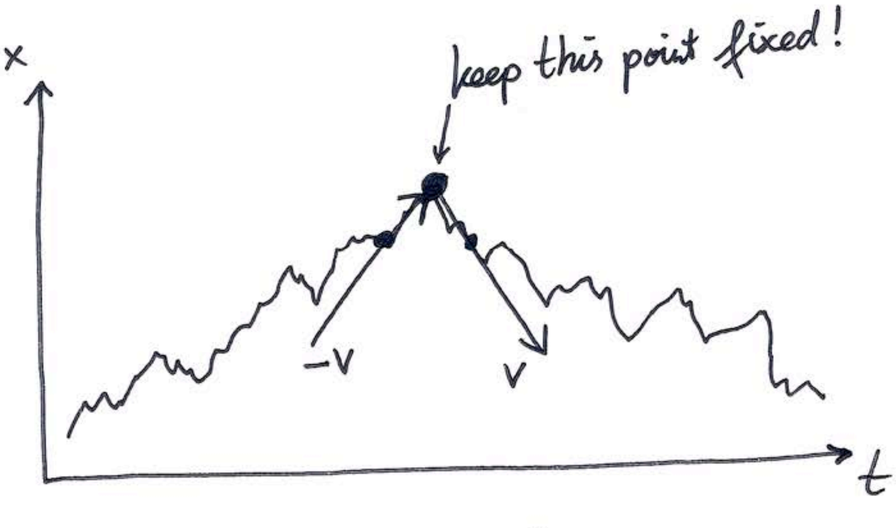
$$E\left(\frac{x(t+\Delta t)-\overset{\text{fixed}}{\overset{\downarrow}{x(t)}}}{\Delta t}\,\middle|\, x(t)=x_t\right) = \ldots = v(x_t)$$

but

$$E\left(\frac{\overset{\text{fixed}}{\overset{\downarrow}{x(t+\Delta t)}}-x(t)}{\Delta t}\,\middle|\, x(t+\Delta t)=x_+\right)$$

$$= E\left(\frac{\Delta x}{\Delta t} + \underbrace{v(x(t))}_{\underset{(!)}{\approx} v(x(t+\Delta t))}\,\middle|\, x(t+\Delta t)=x_+\right)$$

$$\underset{\substack{\text{see}\\\text{above!}}}{=} -2D\underbrace{\frac{\rho'(x_+)}{\rho(x_+)}}_{\frac{1}{D}v(x_+)} + v(x_+) = \underline{\underline{-v(x_+)}}$$



$\Rightarrow$ Two kinds of "derivatives":

$$\frac{d}{dt_{\rightarrow}} f(x(t),t) \equiv E\left(\frac{f(x(t+\Delta t),t+\Delta t)-f(x(t),t)}{\Delta t}\,\middle|\, x(t)\text{ fixed}\right)$$

$$\cancel{\frac{d}{dt_{\leftarrow}} f(x(t),t) \equiv E\left(\ldots \,\middle|\, x(t+\Delta t)\text{ fixed}\right)}$$

$$\frac{d}{dt_{\leftarrow}} f(x(t),t) \equiv E\left(\frac{f(x(t),t)-f(x(t-\Delta t),t-\Delta t)}{\Delta t}\,\middle|\, x(t)\text{ fixed}\right)$$

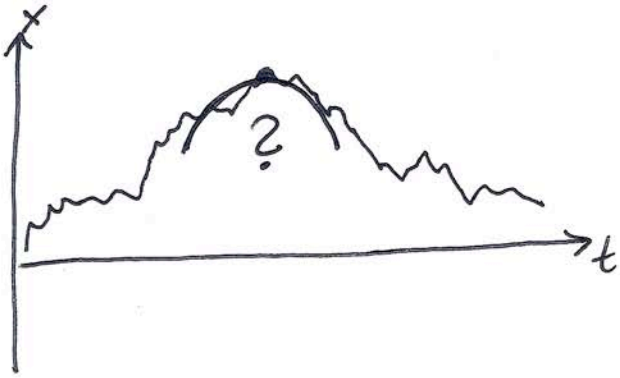
result (for stationary process)

$$\frac{d}{dt_{\rightarrow}} f(x(t),t) = [\partial_t + v(x)\partial_x + D\partial_x^2]\, f(x(t),t)$$

$$\frac{d}{dt_{\leftarrow}} f(x(t),t) = [\partial_t - v(x)\partial_x - D\partial_x^2]\, f(x(t),t)$$

(results are again random processes, since they depend on $x(t)$!)

"Acceleration" of a drift-diffusion process:



Define (symmetric choice) $\quad a(t) = \frac{1}{2}\left[\frac{d}{dt_{\rightarrow}}\frac{d}{dt_{\leftarrow}} + \frac{d}{dt_{\leftarrow}}\frac{d}{dt_{\rightarrow}}\right] x(t)$

$(= a(x(t)))$

~~e.g. Ornstein-Uhl~~

$\Rightarrow \ldots \Rightarrow$ $\boxed{a(x) = -(v(x)\partial_x)v(x) - D\partial_x^2 v(x)}$
(for stationary process)

e.g. for Ornstein-Uhlenbeck: $v(x) = -\gamma x$

$\Rightarrow$ $a = -\gamma^2 x$

[Note: asymmetric def. $a = \frac{d^2}{dt_{\rightarrow}^2} x(t)$ would give opposite sign !]

Now: Find process with prescribed acceleration

$$a = a(x) \stackrel{!}{=} -\frac{1}{m}U'(x)$$

à la Newton $(ma = F = -U'(x))$

$$\Leftrightarrow \quad \frac{1}{m}U'(x) = (v\partial_x)v + D\,\partial_x^2 v$$

~~Use ...~~

$$(v\partial_x)v = \frac{1}{2}\partial_x v^2$$

$$\Rightarrow \quad -\frac{m}{2}v^2 - mD\,\partial_x v + U = \text{const}$$

Use $v(x) = D\,\partial_x \ln \rho$

& define $\rho(x) =: \psi^2(x)$

$$\Rightarrow \quad v(x) = 2D\,\partial_x \ln\psi = 2D\,\frac{\partial_x\psi}{\psi}$$

$$\Leftrightarrow \quad -2mD^2\left(\frac{\partial_x\psi}{\psi}\right)^2 - 2mD^2\underbrace{\partial_x\frac{\partial_x\psi}{\psi}}_{\frac{\partial_x^2\psi}{\psi} - \frac{(\partial_x\psi)^2}{\psi^2}} + U = \text{const}$$

$$\Leftrightarrow \quad -2mD^2\partial_x^2\psi + U\psi = \text{const}\cdot\psi$$

Set $D = \frac{\hbar}{2m}$

$\Rightarrow$ stationary Schrödinger equation (with const $= E$)!

⇒ SEQ appears automatically!

Summary:

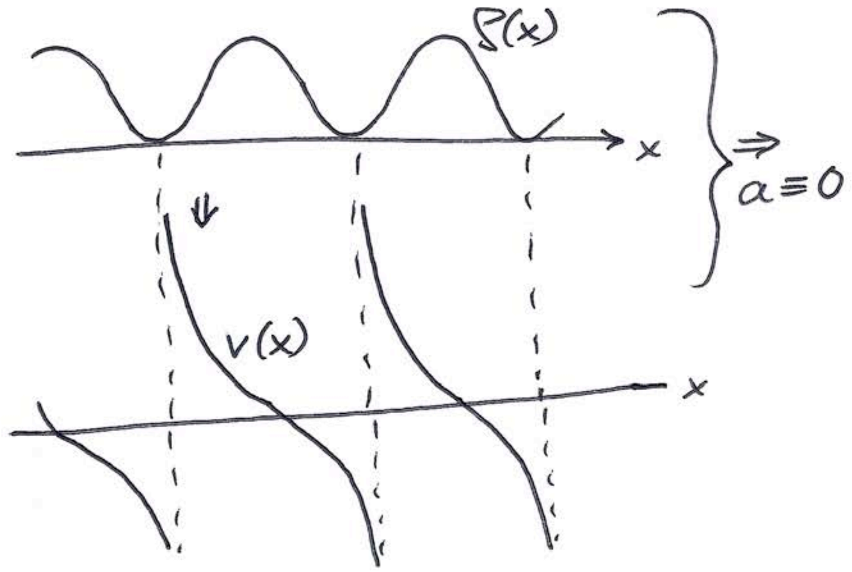
Find processes with
$a(x) = -\frac{1}{m} U'(x)$
and $D = \frac{\hbar}{2m}$!

⟹

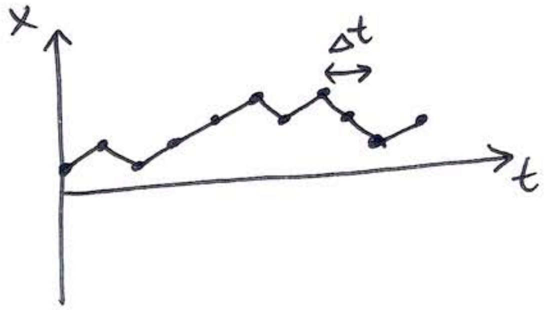
Solve SEQ for $U(x)$
~~Any~~ All processes with
$\rho_n(x) \simeq \psi_n^2(x) \;\; \forall n$
energy eigenfunction
are solutions!

(Nelson, 1966)

e.g. $U(x) \equiv 0$: All standing waves



Note: $D = \frac{\hbar}{2m}$
results from steps $\pm c\Delta t$ with
$\Delta t = \frac{\hbar}{mc^2} \sim$ "Compton time" $\approx 10^{-21}$ s for e^-



Time-dependent case: more freedom for eq. of motion

$$\dot{x} = \tilde{v} + D\frac{\partial_x \rho}{\rho} + \overbrace{\frac{\hbar}{m}\partial_x \varphi}^{\text{extra term}}$$

extra term, assumed to be a gradient

$$\& \quad a \stackrel{!}{=} -\frac{1}{m}U'(x)$$

$$\Rightarrow \quad \psi = \sqrt{\rho}\, e^{i\varphi}$$

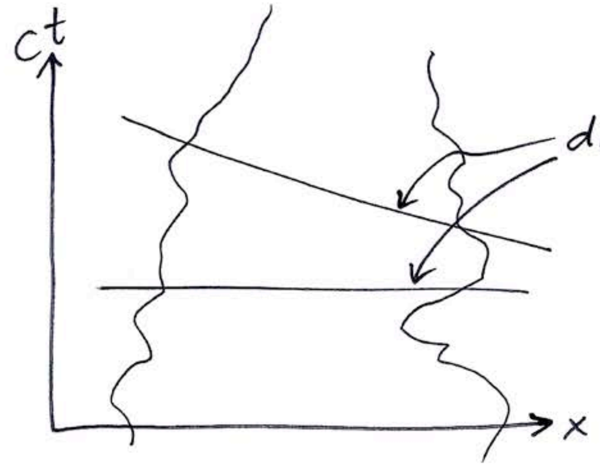
fulfills time-dependent SEQ!

$\Rightarrow$ $\dot{x}$-equation looks like "Bohm + drift-diffusion"

[→ movie]

5.4 Hidden-variable theories: Can they be Lorentz-invariant?

Question: Can we construct __any__ (nonlocal) HV-theory that reproduces QM probabilities in all reference frames?
[i.e. for msmts at different times & places]



different spacelike hypersurfaces
⇒ reproduce $|\psi|^2$ on all of these?

Works for single particle [e.g. Dirac-Bohm], but beyond that?

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Hardy's experiment 1992

(à la Berndl, Dürr, Goldstein, Zanghì) 1996

Consider

particle A

$$|\Psi_{Hardy}\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow\rangle|\downarrow\rangle - \sqrt{2}\,|\leftarrow\rangle|\uparrow\rangle\right)$$

particle B

$|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$

$$= \frac{1}{\sqrt{3}}\left(|\downarrow\rangle|\uparrow\rangle - \sqrt{2}\,|\uparrow\rangle|\leftarrow\rangle\right)$$

$$= \frac{1}{\sqrt{3}}\left(|\uparrow\rangle|\downarrow\rangle - |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{12}}\left(|\rightarrow\rangle|\rightarrow\rangle - |\rightarrow\rangle|\leftarrow\rangle - |\leftarrow\rangle|\rightarrow\rangle - 3|\leftarrow\rangle|\leftarrow\rangle\right)$$

$\Rightarrow$ Conclusions:

$$A_x = +1 \Rightarrow B_z = -1$$

(since $\langle\Psi_{Hardy}|\underset{A}{\rightarrow};\underset{B}{\uparrow}\rangle = 0$)

$$B_x = +1 \Rightarrow A_z = -1$$

$(A_z = -1$ <u>and</u> $B_z = -1)$ never happens

$$P(A_x = +1 \text{ and } B_x = +1) = \frac{1}{12}$$

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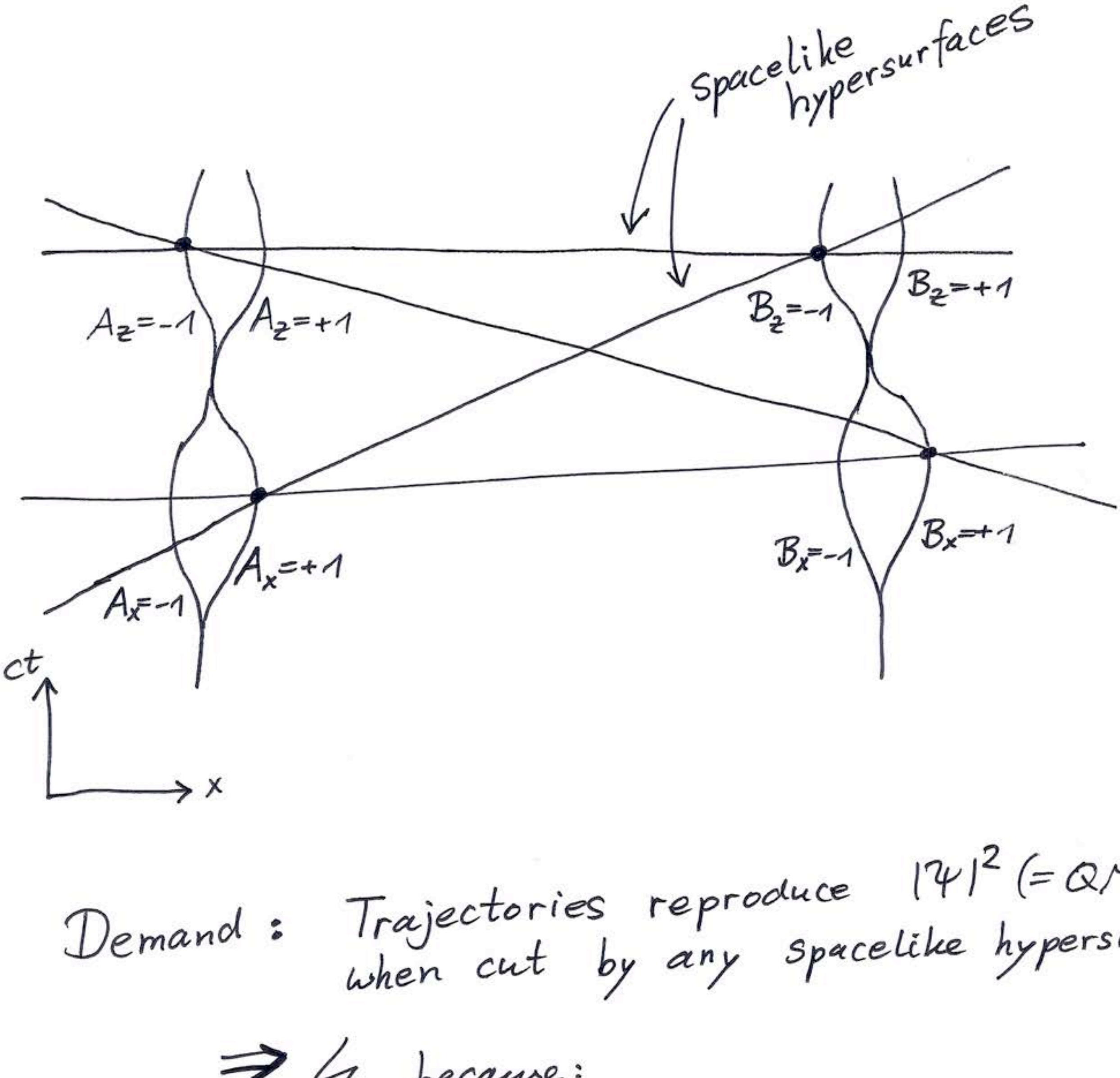
For A, B: ~~Stern-Gerlach~~ "quantum eraser" setup (x-dir.) & then another St. Gel. (z-dir.)



Demand: Trajectories reproduce $|\Psi|^2$ (= QM pred.) when cut by any spacelike hypersurface

$\Rightarrow$ ↯ because:

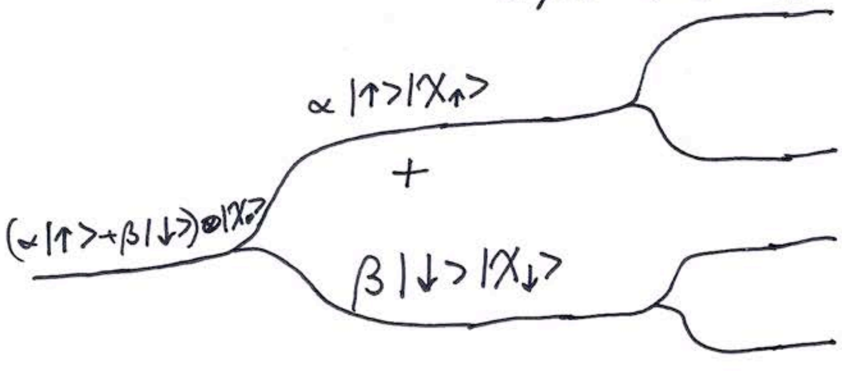
$$A_x=+1 \wedge B_x=+1 \quad \text{(with prob } \tfrac{1}{12}\text{)}$$

$$\Rightarrow \cancel{A_z}\, B_z=-1 \wedge \cancel{B_z}\, A_z=-1 \quad ↯$$

5.5 "Many worlds"

(Everett, DeWitt, ...)

Take QM serious for everything!
($sys_1 + sys_2 + app.1 + app.2 + \ldots$)



"branching"
→ in each branch: may assume ψ to be collapsed (since branches won't interfere again)

Initial problem: How to choose basis?
$[\alpha|\phi\rangle + \beta|\psi\rangle = \alpha'|\phi'\rangle + \beta'|\psi'\rangle \ldots]$

Today: Pick macroscopically distinct, irreversibly separate states → theory of decoherence
(& classical questions of irreversibility & arrow of time)

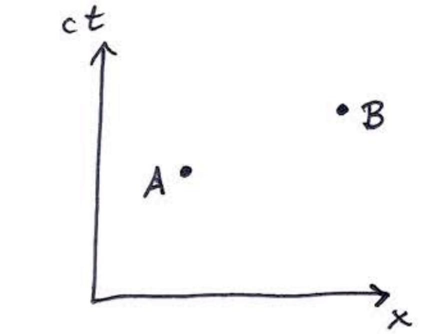
"Is this superposition really there?"
"Yes, if the SEQ is correct"

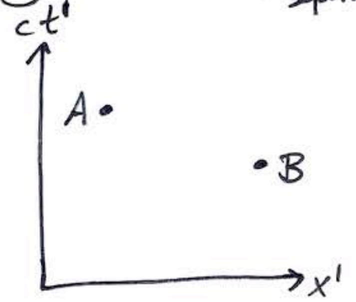
Problem: "Weight" of branches?
→ Postulate (derive?) Born rule, i.e. "$|\psi|^2$"
→ self-consistent, i.e. ~~repea~~ msmts on a string of identically prepared systems yield correct frequencies with near certainty

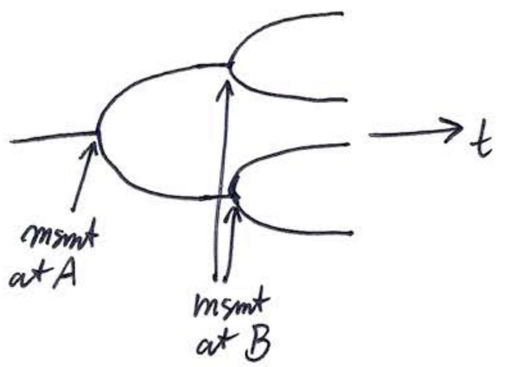
Relativistic description?

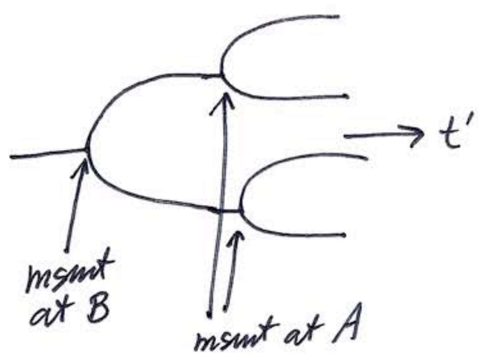
[Note: $i\hbar\partial_t|\Psi\rangle = H|\Psi\rangle$ ~~may~~ does evolve according to relativistic quantum field theory, but that does not guarantee our description is relativ. invariant]

Problem: Time-ordering of branching events! (which are spacelike separated)

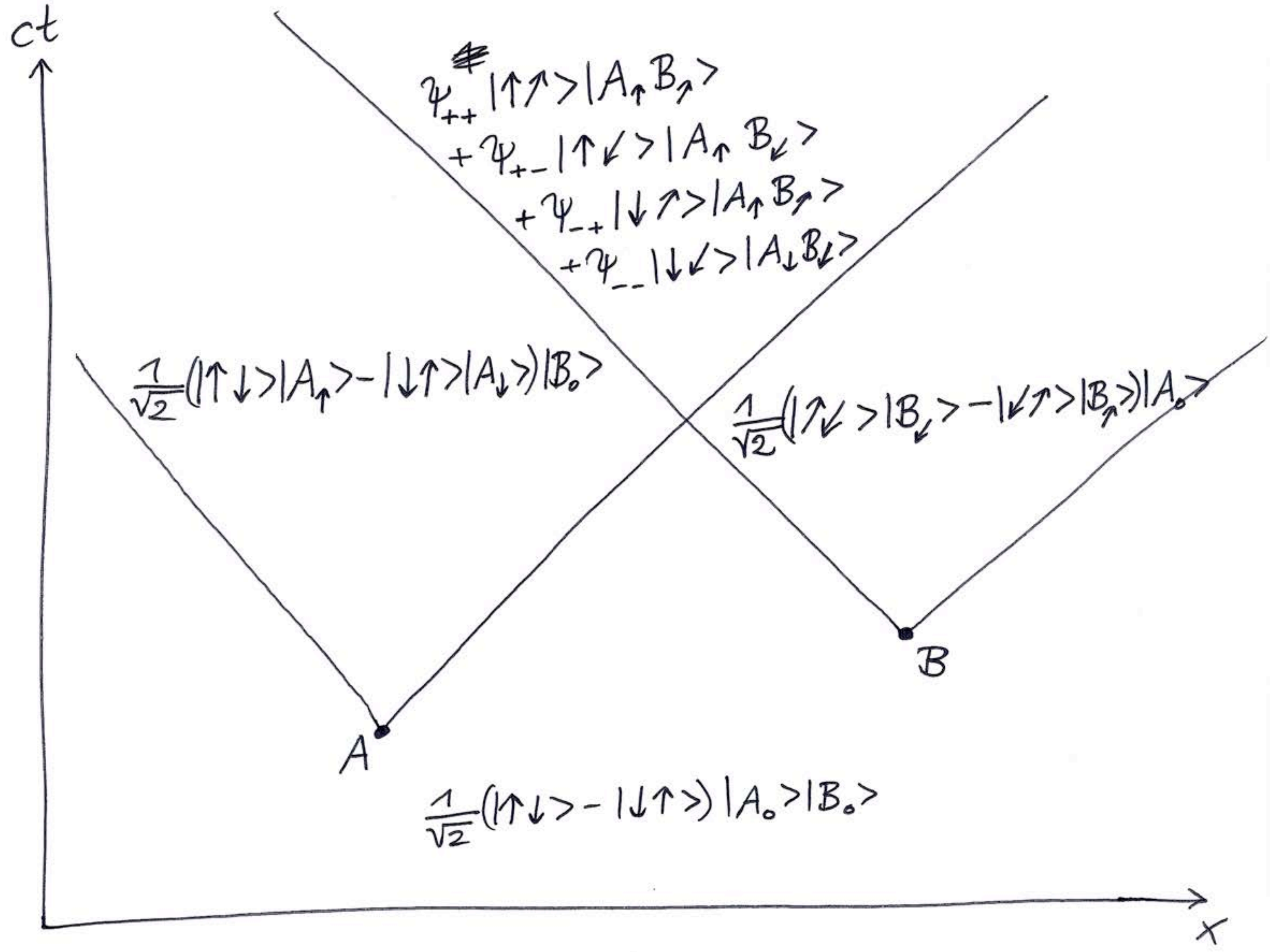








Bell-experiment, in
a relativistic many-worlds picture
(à la S. Saunders ?)



let $|\nearrow\rangle, |\swarrow\rangle$ be some other
basis in the spin-$\frac{1}{2}$ Hilbert space
(msmt basis for B)

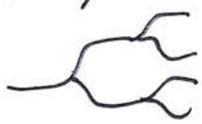
5.6 Consistent histories

(Griffiths '84, Omnès, Gell-Mann, Hartle)

Goal: Assign probabilities to statements like
"The spin pointed along z at time 0 and along x at time t"

Challenge: Incompatible choices of basis give different stories
[remember Kochen-Specker!]

Generalization of branching in many-world's picture of msmt



(now: also interference

can be treated)

Need to:
Avoid problems with description at different times, identified by Leggett-Garg or for Bohm & Nelson hidden variable theories!

Example of a "history":

$$Y = \cancel{F_0 \otimes} F_1 \otimes \ldots \otimes F_n \in \mathcal{H}^n$$

"event" at time t_1 = projector, e.g. $|\phi\rangle\langle\phi|$ or $|\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2|$

event at time t_n

Hilbert space of qu. system (for each time "slice": use one copy)

assume: $t_1 < t_2 < \ldots < t_n$

Calculate probability by defining "chain operator"

$$K(Y) \equiv F_n U(t_n, t_{n-1}) \dots U(t_3, t_2) F_2 U(t_2, t_1) F_1$$

time-evolution operator

then:

$$P(Y) \equiv tr\left(K^+(Y) K(Y)\right)$$

⇒ e.g. for two times:

Born rule

$$Y = |\phi_1\rangle\langle\phi_1| \otimes |\phi_2\rangle\langle\phi_2|$$

$$P(Y) = \dots = |\langle\phi_2|\hat{U}(t_2, t_1)|\phi_1\rangle|^2$$

initial state

~~"Family of histories"~~

$$\cancel{\mathbb{1} = \sum_\alpha Y^\alpha}$$

~~would like to have~~ $\cancel{P(\text{"}Y^2 \text{ or } Y^5\text{"}) = P(Y^2) + P(Y^5)}$

$$\cancel{P\Big(\sum_\alpha \underbrace{\lambda_\alpha}_{=0 \text{ or } 1} Y^\alpha\Big) = \sum_\alpha \lambda_\alpha P(Y^\alpha)}$$

Sample space of histories:

$$Y^\alpha \text{ with}$$

$$Y^\alpha Y^\beta = \delta_{\alpha,\beta}$$

$$\text{and } \mathbb{1} = \sum_\alpha Y^\alpha$$

"Family of histories": all

$$Y = \sum_\alpha \underbrace{\lambda_\alpha}_{0 \text{ or } 1} Y^\alpha$$

(like: "Y^2 or Y^5 or Y^7")

We would like to have

$$P\left(\sum_\alpha \lambda_\alpha Y^\alpha\right) = \sum_\alpha \lambda_\alpha P(Y^\alpha)$$

IF this works, then this family is called consistent!

Sufficient condition:

$$\mathrm{tr}\left(K^+(Y^\alpha)K(Y^\beta)\right) = 0 \text{ for } \alpha \neq \beta$$

Note: Consistency depends on unitary evolution!

Example: Mach-Zehnder interferometer

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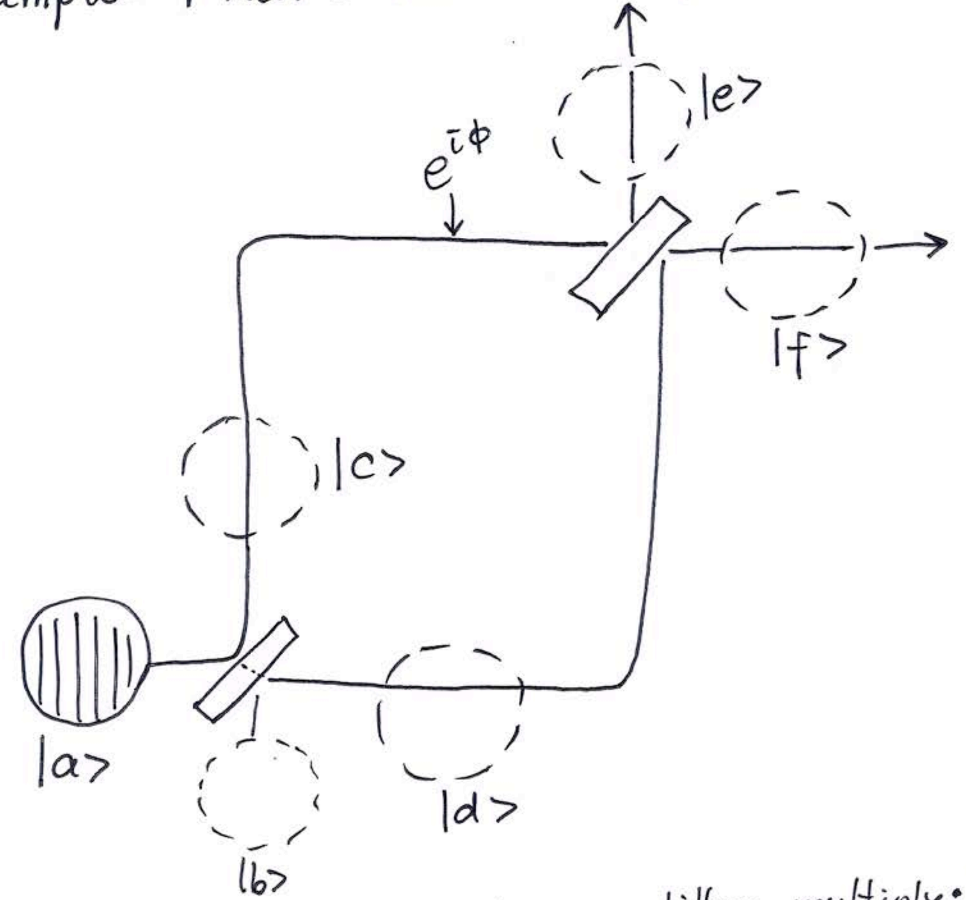

unitary evolution : at beam splitter multiply: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix} = \begin{pmatrix} \psi_c \\ \psi_d \end{pmatrix}$

$$\underset{\text{at } t_1}{|a\rangle} \xmapsto{\hat{U}} \underset{\text{at } t_2}{\frac{|c\rangle + |d\rangle}{\sqrt{2}}} \xmapsto{\hat{U}} \underset{\text{at } t_3}{\frac{e^{i\phi}-1}{2}|e\rangle + \frac{e^{i\phi}+1}{2}|f\rangle}$$

example of consistent family of histories:

$|a\rangle\langle a|_1 \otimes |e\rangle\langle e|_3$

$|a\rangle\langle a|_1 \otimes |f\rangle\langle f|_3$ (subscript $\sim$ time)

(& ~~others with $P=0$~~)

$(\mathbb{1} - |a\rangle\langle a|) \otimes \mathbb{1}$ with $P=0$

Inconsistent family:

$Y^{ce} = |a\rangle\langle a|_1 \otimes |c\rangle\langle c|_2 \otimes |e\rangle\langle e|_3$

$Y^{de} = |a\rangle\langle a|_1 \otimes |d\rangle\langle d|_2 \otimes |e\rangle\langle e|_3$

$\vdots$

inconsistent, since $tr(K^+(Y^{ce}) K(Y^{de})) \neq 0$

Note: Bohm/Nelson trajectories would be of this type

Consistent family:

$$|a\rangle\langle a|_1 \otimes \overbrace{|\psi_+\rangle\langle\psi_+|_2}^{\text{superposition in interferometer}} \otimes |e\rangle\langle e|_3$$

$$|a\rangle\langle a|_1 \otimes |\psi_+\rangle\langle\psi_+|_2 \otimes |f\rangle\langle f|_3$$

$$\vdots$$

where
$$|\psi_+\rangle = \frac{|c\rangle + |d\rangle}{\sqrt{2}}$$

Other example:

$$|a\rangle\langle a|_1 \otimes |c\rangle\langle c|_2 \otimes \overbrace{|\phi_c\rangle\langle\phi_c|_3}^{\text{superposition outside of interferom.}}$$

$$|a\rangle\langle a|_1 \otimes |d\rangle\langle d|_2 \otimes |\phi_d\rangle\langle\phi_d|_3$$

$$\vdots$$

with
$$|\phi_c\rangle = e^{i\phi}\,\frac{1}{\sqrt{2}}\left(|e\rangle + |f\rangle\right)$$
$$|\phi_d\rangle = \frac{1}{\sqrt{2}}\left(-|e\rangle + |f\rangle\right)$$

Chapter 6. Extensions of Quantum Mechanics

6.1 "Spontaneous localization"

Ghirardi - Rimini - Weber 1986

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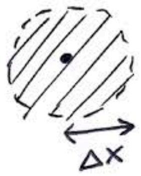
Hypothesis: Fundamental extra decoherence, always present,

weak enough $\rightarrow$ microscopic coherence & macroscopic classical dynamics (almost) unchanged

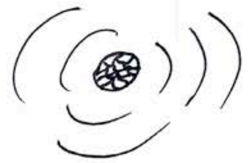
strong enough $\rightarrow$ ~~to~~ destroy coherence of "macroscopic superpositions"

Localize to precision Δx, at rate Γ



Example (see Schr.-cat discussion) of a real decoherence process of this kind:

thermal emission from molecules

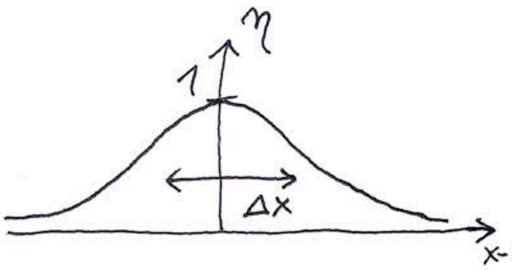


$\Delta x \sim \lambda_{\text{thermal radiation}}$

Remember: for one emission

$$\rho^{new}(x,x') = \langle e^{ikx} \rho(x,x') e^{-ikx'} \rangle_k$$

$$= \langle e^{ik(x-x')} \rangle_k \rho(x,x')$$



$$= \eta(x-x')\, \rho(x,x')$$

$\eta \rightarrow = 1$ for $x = x'$

$\rightarrow 0$ for $|x-x'| \rightarrow \infty$

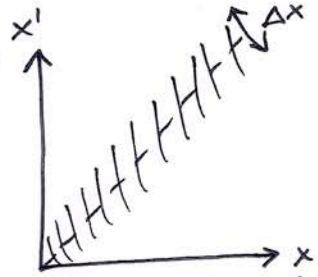
GRW assume simple model:

$$\eta(x-x') = \exp\left[-\frac{(x-x')^2}{4\Delta x^2}\right]$$

$\Rightarrow$ Two parameters in total: $\Gamma, \Delta x$

master equation:

$$\frac{d\rho(x,x')}{dt} = \left(\frac{1}{i\hbar}[\hat H, \hat\rho]\right)_{(x,x')} - \Gamma(1-\eta(x-x'))\rho(x,x')$$



limit $\Gamma \to \infty$, $\frac{\Gamma}{\Delta x^2} = \text{const} \Rightarrow \frac{d\hat\rho}{dt} = \ldots - \frac{\Gamma}{4\Delta x^2}[\hat x, [\hat x, \hat\rho]]$

Note: View as msmt (without regard to result)

$$\cancel{\hat\rho^{new} = \frac{1}{\sqrt{\pi \Delta x^2}}\int_{-\infty}^{+\infty} d\tilde x\, e^{-\frac{1}{2\Delta x^2}(\hat x - \tilde x)^2}\hat\rho\, e^{-\frac{1}{2\Delta x^2}(\hat x - \tilde x)^2}}$$

$$\rho^{new}(x,x') = \frac{1}{\sqrt{\pi}\,\Delta x}\int_{-\infty}^{+\infty} dq\; e^{-\frac{1}{2\Delta x^2}(x-q)^2}\rho(x,x')\, e^{-\frac{1}{2\Delta x^2}(x'-q)^2}$$

= Kraus-representation!

measured position (up to precision Δx)

[Check: $\int \exp[-[(x-q)^2+(x'-q)^2]]$

$= \int \exp[-[2q^2 - 2(x+x')q + (x^2+x'^2)]]$

$= \int \exp\Big[-2\left(q - \frac{x+x'}{2}\right)^2 + \underbrace{\tfrac{1}{2}(x+x')^2 - (x^2+x'^2)}_{-\frac{1}{2}(x-x')^2}\Big]$

$= (\ldots)\cdot \exp\left[-\tfrac{1}{2}(x-x')^2\right]$

Many-particle system (macroscopic body):

each atom (number j) localized at rate Γ_j

center-of-mass : $\hat{Q} = \frac{\sum_j m_j \hat{x}_j}{\sum_j m_j}$

relative coordinates: $\hat{r}_\ell$ $(\ell = 1, 2, \ldots, N-1)$

$$\hat{x}_j = \hat{Q} + \sum_{\ell=1}^{N-1} C_{j\ell}\, \hat{r}_\ell$$

[Note: Choice is a bit arbitrary]

$\Rightarrow$ Evolution of CM-state?

$$\rho_{CM}(Q, Q') = \int dr_1 \ldots dr_N\, \rho(x_1, x_2, \ldots ; x_1', x_2', \ldots)$$

where $x_j = Q + \sum_\ell C_{j\ell} r_\ell$, $x_j' = Q' + \sum_\ell C_{j\ell} r_\ell$

Localize $x_j \rightarrow$

$$\rho_{CM}^{new}(Q, Q') = \int dr_1 .. dr_N\, e^{-\frac{1}{4\Delta x^2}\underbrace{(x_j - x_j')^2}_{Q-Q'}} \rho(\ldots)$$

$$= e^{-\frac{1}{4\Delta x^2}(Q-Q')^2} \rho_{CM}(Q, Q')$$

$\Rightarrow$ CM localizes on same ~~sca~~ scale Δx, at rate

$$\Gamma_{CM} = \sum_j \Gamma_j$$

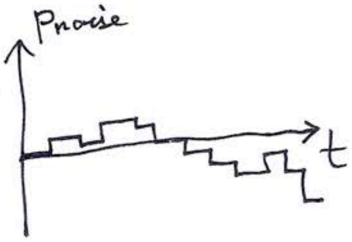
$$\Rightarrow \quad \Gamma_{CM} = \underbrace{N}_{\text{number of atoms}} \cdot \underbrace{\Gamma}_{\text{microscopic rate}}$$

$\Rightarrow$ arrange "Γ_{CM} large, Γ small"

How to choose Γ and Δx?

"Side-effects" (unwanted)

Momentum diffusion for one atom:



$$\Delta p = \frac{\hbar}{\sqrt{2}\Delta x}$$

[due to definition of Δx]

$\Rightarrow$ In time t: $p_{noise} = \sum_j \Delta p_j$

$$\mathrm{Var}\, p_{noise} = \underbrace{(\Gamma t)}_{\#\text{ of events}} \cdot \Delta p^2$$

[For CM: $\Gamma \to \Gamma_{CM}$]

$\Rightarrow$ Energy increase:

$$\delta E(t) = \frac{\mathrm{Var}\, p_{noise}}{2m} = (\Gamma t)\frac{\Delta p^2}{2m}$$

(Extra)
Position spread:

$$x_{noise}(t) = \int_0^t \frac{p_{noise}(t')}{m}\, dt'$$

$$\langle x_{noise}(t)^2 \rangle = \frac{\hbar^2}{6 m^2 \Delta x^2} \Gamma t^3$$

$$= \frac{1}{3}\frac{\Delta p^2}{m^2} \Gamma t^3$$

Example of reasonable parameters:
(given by GRW)

$$\boxed{\begin{aligned}&\Gamma_{(micro)} \sim 10^{-16}\,\mathrm{Hz}\\ \\ &\Delta x \sim 100\,\mathrm{nm}\end{aligned}}$$

($\rightarrow$ once in $\sim 10^{8}$ yrs)

$\Rightarrow$ microscopic dynamics not changed (almost)

$$\text{e.g. } \delta E(t) \sim 10^{-16}\,\mathrm{Hz}\cdot t\cdot\left(\frac{10^{-34}\,\mathrm{Js}}{\cancel{100}\,10^{-7}\,\mathrm{m}}\right)^2\cdot\frac{1}{10^{-25}\,\mathrm{kg}}$$

$$\sim \underbrace{10^{9+14-68}}_{10^{-45}}\,\frac{\mathrm{J}}{\mathrm{s}}\cdot t$$

$$\hat{=}\ 10^{-10}\,\mathrm{eV}\cdot\underbrace{\left(\frac{t}{10^{16}\,\mathrm{sec}}\right)}_{=\frac{t}{10^{8}\,\mathrm{yrs}}} \quad \text{tiny!}$$

gas of such atoms:

$\sim 10^{-15}\,\frac{\mathrm{K}}{\mathrm{yr}}$ temperature increase: tiny!

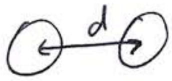
Macroscopic object, e.g. $N \sim 10^{23}$ (about 1g)

$$\Rightarrow \Gamma_{CM} \sim 10^{7}\,\mathrm{Hz}$$

let $\Delta Q_{initial} \sim 100\,\mathrm{nm}$

$$\Rightarrow \langle Q^2_{noise}\rangle < \Delta Q^2_{initial}$$

for times up to 100 yrs!

But: superpositions  $d > 100\,\mathrm{nm}$ destroyed in time $\leq \mu\mathrm{sec}$!

Goal today: Establish bounds!

e.g. sphere of $r \sim 100\,nm$

$\Rightarrow N \sim 10^9$ atoms

$\Rightarrow \Gamma_{CM} \sim 10^{-7}\,Hz$

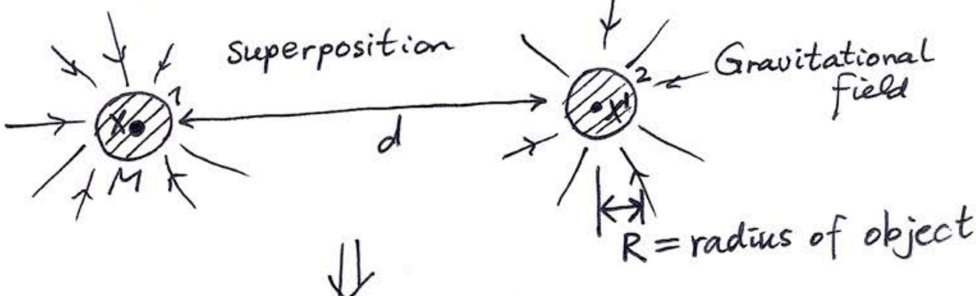
too small to measure —

but maybe $\Gamma_{(micro)}$ is larger?

6.2 Gravitationally-induced decoherence

Penrose
Diosi

So far: No quantum theory of gravity
→ speculate!



$\Downarrow$

decoherence?

"NO" in QED,
but perhaps for QGrav?

Penrose suggests: for a superposition with **mass** densities ρ_1, ρ_2 at the two places:

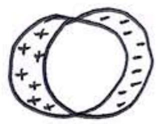
$$\Gamma_\varphi(x,x') = (\text{const.}) \frac{1}{\hbar} \cdot G \int dr\, dr' \, \frac{\delta\rho(r)\,\delta\rho(r')}{|r-r'|}$$

$[G \approx 6.7 \cdot 10^{-11} \, N \frac{m^2}{kg^2}]$

where

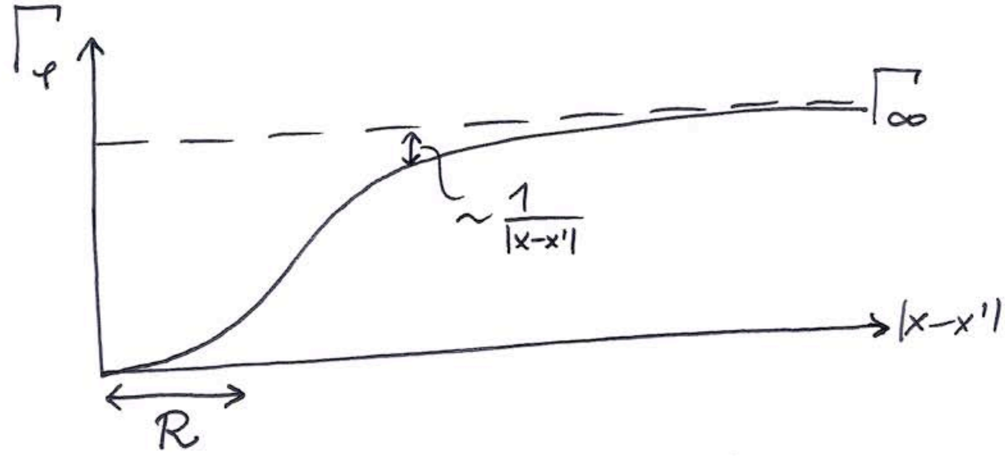
$$\delta\rho(r) = \rho_1(r) - \rho_2(r)$$
and $\rho_j(r) = \rho(r - x_j)$

$|x-x'| = 0$: $\delta\rho = 0$

$|x-x'| < R$:  $\delta\rho$

$|x-x'| \gg R$: 

$\Rightarrow$



$$\Gamma_\infty = \lim_{|x-x'|\to\infty} \Gamma_\varphi(x,x') = 2\frac{G}{\hbar}\int dr\,dr'\,\frac{\rho(r)\rho(r')}{|r-r'|}$$

$$\sim \frac{1}{\hbar}\text{ gravitational self-energy}$$

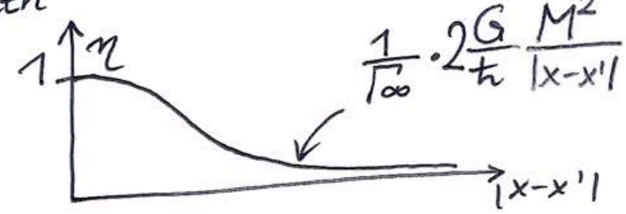
$$\sim \frac{1}{\hbar}\frac{GM^2}{R}$$

$\Rightarrow$ can write

$$\dot{\rho}(x,x') = \dots \quad -\underbrace{\Gamma_\varphi(x,x')}_{\equiv\, \Gamma_\infty(1-\eta(x-x'))}\rho(x,x')$$

[analogous to before]

with



Typical numbers: 1g, ~~|x−x'|≫1cm~~, $R\sim 1\,\text{cm}$

$$\Rightarrow \Gamma_\infty \sim \frac{G}{\hbar}\frac{M^2}{R} \sim \frac{10^{-10}}{10^{-34}}\,\frac{10^{-6}}{10^{-2}}\,\text{Hz} \sim 10^{20}\,\text{Hz} \quad (!)$$

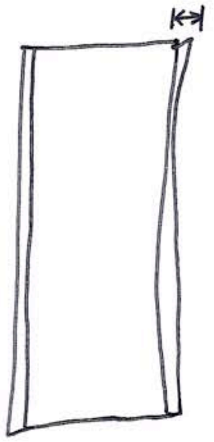
$R\sim 100\,\text{nm}$ sphere; $M\sim 10^{-15}\,g$ $\Rightarrow \Gamma_\infty \sim \frac{10^{-10}}{10^{-34}}\,\frac{10^{-36}}{10^{-7}} \sim 10^{-5}\,\text{Hz}$ (!)

$\Rightarrow$ Very strong dependence on size; at fixed density: $\Gamma_\infty \sim R^5$

[GRW has $\Gamma\sim R^3$]

Estimate for <u>small</u> displacement

(e.g. mirror in optomechanics)



Assume
$|x-x'| \sim 10^{-13}\,m$
$R \sim 10^{-5}\,m$
$M \sim 5 \cdot 10^{-12}\,kg$
$\Rightarrow \Gamma_\varphi \sim 10^{-9}\,Hz$!

[Adler 2007]

Diosi: This gravitational decoherence would follow from white noise in gravit. potential:

$$\langle \phi(r,t)\phi(r',0)\rangle = 2\frac{\hbar G}{|r-r'|}\,\delta(t-t')$$

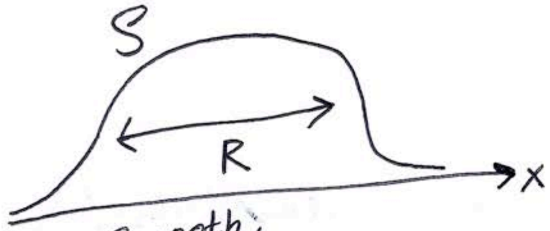
where $\delta E_{Grav}^{(t)} = \int dr\; \phi(r,t)\,\rho(r,t)$

[Note: Diosi has no "2" on rhs, but needed to get Penrose's choice of Γ_φ]

Warning! In usual QFT, it is <u>very wrong</u> to treat quantum fluctuations like classical fluctuations!

(example: e^- traveling through QED vacuum at constant velocity $\rightarrow$ <u>no</u> dephasing!)

Problem: micro-structure of ρ?

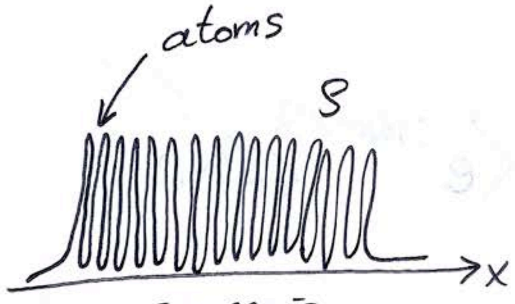


Smooth, macroscopic density

$\Rightarrow$

$\cancel{\hbar\Gamma_\infty \sim G\frac{M^2}{R}}$

$$\Gamma_\varphi \sim G\frac{M^2}{R}\,\frac{|x-x'|^2}{R^2}$$



microscopic density

$\Rightarrow$

$\cancel{\hbar\Gamma_\infty \sim G N \cdot \frac{m^2}{a}}$

~~mass of atom~~

~~number of atoms~~

~~diameter of atom (or nucleus?)~~

$$\Gamma_\varphi \sim \frac{M}{m}\,\frac{Gm^2}{r}\,\frac{|x-x'|^2}{r^2}$$

mass of atom (nucleus?)

size of atom (nucleus?)

$\sim 10^{12}$ times larger for typical density (& nuclei)

Penrose's idea:

stationary state:

$$i\hbar\partial_t|\psi\rangle = E|\psi\rangle$$

time-like translation

superposition:

$$|\psi_1\rangle + |\psi_2\rangle$$

might include object + surrounding grav. field

different spacetimes

$\Rightarrow$ "∂_t" not defined simultaneously

$\Rightarrow$ E uncertain

degree of incompatibility of spacetimes?

$$\int (\vec{f}_1 - \vec{f}_2)^2 \, d^3r = \int (\vec{\nabla}\Phi_1 - \vec{\nabla}\Phi_2)^2 d^3r$$

gravitational acceleration in space-time 1

$$= -\int (\Phi_1 - \Phi_2)\, \Delta(\Phi_1 - \Phi_2)\, d^3r$$

$$\Delta\Phi = 4\pi G\rho$$

$$= -4\pi G \int (\Phi_1 - \Phi_2)(\rho_1 - \rho_2)\, d^3r$$

$$\Phi(r) = -G\int \frac{\rho(r')}{|r-r'|} d^3r'$$

$$= +4\pi G^2 \iint \frac{\delta\rho(r)\,\delta\rho(r')}{|r-r'|}\, d^3r\, d^3r'$$

6.3 "Schrödinger-Newton equation"

Treat gravitational effects in QM?

Simple approach:

$$V_{int}(x_1, x_2) = -G \frac{m_1 m_2}{|x_1 - x_2|}$$

$\Rightarrow$ Solve many-body SEQ with this potential

$\Rightarrow$ No problems

e.g.: molecule interference
$\rightarrow$ COM motion unaffected ✓

"slightly"* ~~strange~~ more strongly bound molecule

[*Note: $V^{grav} \sim 10^{-50}\,J$
$\sim 10^{-30}\,eV$!]

(need $\sim 10^8$ yrs to measure this energy shift)

~~But~~
Alternative: Field formulation

$$\Delta\Phi = 4\pi G\, \rho(r)$$

$\rho(r)$: mass density

Φ: gravitational potential

$(\Rightarrow V(x) = m\Phi(x))$

$$\hat\rho(r) = \sum_j m_j (r - \hat x_j)$$

QM? $\quad \Delta\hat\Phi = 4\pi G\hat\rho$

$\hat\Phi$, $\hat\rho$: operators $\Rightarrow$ QGrav ?? ...

Semiclassical approach:

$$\Delta\Phi(r) = 4\pi G \langle \Psi | \hat{\varrho}(r) | \Psi \rangle$$

many-body wavefct

density operator

$$\hat{\varrho}(r) = \sum_j m_j \, \delta(r - \hat{x}_j)$$

mostly good for large masses,
but: effect on COM motion! ⇒ ?

≙ Hartree theory with self-interaction!

Limit of 1 particle:

$$\Delta\Phi(r) = 4\pi G \, m \, |\Psi(r)|^2$$

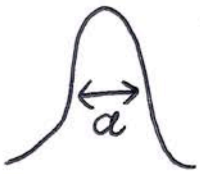
$$\Rightarrow \quad i\hbar \partial_t \Psi = -\frac{\hbar^2}{2m} \Delta \Psi(r) - G m^2 \left[\int \frac{|\Psi(r')|^2 dr'}{|r - r'|} \right] \Psi(r)$$

Schrödinger-Newton-eq. (for 1 particle)

[structure similar to Gross-Pitaevskii] (Møller, Rosenfeld, Diosi, Penrose, Runi, Bonazzola)

⇒ Tendency to localize wave function!

Self-consistent "soliton"-like solution:



Variational ansatz

Estimate, using Heisenberg $\Delta p \sim \frac{\hbar}{2\Delta x}$ $\Delta x = a$

$$\Rightarrow E \approx \frac{\hbar^2}{Ma^2} - G\frac{M^2}{a} \overset{!}{=} \min$$

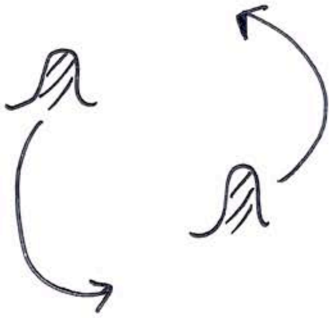
$$\Rightarrow \quad a \sim \frac{\hbar^2}{GM^3}$$

e.g. $M = 10^{-25}\,\mathrm{kg} \Rightarrow a \sim 10^{17}\,\mathrm{m}$

$M = 10^{-20}\,\mathrm{kg} \Rightarrow a \sim 10^{2}\,\mathrm{m}$ (10^5 atoms)

$M = 1\,\mathrm{g} \Rightarrow a \sim 10^{-31}\,\mathrm{m}$

But also: solutions like



orbiting gravitating wave packets

6.4 Other models:

Karolyhazy, 1960's

(uncertainty in quantum objects → fluctuations in space-time structure)

$$\Lambda_K \equiv \frac{\Gamma}{4\Delta x^2} = \frac{\hbar}{8 M a_c^4}$$

(notation of GRW)

$$a_c = \frac{\ell_c^3}{\ell_p^2}$$

with $\ell_c = \frac{\hbar}{Mc}$ = Compton wavelength

$\ell_p = \sqrt{\frac{G\hbar}{c^3}}$ = Planch length $\approx 1.6 \cdot 10^{-35} m$

$\Rightarrow$ for sphre $\sim \mu m$, $|x-x'| \sim 1\mu m$: coherence time $\sim ms$

[see O. Romero-Isart 2011]

Ellis, Mohanty, Mavromatos, Nanopoulos

"wormhole fluctuations" $\Rightarrow$

$$\Lambda_{QG} = \frac{\Gamma}{4\Delta x^2} = \frac{c^4}{\hbar^3} \frac{m_0^6}{m_P^3}$$

m_0 = mass of a nucleon

m_P = Planch mass $= \sqrt{\frac{\hbar c}{G}} \approx 2 \cdot 10^{-8} kg$

6.5 Modified commutator relations

Quantum gravity $\Rightarrow \overset{?}{\ldots} \Rightarrow$

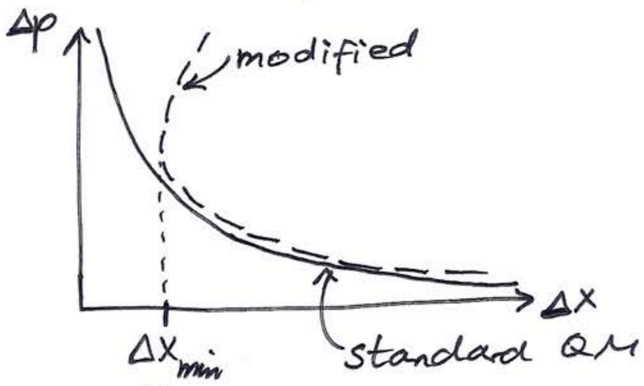
for example: $[\hat{x}, \hat{p}] = i\hbar \left(1 + \beta_0 \cancel{\text{const}} \left(\frac{\hat{p}}{M_p c}\right)^2\right)$

M_p = Planck mass $\approx 22 \mu g$

β_0 = const $< 10^{33}$ according to current experiments

$\Rightarrow$ modified uncertainty relation:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta_0 \left(\frac{\Delta p}{M_p c}\right)^2\right)$$



"minimum measurable length scale"

$\Delta x_{min} = L_p \sqrt{\beta_0}$

L_p ↳ Planck length

(currently $\Delta x_{min} < 10^{-19} m$, from expts.)